



Grenoble INP



Physics at the Nanoscale and applications

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Physics at the Nanoscale

I Basics of quantum mechanics

II **Statistical physics**

III Forces at the nanoscale

IV Electron tunneling and applications

V Quantum electronic transport

Statistical Physics

Large numbers and fluctuations

Macroscopic world:
ensembles of $N \approx 10^{23}$ particles

Fluctuations concern $\approx \sqrt{N}$ particles

Relative fluctuations amount to $\sqrt{N}/N = 1/\sqrt{N} \approx 10^{-11}$

« In a big group, you only care about the mainstream »

In nanoscience, there's room for fluctuations.

Statistical Physics

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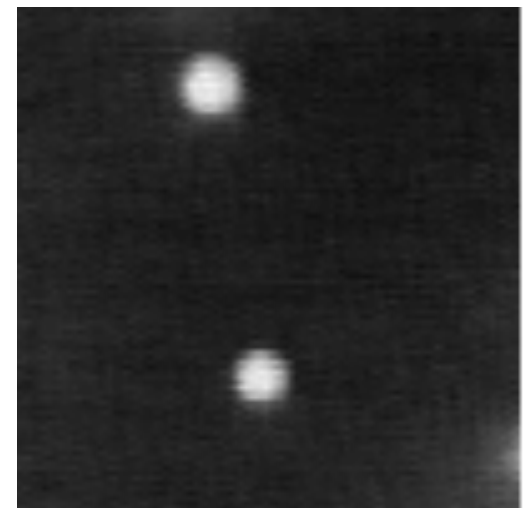
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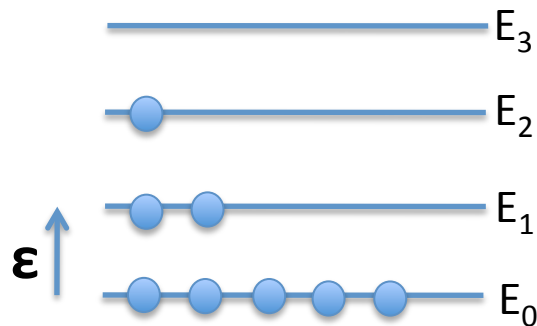
Brownian motion of 2 μm
spheres in water.

Courtesy Professor Eric Weeks, Emory University



Statistical Physics

Boltzmann distribution



At fixed temperature, probability of a given particle to be in a given state of energy E

$$P(E) = \frac{1}{Z} \exp\left(-\frac{E}{k_B T}\right)$$

Exercise:

In the above drawing, assume $T=300\text{K}$ and $\epsilon=100\text{ meV}$. How much more is state 0 populated than state 1 ?
How about if $\epsilon=1\text{ eV}$?

Boltzmann constant
 $k_B=1.38 \cdot 10^{-23}\text{ J/K}$

At room temperature (300K)

$$\begin{aligned} k_B T &= 4.1 \cdot 10^{-21}\text{ J} \\ &= 26\text{ meV} \\ &= 4.1\text{ pN}\cdot\text{nm} \end{aligned}$$

Statistical Physics

Quantum statistics

○ Quantum particles are entirely described by their quantum state

-classically, 5 ways of realizing



- in the quantum world, they're one and the same thing

- How many particles can be in the same quantum state?

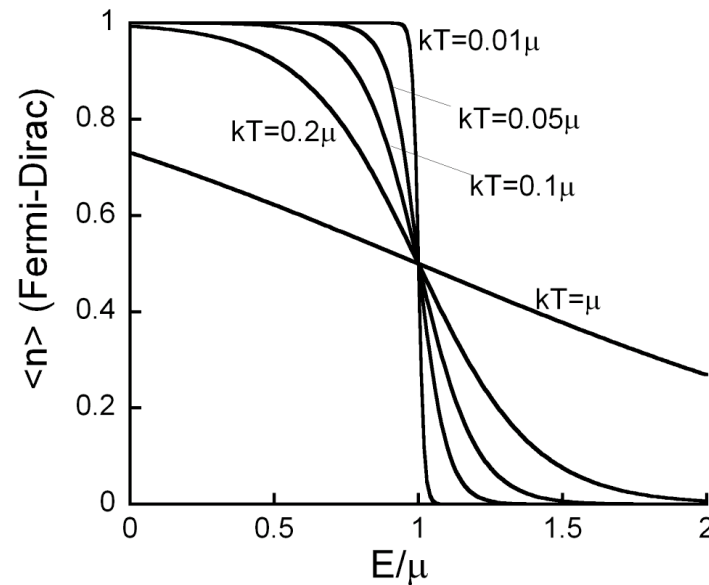
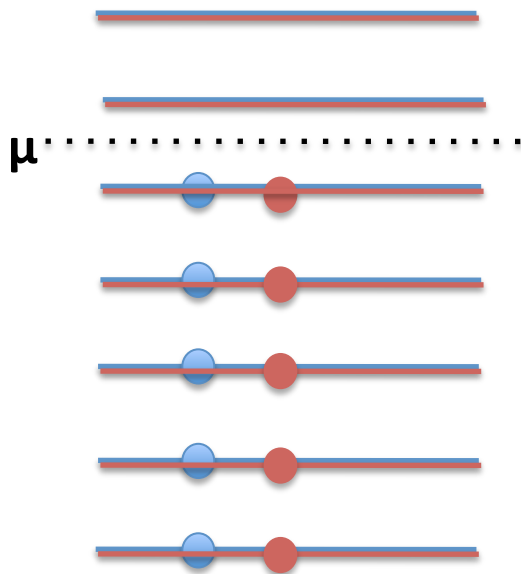
➡ Pauli exclusion principle: Two fermions cannot be in the same quantum state.

