Quantum electronic transport

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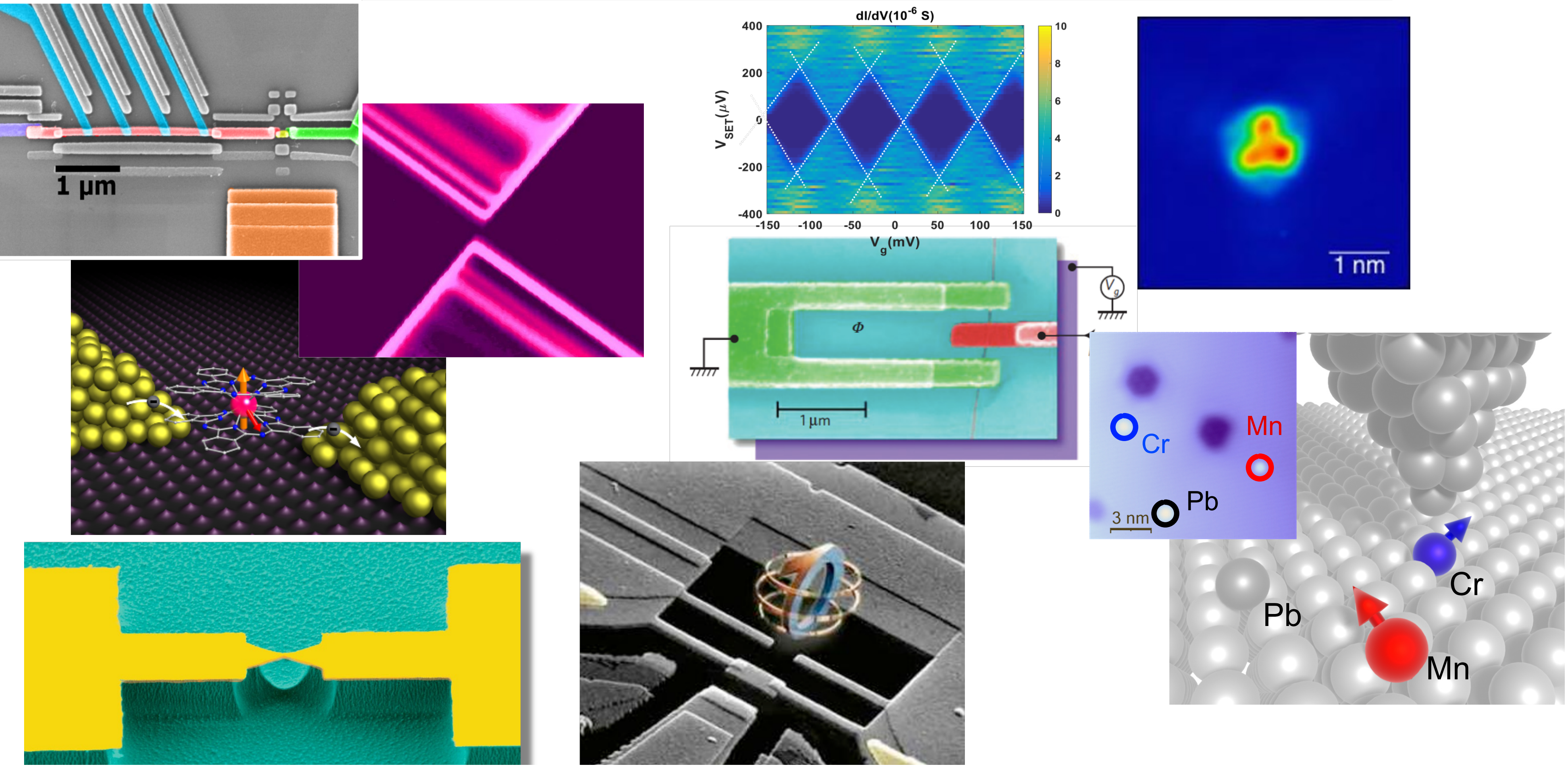
Quantum conductors



Menu

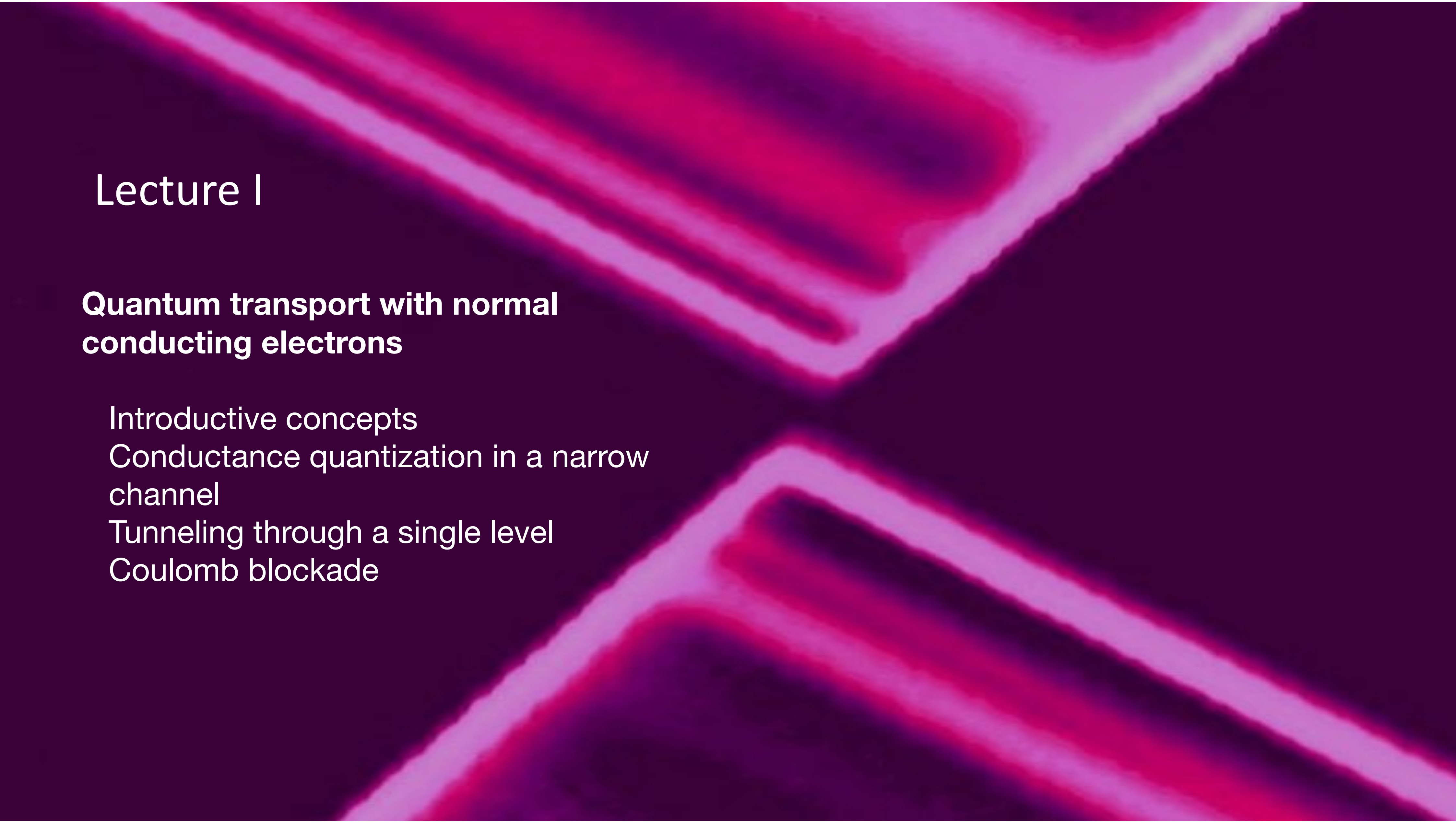
Lecture I: Quantum transport with normal conducting electrons

Introductive concepts
Conductance quantization in a narrow channel
Tunneling through a single level
Coulomb blockade

Lecture II: Superconducting nanoelectronics

Introductive concepts
Josephson effect
Andreev reflection
Selected recent results

Recommended reading:
Quantum transport, Y. Nazarov & Y. Blanter
Introduction to superconductivity, M. Tinkham
E. Scheer, ESONN slides



Lecture I

Quantum transport with normal conducting electrons

Introductive concepts
Conductance quantization in a narrow channel
Tunneling through a single level
Coulomb blockade

Length scales

$$\lambda_F \ll l_e \ll l_\phi < l_{in}$$

λ_F	l_e	l_ϕ	l_{in}
Fermi wavelength	Elastic mean free path	Phase relaxation length	Inelastic scattering length

Classical versus quantum currents

Drude model

$$\mathbf{j} = \sigma \nabla \mu$$

$$\sigma = \frac{ne^2\tau}{m}$$

Quantum currents

$$\frac{\partial \mid \psi \mid^2}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\mathbf{j} = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{q}{m} \mid \psi \mid^2 \mathbf{A}$$

$$= \frac{\hbar}{m} \mid \psi \mid^2 \left(\nabla \varphi - \frac{q}{\hbar}\mathbf{A} \right)$$

