Eur. Phys. J. B **24**, 405–408 (2001)
DOI:

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Side wall effects in Rayleigh Bénard experiments

P.-E. Roche[1,3,a], B. Castaing[1,2], B. Chabaud[1], B. Hébral[1], and J. Sommeria[4]

[1] Centre de Recherches sur les Très Basses Températures, BP 166, 38042 Grenoble Cedex 9, France
[2] École Normale Supérieure de Lyon, 46 Allée d'Italie, 69364 Lyon Cedex 07, France
[3] Laboratoire de Physique de la Matière Condensée, E.N.S., 24 rue Lhomond, 75231 Paris Cedex 05, France
[4] LEGI - Plateforme Coriolis, 21 avenue des Martyrs, 38000 Grenoble, France

Received 26 April and Received in final form 1st October 2001

Abstract. In Rayleigh Bénard experiments, the side wall conductivity is traditionally taken into account by subtracting the empty cell heat conductivity from the measured one. We present a model showing that the correction to apply could be considerably larger. We compare to experiments and find good agreement. One of the consequences is that the Nusselt behavior for $Ra < 10^{10}$ could be closer to $Nu \propto Ra^{1/3}$ than currently assumed. Also, the wall effect can appear as a continuous change in the γ exponent $Nu \propto Ra^{\gamma}$.

PACS. 47.27.Te Convection and heat transfer – 44.25.+f Natural convection – 67.90.+z Other topics in quantum fluids and solids; liquid and solid helium

1 Introduction

Understanding high Rayleigh number turbulent convection is a long standing challenge. Experimental efforts in the few past decades were oriented towards large ranges in Rayleigh numbers Ra for evidencing the asymptotic regimes predicted by the various models [1,2]. Recently [3], an experimental study with classical fluids succeeded in increasing the precision on the Nusselt numbers by nearly one decade, allowing some test of the theories within a limited range of Ra. Obviously, one then needs to consider spurious effects previously neglected, as their incidence on the Nusselt number Nu was estimated to be smaller than the precision.

The influence of lateral wall conduction is one of these. One consequence is that part of the heat power supplied at the bottom plate is absorbed by the lateral wall. This is traditionally taken into account by measuring the heat conductivity of the empty cell, and subtracting it from the observed effective heat conductivity. It is equivalent to assume that the temperature gradient in the wall remains linear whatever the fluid state is. However, the fluid temperature is nearly uniform, equal to the average temperature between the plates, except close to them, in the thermal boundary layer of depth $h/2Nu$ (h being the height of the cell). As will be shown in the next section, the thermal contact between the wall and the fluid increases the temperature gradient in the wall close to the plates and thus the heat flux in it.

[a] e-mail: Philippe.Roche@lpmc.ens.fr

2 The model

Assuming that the fluid close to the lateral wall is nearly uniform in temperature, makes easy to estimate the temperature profile in the wall and the heat flux in it. However, the correction to apply to the heat supplied at the bottom plate is not simply equal to this spurious heat flux [4]. The heated wall warms up the adjacent fluid and reduces the heat flux from the plate by thickening the thermal boundary layer. On the other hand, the presence of the heated wall could enhance the convection and thus the heat exchange on the whole plate.

Such an intricated situation can be made clearer through a slightly different point of view. Due to its conductivity, the lateral wall close to the bottom plate is warmer than the average fluid temperature on a height λ which we estimate soon. This part of the wall acts as an additional heat exchange area, a vertical one indeed. Experimental studies [5,6] show that the Nusselt number, when high enough, poorly depends on the angle between the plate and horizontal. It is also poorly dependent on the cell aspect ratio. Thus considering the height λ of the wall as an additional heat exchange area allows to take into account most of the aspects mentioned above. The heat power Q_{cor} passing through the plates boundary layers differs from the applied one Q_{mea} to the bottom plate as:

$$Q_{\mathrm{cor}} = \frac{\pi R^2}{\pi R^2 + 2\pi R\lambda} Q_{\mathrm{mea}} = \frac{Q_{\mathrm{mea}}}{1 + 2\lambda/R}$$

for a cylindrical cell of radius R. Now to estimate λ, we have to estimate the temperature profile $T(z)$ in the wall.