➡ **Fermi-Dirac distribution**
$$n(E) = \frac{1}{\exp\left(\frac{E - \mu}{k_B T}\right) + 1}$$

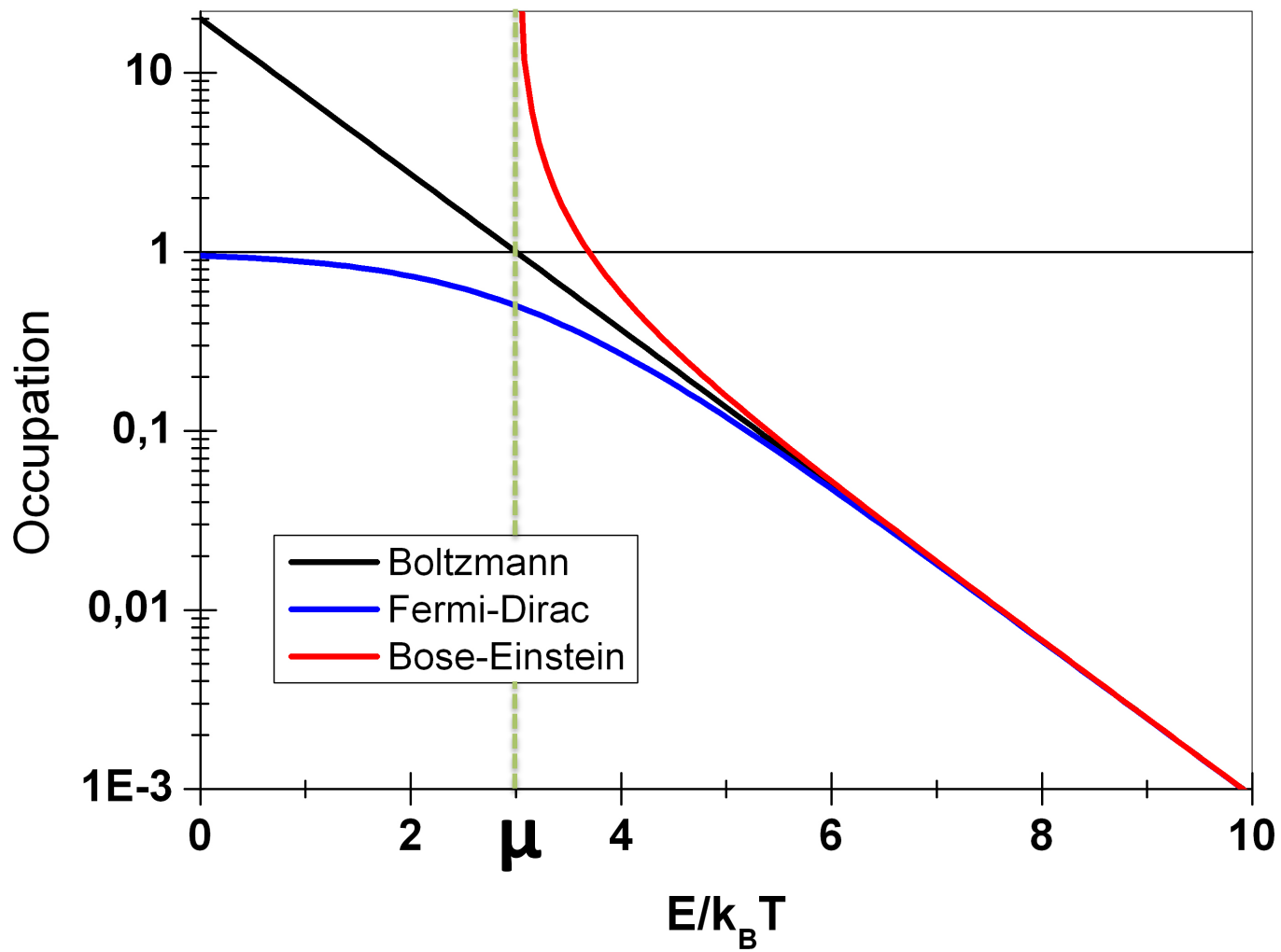
Statistical Physics

Fermi-Dirac distribution

$$f(E) = \frac{1}{\exp\left(\frac{E - \mu}{k_B T}\right) + 1}$$



- Low-energy states are filled with probability ≈ 1 .
- For $E > \mu$, occupation probability decays similarly to Boltzmann distribution.
- Orbital states may be spin degenerate.



Statistical Physics

Thermal distribution of fermionic ensembles

- Fermions: electron, proton, neutron, ^3He , ...
- At $T = 0$, all states below μ are filled, all above are empty.
- At $T > 0$, thermal excitations around $E = \mu$.

States filled with finite probability $0 < p < 1$.

Thermal smearing: df/dE

Full width at half height $\approx 3.5k_B T$

- At high $T \approx \mu/k_B$, recover Boltzmann statistics.