Expression of current

$$\mathbf{j} = \int 2 \frac{d^3\mathbf{k}}{(2\pi)^3} q\mathbf{v}(\mathbf{k}) f(\mathbf{k}) \qquad \mathbf{v}(\mathbf{k}) = \frac{\partial E}{\partial \mathbf{k}} = \frac{\hbar \mathbf{k}}{m}$$

1D wave guide

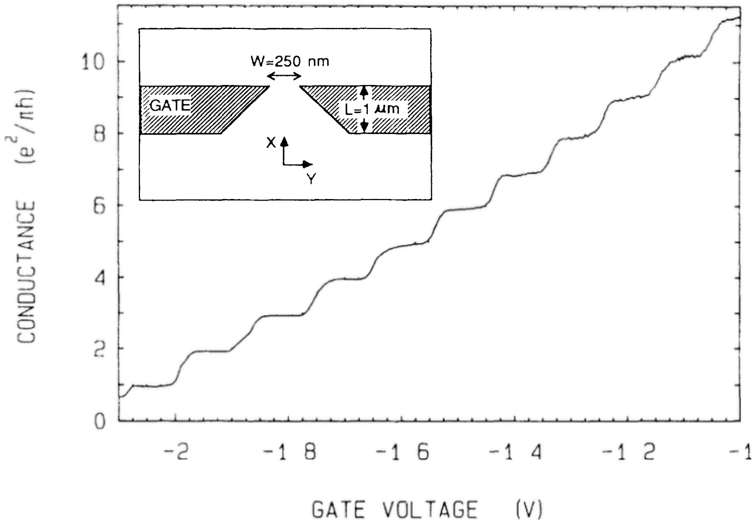
$$E_n(k_x) = \frac{\hbar^2 k_x^2}{2m} + \frac{\pi^2 \hbar^2}{2m}\left(\frac{n_y^2}{a^2} + \frac{n_z^2}{b^2}\right)$$

$$j = \int 2 \sum_n \frac{dk_x}{2\pi} \frac{1}{ab} q \frac{\hbar k_x}{m} f_n(k_x)$$

$$I = \frac{2e}{h} \int dE \sum_{\text{n:open}} \left(f_L(E) - f_R(E)\right) \quad = \frac{2e^2}{h} NV$$

Quantum point contacts

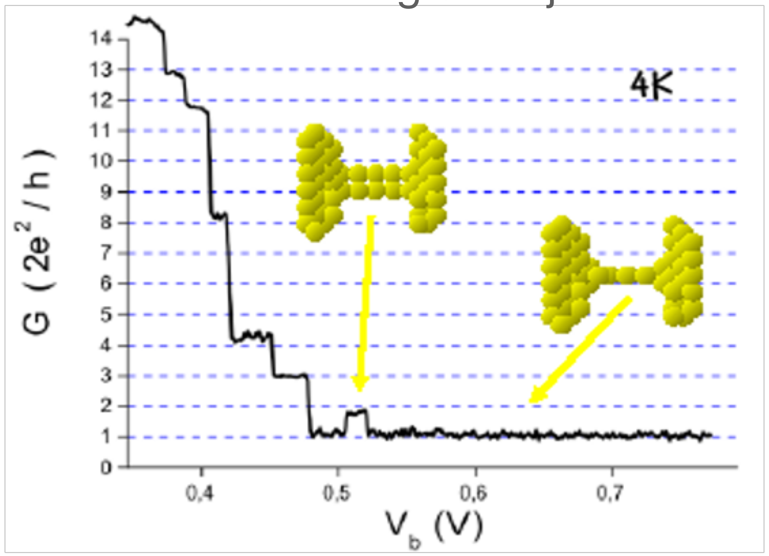
$$G = G_Q \sum_i \tau_i$$



Constriction in a 2DEG

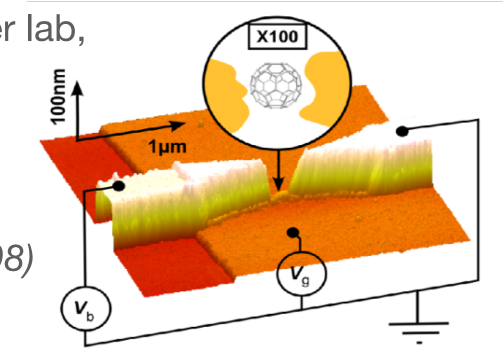
van Wees et al., PRL (1988)

Electromigration junction

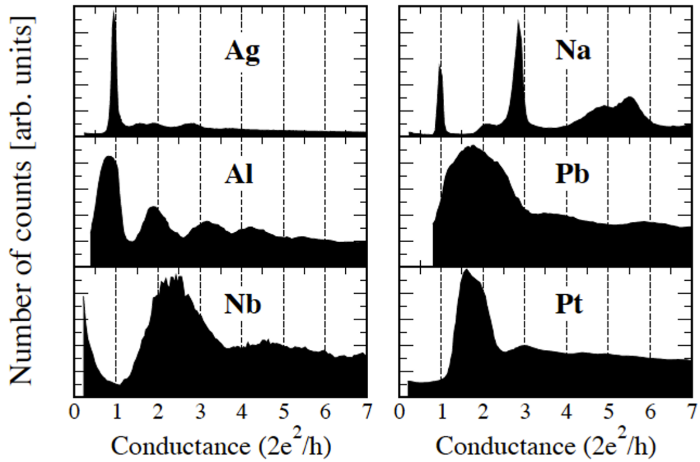


Wernsdorfer lab, Grenoble

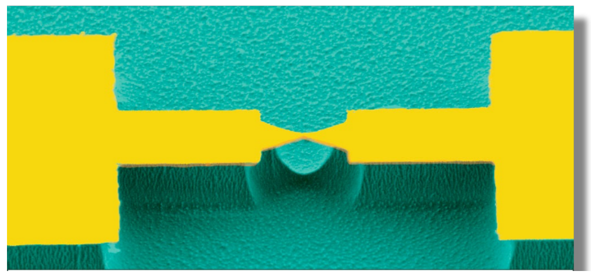
Roch et al., Nature (2008)



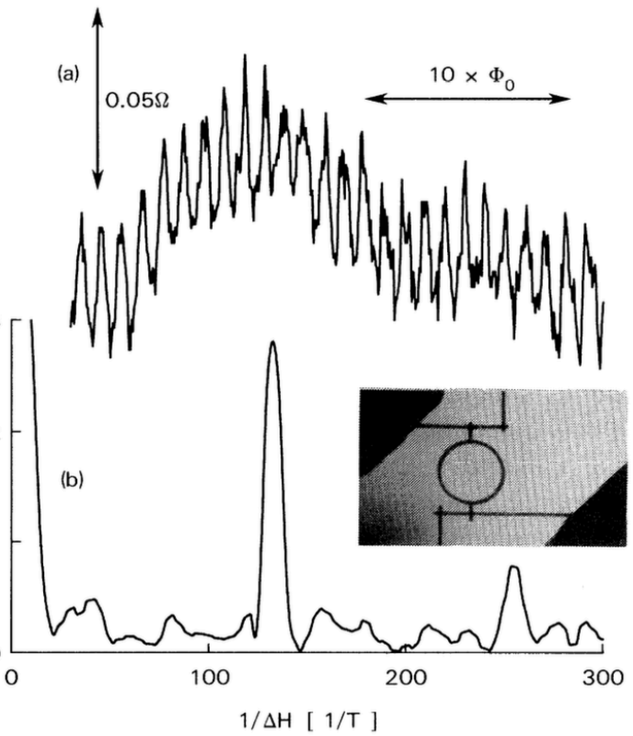
Mechanical break junction



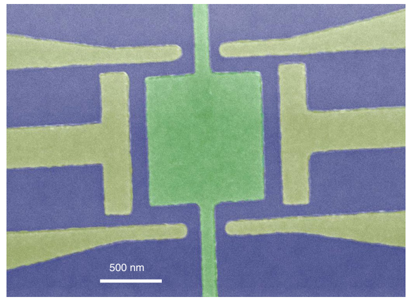
Scheer lab, Konstanz
van Ruitenbeek lab, Leiden