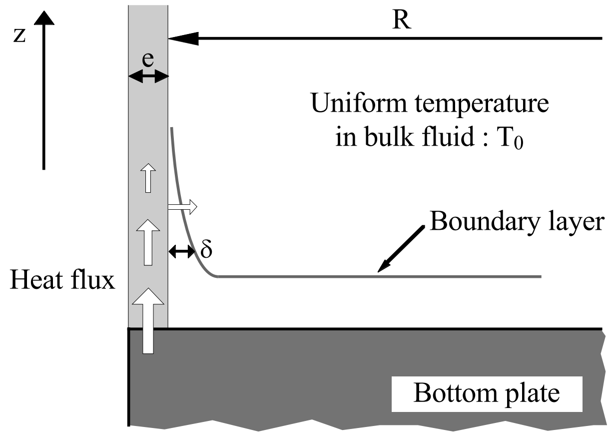

Fig. 1. Heat balance in the wall.

For a wall unit horizontal length, the heat balance of a vertical height dz can be evaluated as (see Fig. 1):

$$\chi_{\mathrm{W}} e \partial^2 T/\partial z^2 - \chi_{\mathrm{f}}(T - T_o)/\delta = 0$$

where χ_{f} (resp. χ_{W}) is the intrinsic heat conductivity of the fluid (resp. wall material), e is the wall thickness, δ is the wall thermal boundary layer width, and T_o the middle fluid temperature [7]. This gives an exponential temperature profile and a characteristic length:

$$(\chi_{\mathrm{W}} e\delta/\chi_{\mathrm{f}})^{1/2}.$$

We have considered δ as constant with z which certainly is an approximation. However, we simply need a characteristic length. Let us assume that, constant or not, δ scales with the thermal boundary layer on the plates: $h/2Nu$. Then λ has to be proportional to:

$$(\chi_{\mathrm{W}} eh/2\chi_{\mathrm{f}} Nu)^{1/2}.$$

The empty cell heat conductivity is $\chi_{\mathrm{W}} e 2\pi R/h$ while the quiescent fluid heat conductivity is $\chi_{\mathrm{f}} \pi R^2/h$. Their ratio defines the wall number:

$$W = 2\chi_{\mathrm{W}} e/\chi_{\mathrm{f}} R$$

and we can write:

$$2\lambda/R = 2A\left(\frac{1}{2Nu}\frac{2\chi_{\mathrm{W}} e}{\chi_{\mathrm{f}} R}\frac{h}{2R}\right)^{1/2} = A\sqrt{2}\left(\frac{W}{\Gamma Nu}\right)^{1/2}$$

where $\Gamma = 2R/h$ is the cell aspect ratio and A a constant of order unity which may slightly vary with the aspect ratio Γ.

Defining the measured Nusselt Nu_{mea} as the one based on the measured power supplied to the hot plate, Q_{mea}, the corrected Nusselt, Nu_{cor}, should be:

$$Nu_{\mathrm{cor}} = Nu_{\mathrm{mea}}/(1 + A\sqrt{2}(\frac{W}{\Gamma Nu_{\mathrm{cor}}})^{1/2}).$$

This must be compared to the value generally published Nu_{pub}:

$$Nu_{\mathrm{pub}} = Nu_{\mathrm{mea}} - W.$$

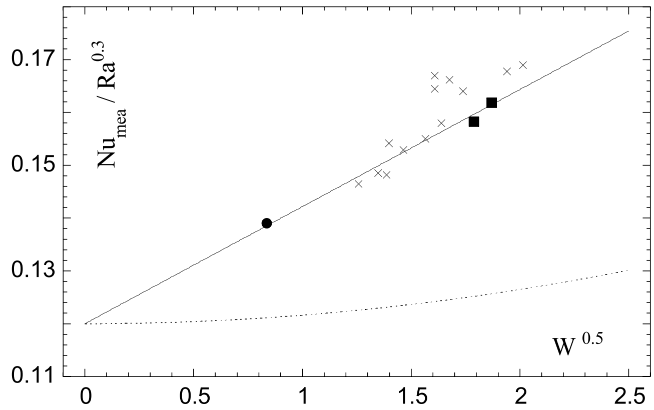

Fig. 2. Dependence of Nusselt with the wall number W. $\times$: 2 cm cell; squares: 20 cm cell, thick wall; circle: 20 cm cell, thin wall; dashed line: traditional correction; full line: high $W\Gamma Nu$ limit of the present model.