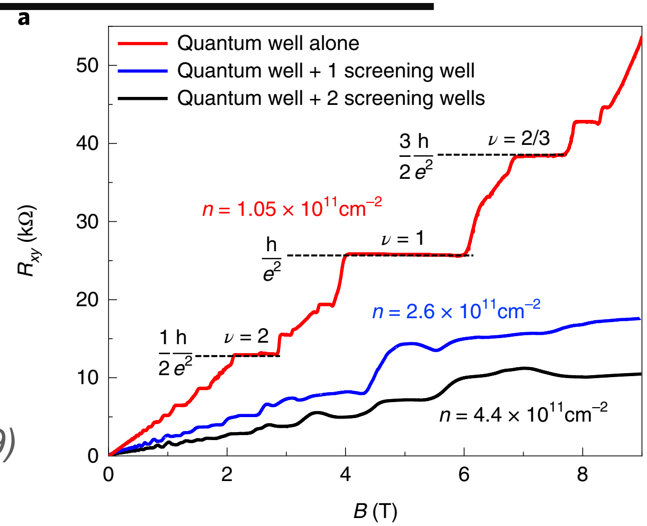
Aharonov-Bohm oscillations

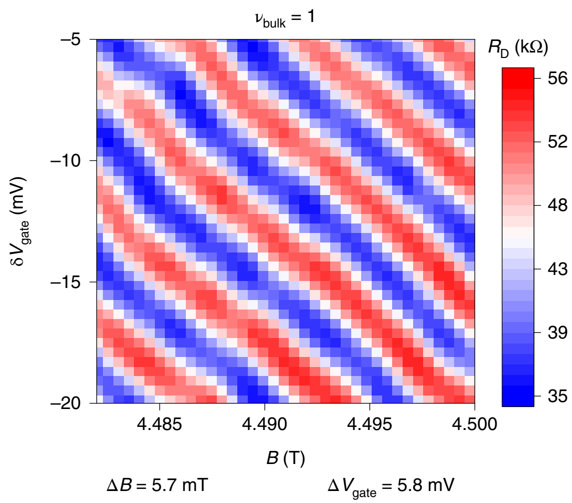


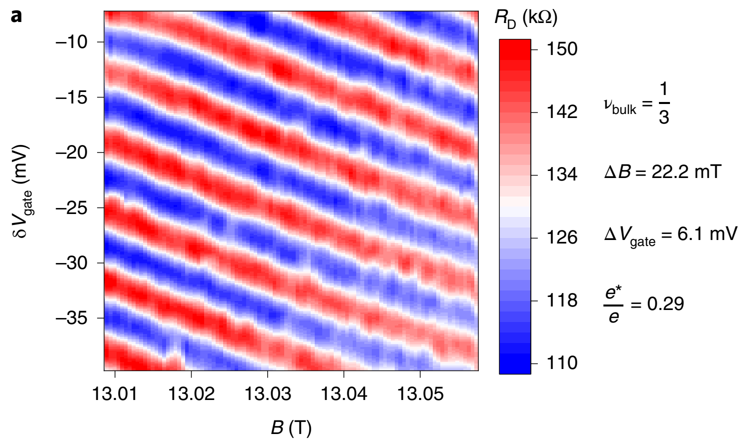
Webb et al., PRL (1985)



Nakamura et al., Nature Phys. (2019)

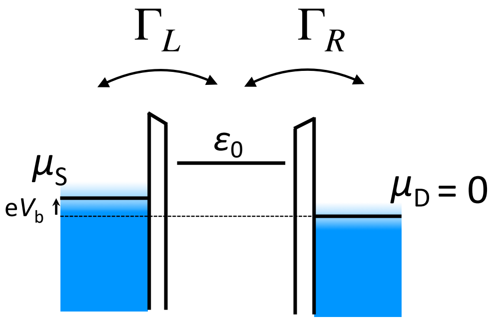






Measuring a fractional charge with AB oscillations

Transport through a single quantum level



Current

$$I = \frac{e}{\hbar}\int dE\,\rho_{QD}(E)\,\frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}\,\big(f_L(E+eV)-f_R(E)\big)$$

Thijssen and van der Zant, Phys. Stat. Sol. B (2008)

Bare level

$$\rho_{QD} = \delta(E-\varepsilon_0)$$

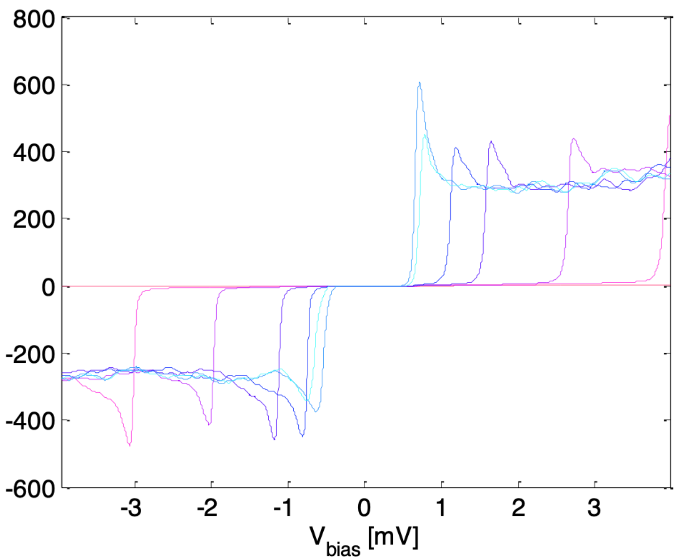
Hybridised level

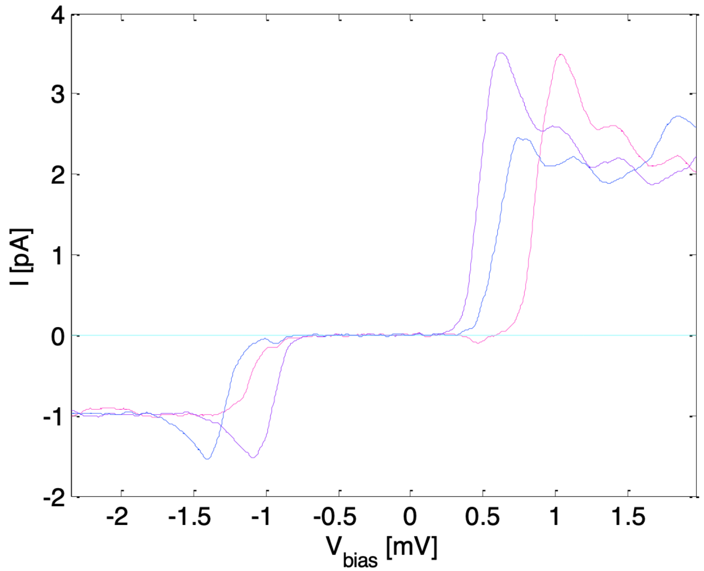
$$\rho_{QD} = \frac{1}{2\pi}\,\frac{\Gamma}{(E-\varepsilon_0)^2+(\Gamma/2)^2}$$

$$\Gamma = \Gamma_L + \Gamma_R$$

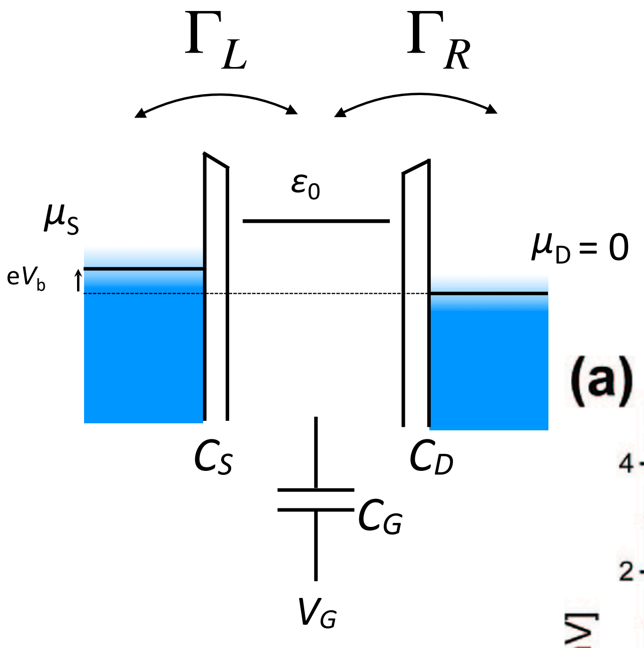
Asymmetry

$$\frac{|I_+|}{|I_-|} = \frac{2\Gamma_L+\Gamma_R}{\Gamma_L+2\Gamma_R}$$

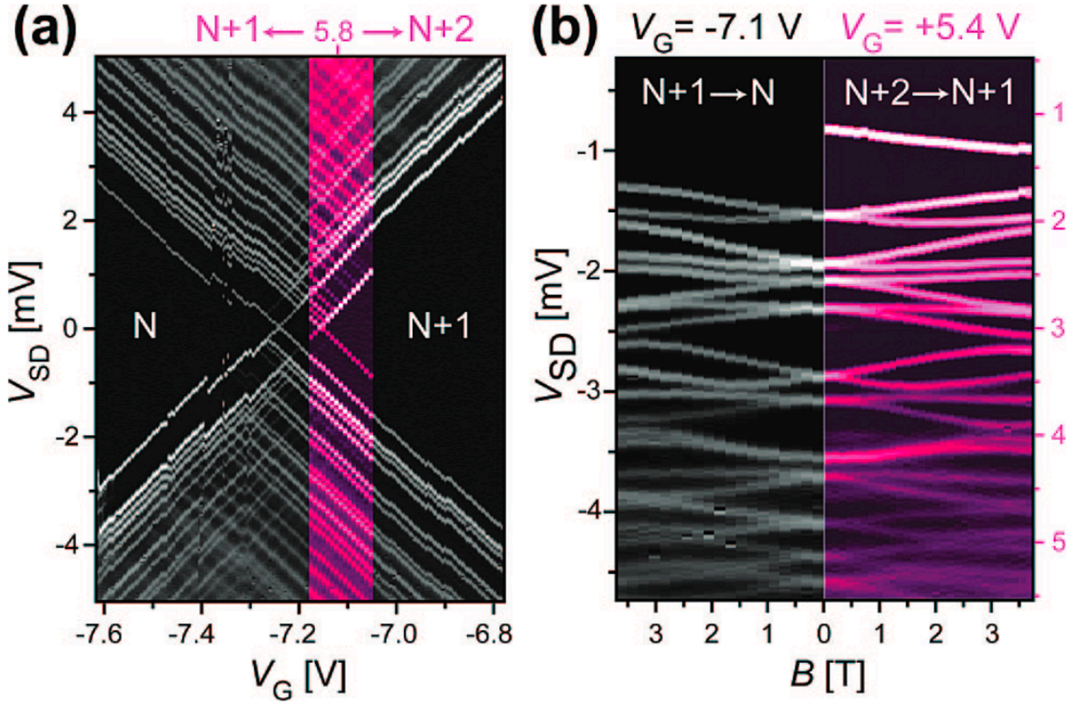


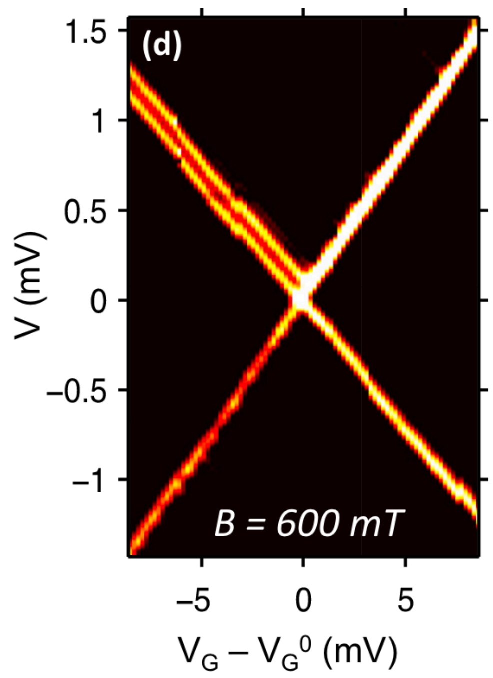


Coulomb diamonds



Kuemmeth et al., Nano Lett. (2008)





Van Zanten et al., PRB (2015)

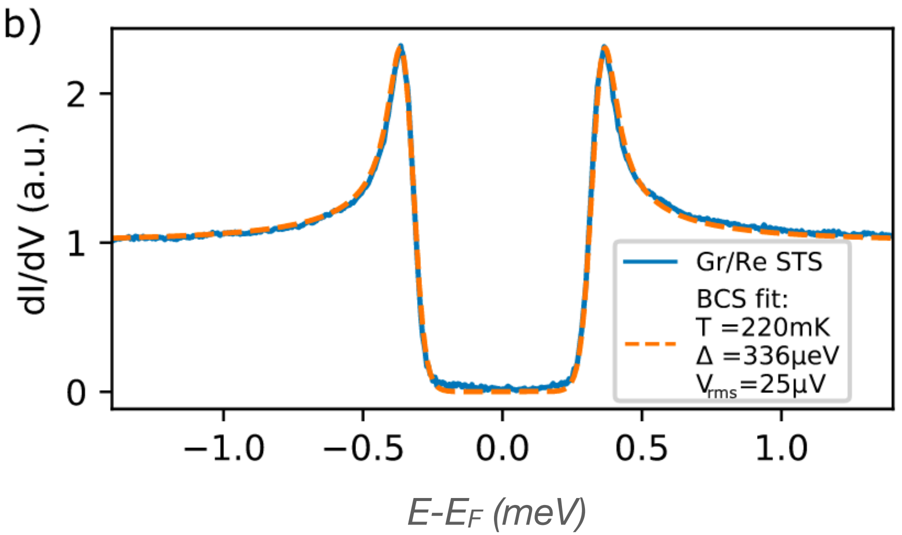
Lecture II

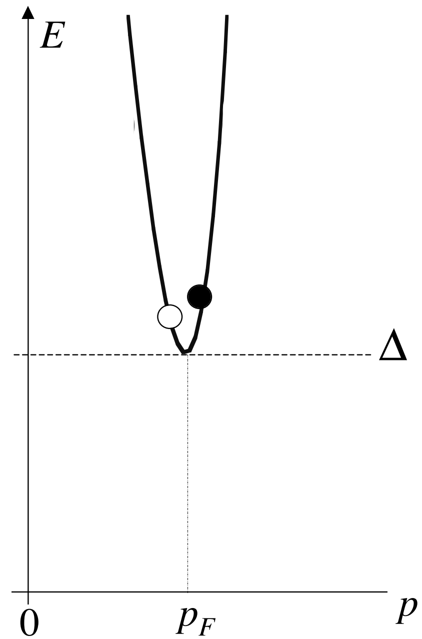
Superconducting nanoelectronics

Introductive concepts
Josephson effect
Andreev reflection
Selected recent results

Superconducting condensate and quasiparticles

Condensate $\psi(\mathbf{r}) = \sqrt{N}\, e^{i\varphi}$ at energy $E = E_F$

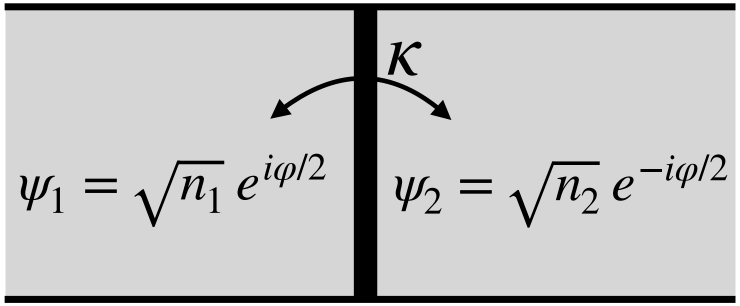


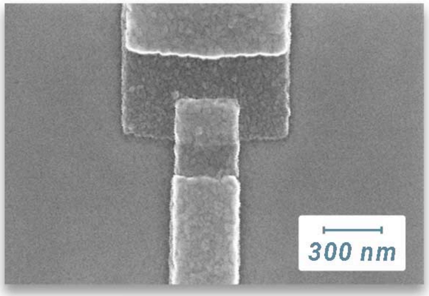


Quasiparticle spectrum

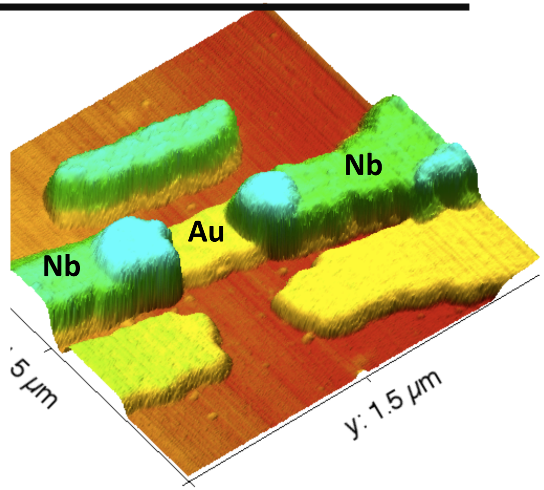
$$\frac{\rho_s(E)}{\rho_n} = \mathscr{Re}\left(\frac{|E|}{\sqrt{E^2 - \Delta^2}}\right) \qquad E = \sqrt{\xi^2 + \Delta^2} \qquad \xi = \hbar v_F (k - k_F)$$

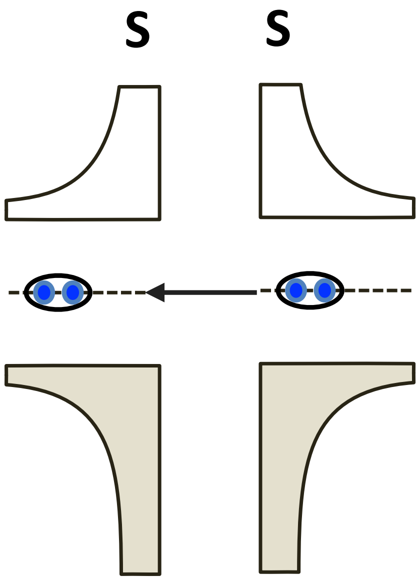
The Josephson effect





SEM image courtesy of the Institute for Quantum Computing (IQC) at the University of Waterloo

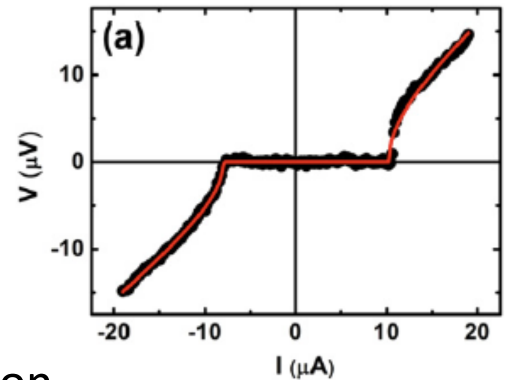




The Josephson relations

$$I_s = I_0 \sin(\varphi)$$

$$\frac{\partial\varphi}{\partial t} = \frac{2\pi}{\Phi_0} V$$

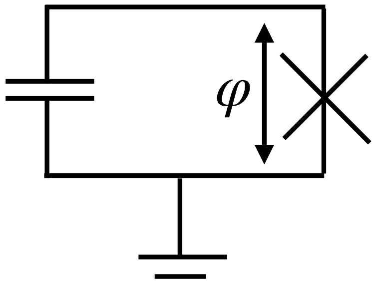


Energy of a Josephson junction

$$H_J = -E_J \cos\varphi$$

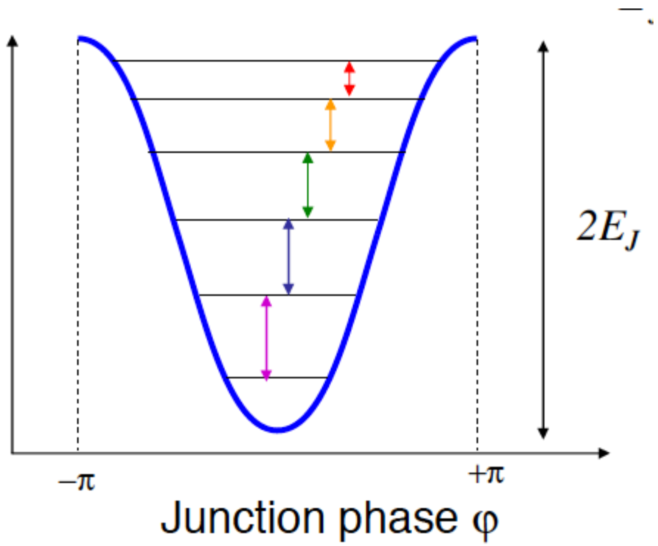
De Cecco et al., PRB (2016)