3 Comparison with experiments

To test this model we used cryogenic helium gas cells with aspect ratio $\Gamma = 1/2$. The walls are stainless steel tubes. One cell is 2 cm high and the others are 20 cm high. One of the 20 cm high cells has thicker walls than the others to make its wall number comparable to the 2 cm high cell. Plates are of high conductivity copper and much care has been taken to ensure that the wall thickness is constant along the whole height of the cell. Complete description of the apparatus is given in reference [7].

We compared measurements in the range $10^9 < Ra < 5 \times 10^9$. According to most authors [1], the Nusselt dependence on Ra can be fitted in this range by a power law: $Nu \propto Ra^{\gamma}$, with γ close to 0.3. All reasonable values of γ give $5^{(\gamma-0.3)}$ equal to 1 within 2% which means that $Nu/Ra^{0.3}$ should be constant in the considered range of Ra with the same accuracy.

Figure 2 shows $Nu_{\mathrm{mea}}/Ra^{0.3}$ *versus* $x = \sqrt{W}$. The values cannot be considered as constant. Yet, the traditional correction is too small ($Nu_{\mathrm{cor}} = Nu_{\mathrm{mea}} - W$) to account for the observed variation.

The model discussed in the preceding section can be fitted with the observed variation with $A = 0.8$. However, it is valid only when $\lambda \gg \delta$ which can be written $W\Gamma Nu \gg 1$. For the lowest values of Nu and small values of W, this condition could not be satisfied. On the other hand, when the wall conduction is poor, its temperature is imposed by the fluid, and the correction then corresponds to:

$$Nu_{\mathrm{mea}} = Nu_{\mathrm{cor}}(1 + W).$$

One can see that the formula:

$$Nu_{\mathrm{mea}} = Nu_{\mathrm{cor}}(1 + f(W))$$

with

$$f(W) = \frac{A^2}{\Gamma Nu}\left(\sqrt{1 + \frac{2W\Gamma Nu}{A^2}} - 1\right)$$

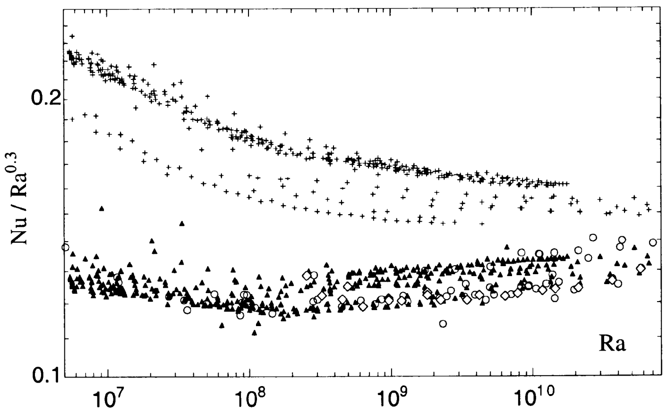

Fig. 3. Data for four helium cells. Uncorrected: + Small cell (2 cm). Corrected: ▲ Small cell ($W \simeq 3$), ◇ Large cell thick walls (20 cm, $W \simeq 3$), o Large cells thin walls ($W \simeq 0.6$).