Capacitive shunt



$$H = \frac{2e^2}{C}\hat{n}^2 - E_J \cos\hat{\varphi}$$

$$\left[\hat{\varphi}, \hat{n}\right] = i$$



$$\varphi \rightarrow x$$

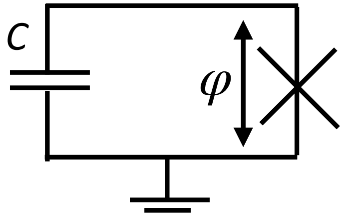
$$n \rightarrow p_x$$

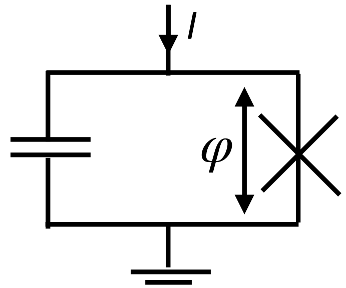
$$V \rightarrow \dot{x}$$

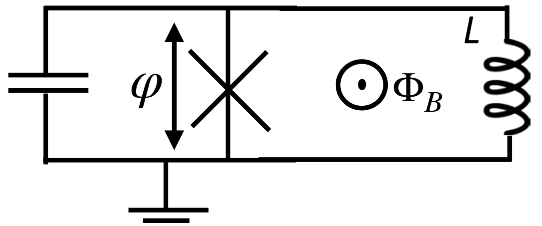
$$C \rightarrow m$$

M. Devoret, *Quantum circuits and signals,* Lectures at Collège de France (2008-09).

Biasing the Josephson junction



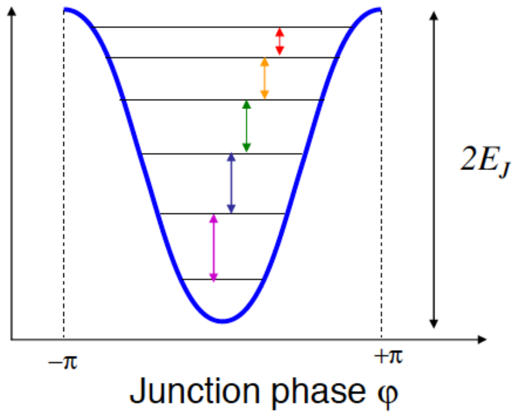


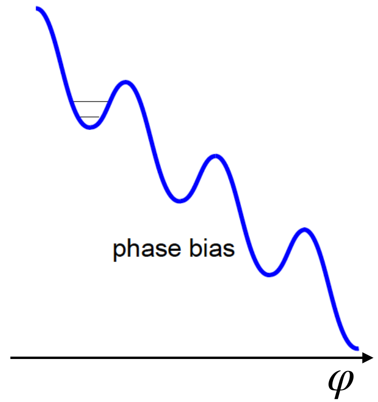


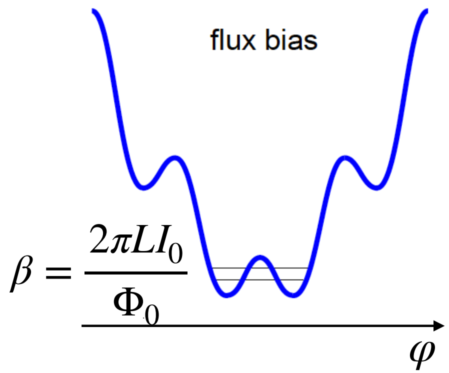
$$H_J = 4E_c\hat{n}^2 - E_J\cos\hat{\varphi}$$

$$H = H_J - E_J\frac{I}{I_0}\hat{\varphi}$$

$$H = H_J + \frac{E_L}{2}\left(\hat{\varphi} - 2\pi\frac{\Phi_B}{\Phi_0}\right)^2$$

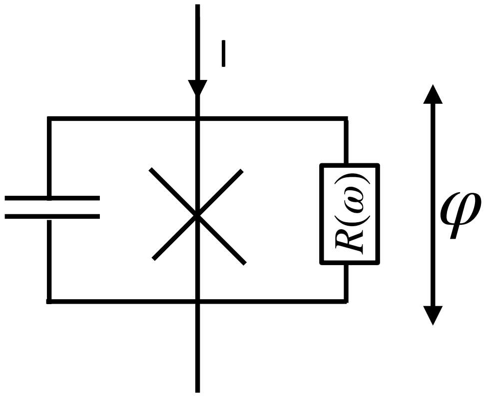




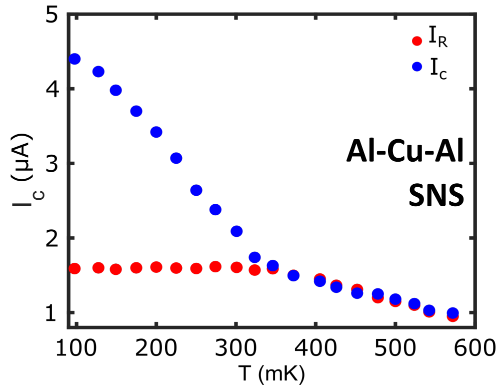


M. Devoret, *Quantum circuits and signals,* Lectures at Collège de France (2008-09).

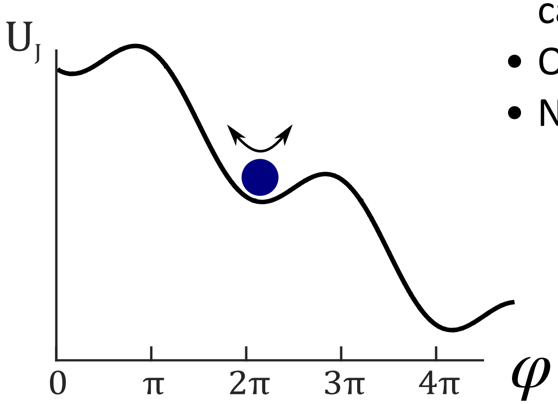
Friction: the resistively shunted junction



Tunnel junction
- Large shunt resistance and capacitance
- Underdamped phase dynamics
- Hysteresis



SNS junction
- Small shunt resistance and capacitance
- Overdamped phase dynamics
- No hysteresis (?)



$$\varphi \rightarrow x$$

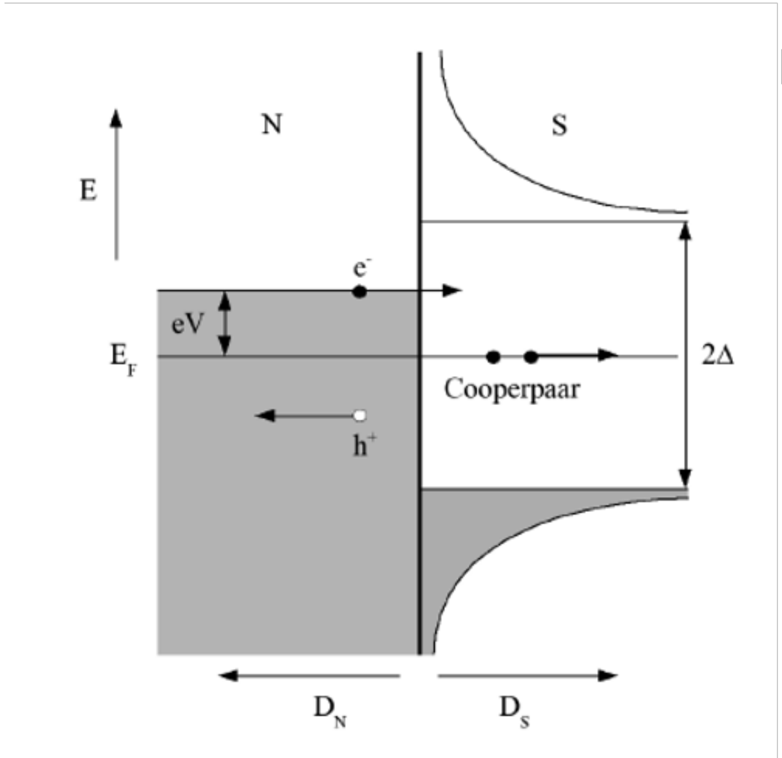
$$n \rightarrow p_x$$

$$V \propto \dot{\varphi} \rightarrow \dot{x}$$

$$C \rightarrow m$$

$$R^{-1} \rightarrow \text{friction}$$

Andreev reflection



Evanescent wave

$$\tilde{\psi}(x) \propto e^{-\frac{x}{\hbar v_F}\sqrt{\Delta^2 - E^2}}$$

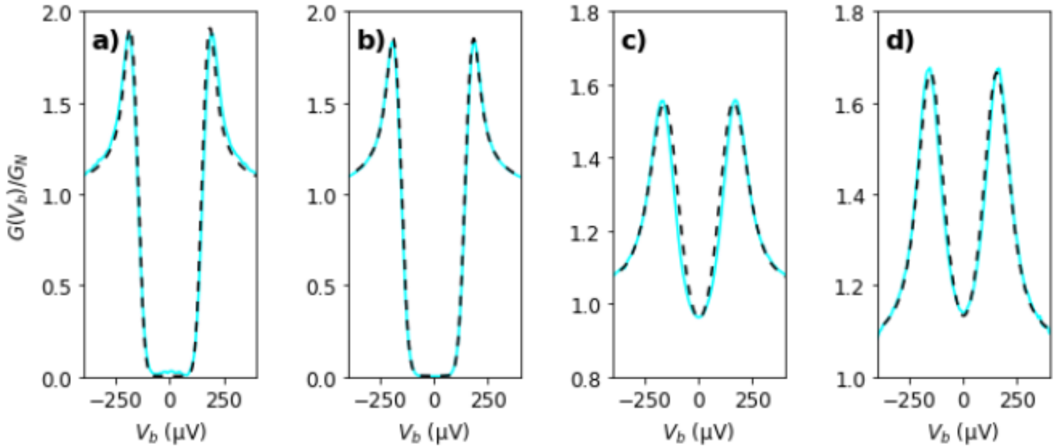
e-h scattering phase

$$\theta = -\arccos\left(\frac{E}{\Delta}\right) - \varphi$$

Conductance

$$G_A = 2G_Q \sum_p \frac{\tau_p^2}{(2-\tau_p)^2}$$

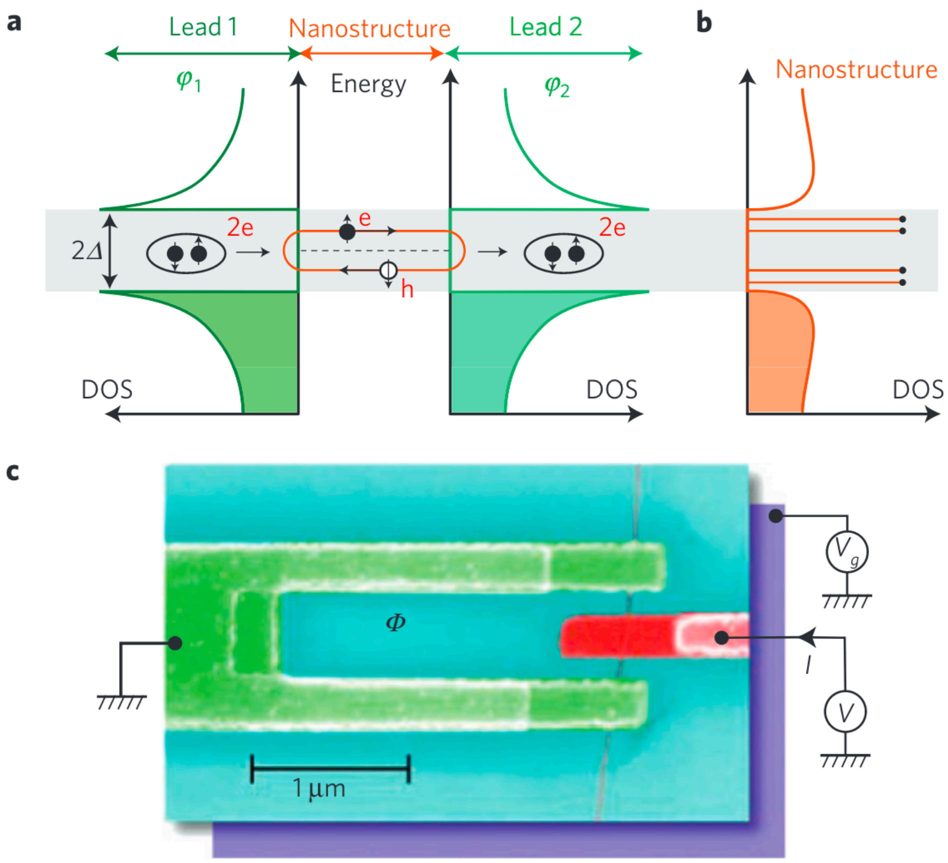
Pt-Ga Andreev point contact spectroscopy



Andreev, Sov. Phys. JETP (1964)

Blonder, Tinkham, Klapwijk, PRB (1982)

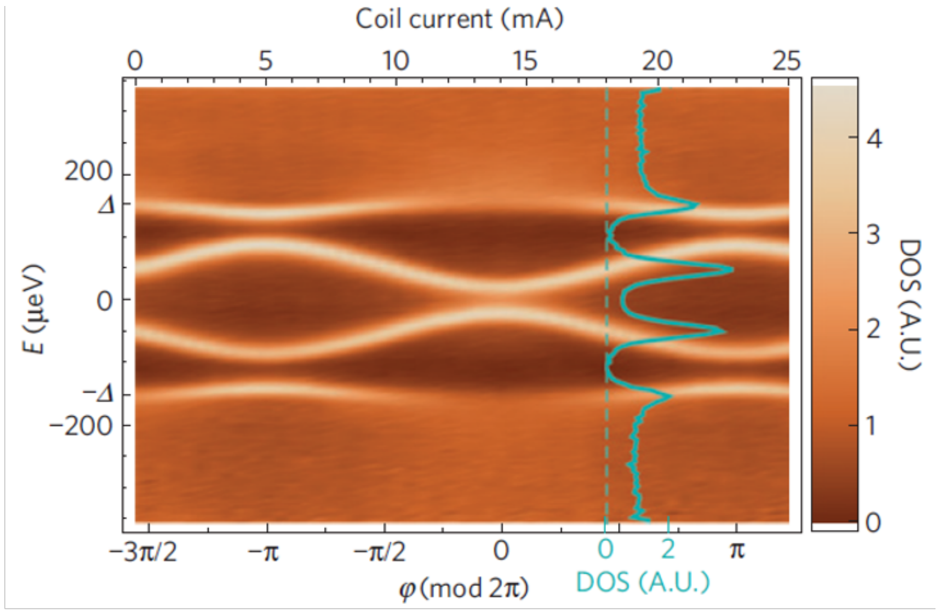
Andreev bound states



Pillet et al., Nature Phys. (2010)

$$\frac{E}{\Delta} = \sqrt{1 - \sin^2(\varphi/2)}$$

$$\frac{E_p}{\Delta} = \sqrt{1 - \tau_p \sin^2(\varphi/2)}$$



Back to the supercurrent

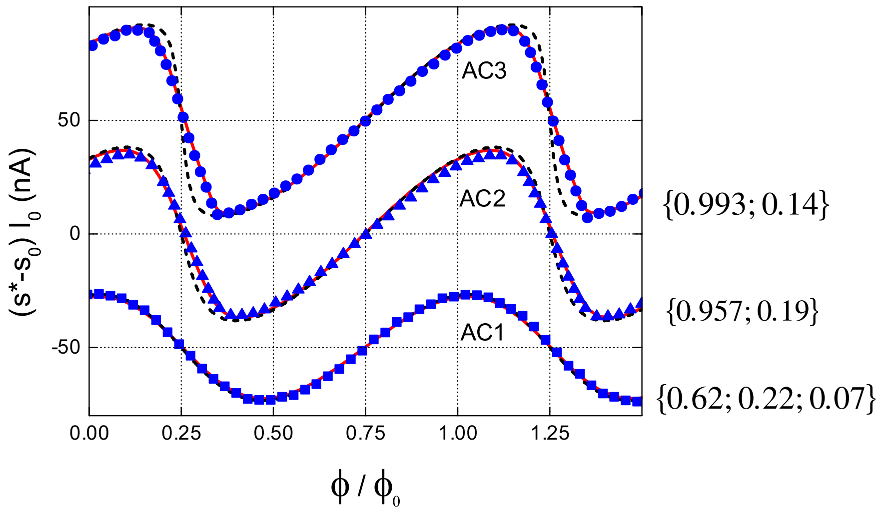
$$I = \frac{2e}{\hbar}\frac{\partial E}{\partial \varphi}$$

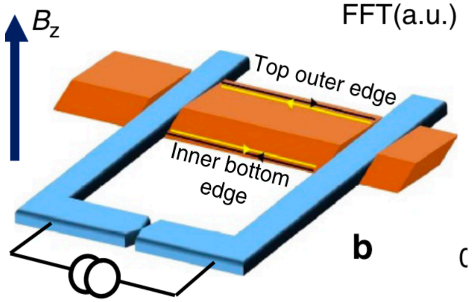
$$I = I_c \sin\varphi$$

$$I_c = \frac{\pi\Delta}{2eR_n}$$

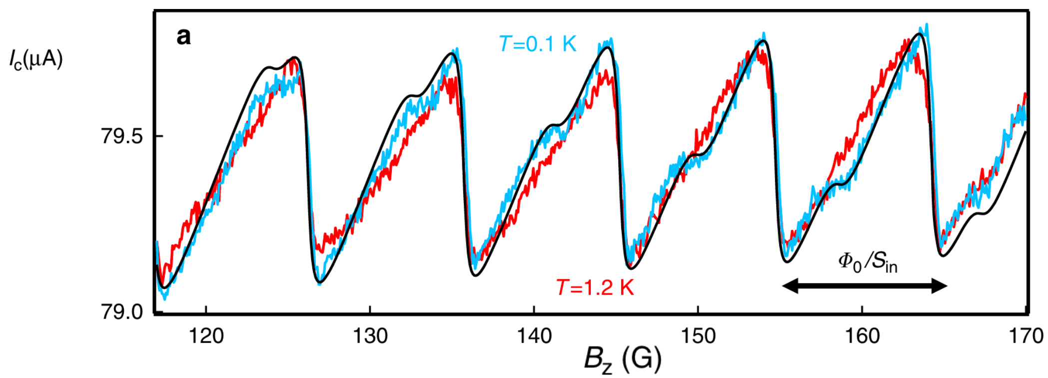
Ambegaokar-Baratof formula

Della Roca et al., PRL (2007)



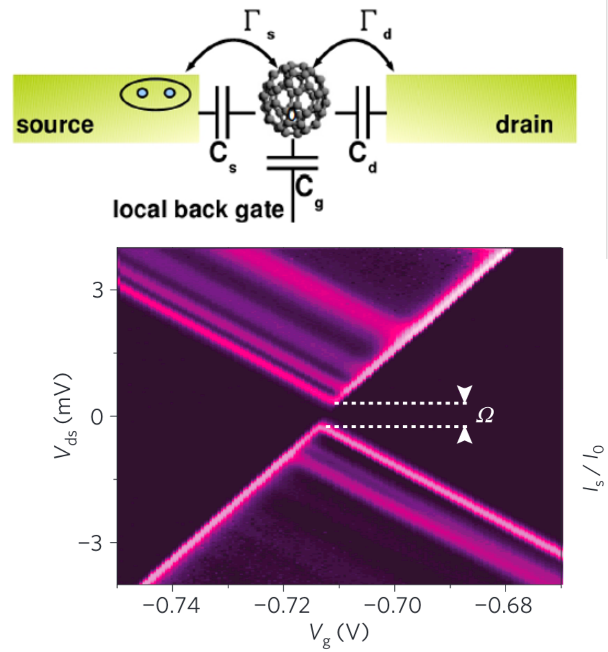


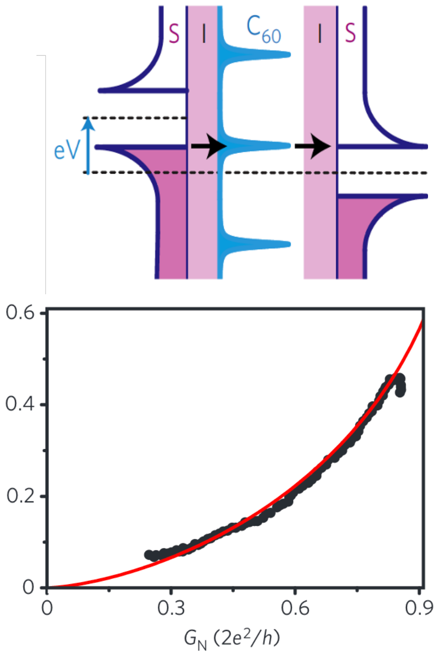
Murani et al., Nature Commun. (2017)

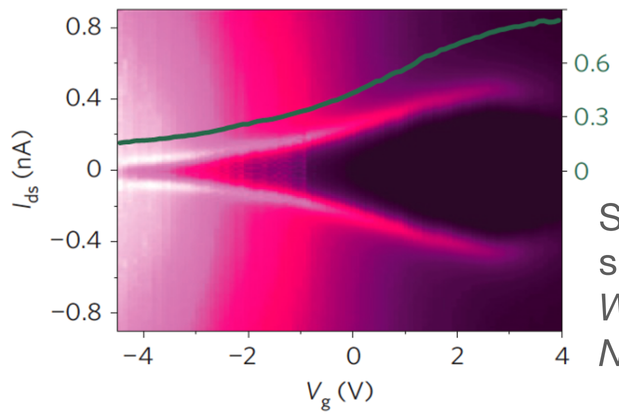


A few results from my works

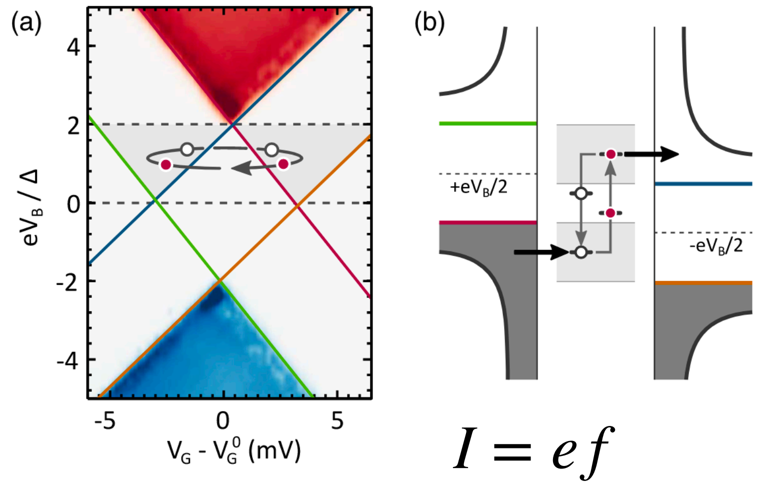
S - Quantum Dot - S junctions





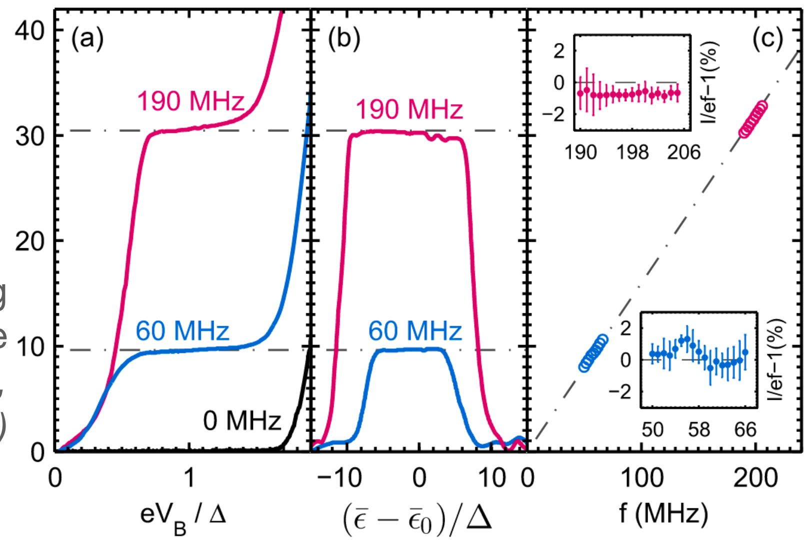


Superconducting single-C_{60} transistor
Winkelmann et al., Nature Phys. (2009)

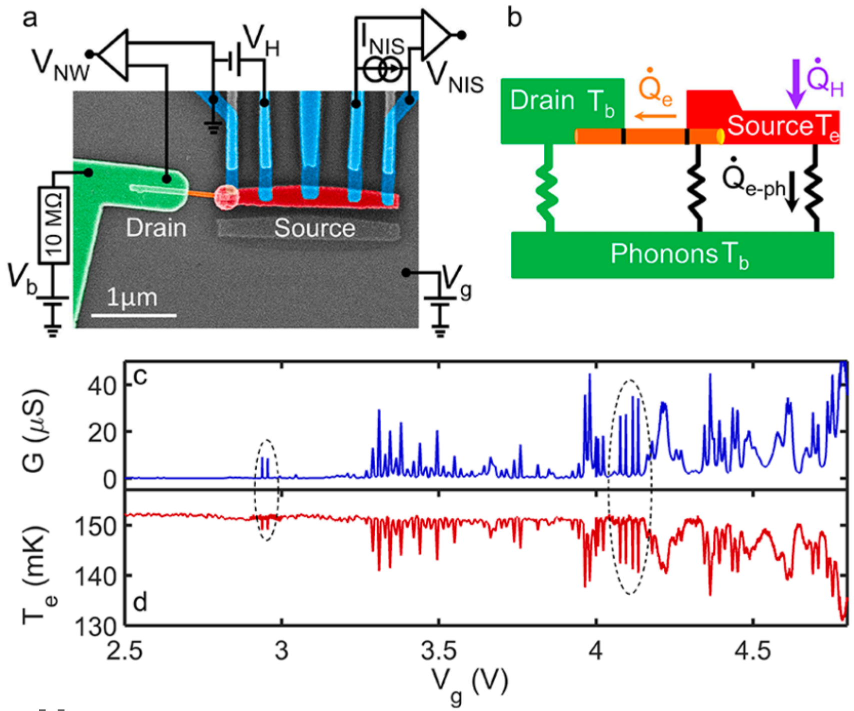


$$I = ef$$

Superconducting single electron turnstile
van Zanten et al., PRL (2016)



Thermodynamics of quantum electronic transport

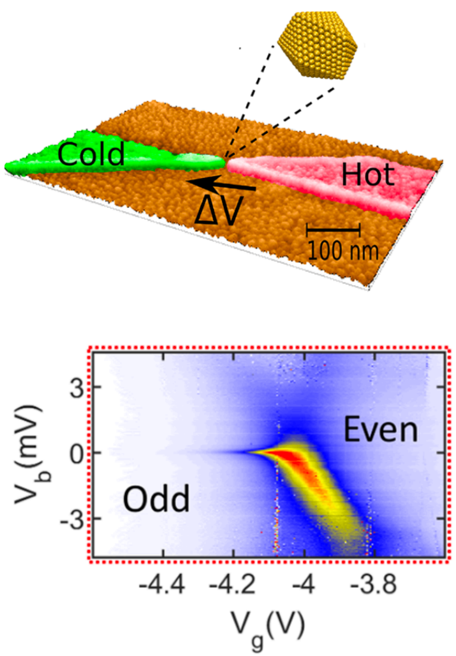


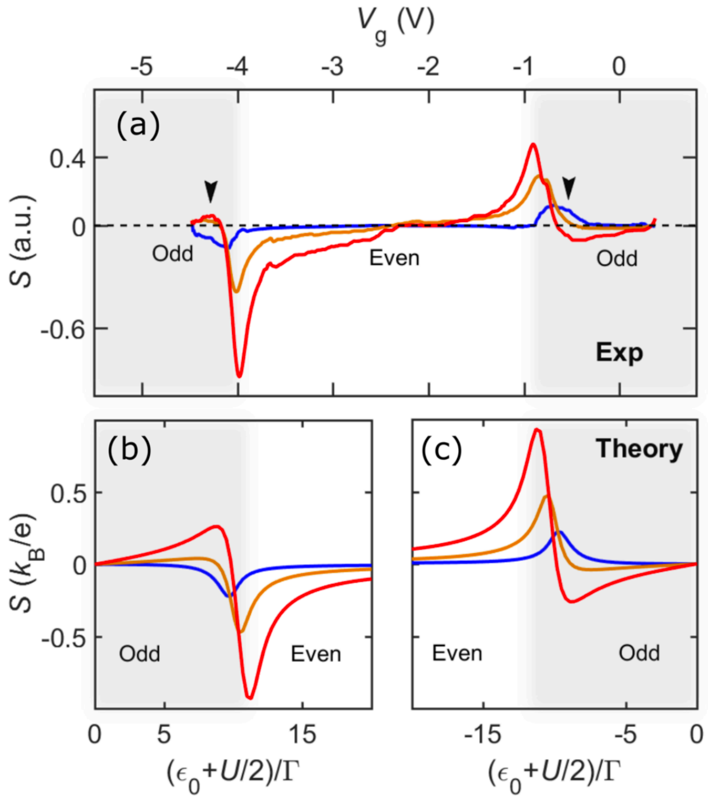
Heat transport

Role of Coulomb blockade
Dutta et al., PRL (2017)
Role of energy quantisation
Dutta et al., PRL (2020),
Majidi et al., Nano Lett. (2022)

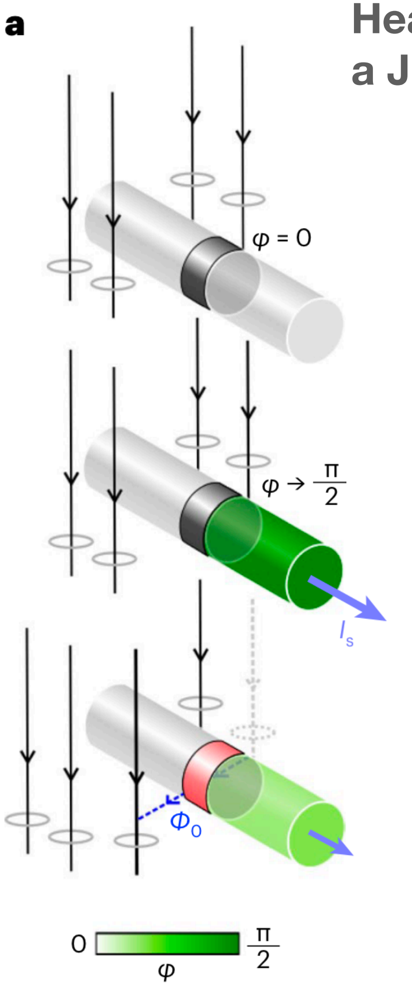
Thermopower

Role of Kondo correlations
Dutta et al., Nano Lett. (2019)



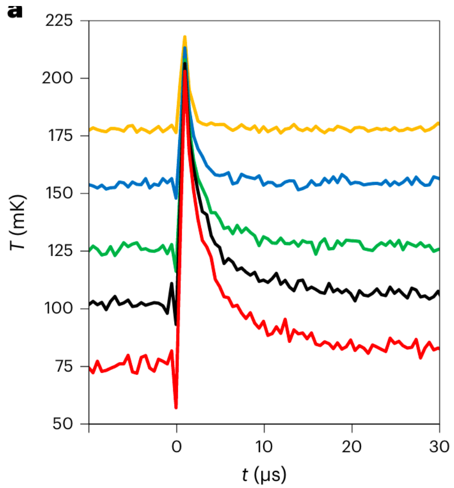


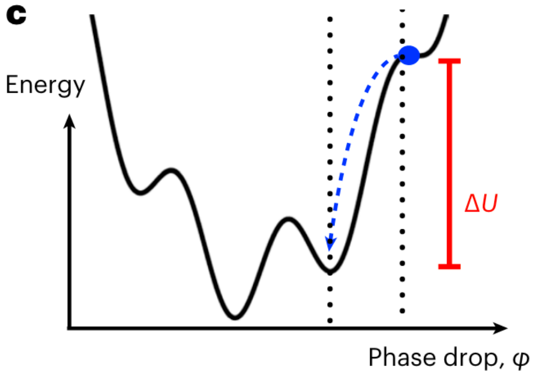
Friction and dissipation in Josephson devices

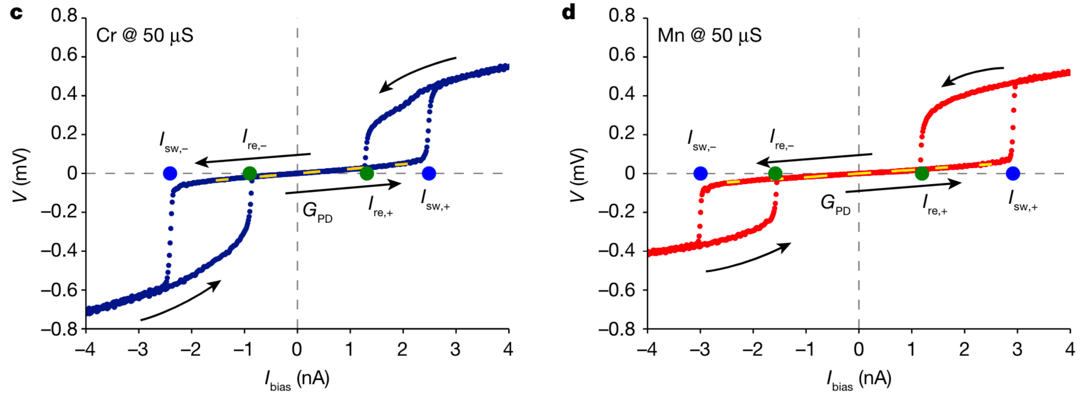


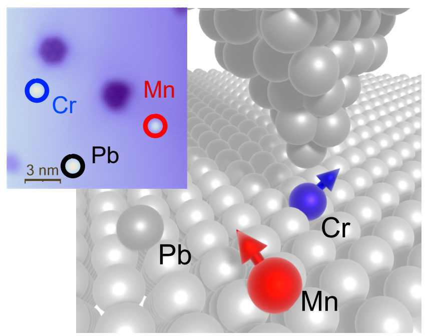
Heat released by a phase slip in a Josephson junction

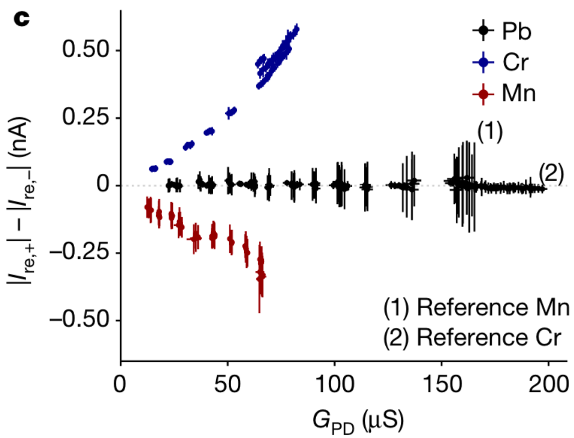
Gümüş et al., Nature Phys. (2023)











Josephson diode effect from asymmetric phase friction

Trahms et al., Nature (2023)