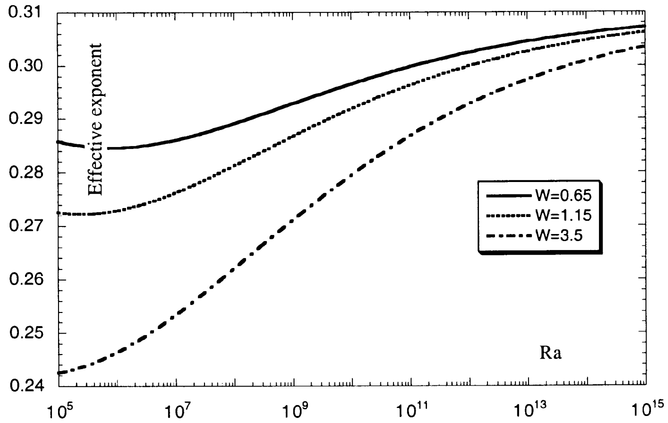

Fig. 4. Influence of the wall effect on the observed effective exponent: $W = 0.65, 1.15$, and 3.5.

interpolates between $f(W) = W$ (small $W\Gamma Nu$) and $f(W) = A\sqrt{2}(\frac{W}{\Gamma Nu_{\rm cor}})^{1/2}$ (large $W\Gamma Nu$). We propose it as a general correction formula.

Figure 3 compares the corrected data of four different cells as $Nu/Ra^{0.3}$ *versus Ra*. All these cells have 1/2 aspect ratio and work with low temperature gaseous helium. One is 2 cm high, and has $W \simeq 1.5 - 4$. Uncorrected data for this cell are also shown. The others are 20 cm high. One of them has $W \simeq 3.3$. The two others have $W \simeq 0.6$. The comparison shows that all these results nearly agree after the correction.

4 Concluding remarks

The present analysis has numerous consequences. First, it can explain some surprising results and discrepancies. For instance, published experiments seem to show that, for $Ra = 10^9$, the Nusselt number is constant if not slightly increasing when the aspect ratio decreases, down to $\Gamma = 1/2$. Introducing crossed insulating walls in a 1 aspect ratio cell can turn it into four 1/2 aspect ratio cells. The preceding result means that doing so, the total heat flux does not decrease, which is surprising. Indeed, $f(W)$ increases when Γ decreases, which can explain this spurious result. In addition, the discrepancies between helium and water results on the Nusselt value for $Ra = 10^9$ is intriguing. The helium values are systematically higher than the water one ($\Gamma = 1; Nu \simeq 60$; [10–13]), the highest being the one reported by the Chicago group for $\Gamma = 1$ ($Nu = 80$ [8]). Indeed, the wall number for this experiment was 3.5, which gives $f(W) = 0.27$ while the standard value of $W = 0.6$ for other ($\Gamma = 1/2$) helium cells [8,9,14] gives $f(W) = 0.14$. On the other hand, the high thermal conductivity of water makes W to be generally small.

Second, going back to Figure 3, one can observe that beyond the soft-hard turbulence regime ($Ra > 10^8$ for $\Gamma = 0.5$, according to [15]) the corrected data are fitted with a γ exponent $Nu \propto Ra^{\gamma}$, $\gamma \simeq 0.31 > 0.3$, while the uncorrected data give $\gamma < 0.3$. Indeed, and this is the most important point in the present debate about high Rayleigh numbers convection, a pure power law $Nu_{\rm cor} \propto Ra^{0.31}$ gives a non linear log-log plot for $Nu_{\rm mea}$ *versus Ra*. This is important, as such non linear plots are presented [3] as decisive arguments supporting a recent theory [2].

In Figure 4, we present the effective exponent

$$\gamma_{\rm eff} = \frac{{\rm dln}Nu_{\rm mea}}{{\rm dln}Ra}$$

versus the Rayleigh number, assuming $Nu_{\rm cor} = 0.1Ra^{0.31}$, for various wall numbers W. This shows how controlling the wall effect is important.

It should be noted that the corrected data are compatible with recent numerical simulations (W=0) [16]. However, experiments exist, claiming a 2/7 value for γ, in which the wall conduction cannot be incriminated (direct measurement of the thermal layer). This point needs a clarification [17,18] .

In summary, we propose a simple model for a realistic estimate of the wall effect in Rayleigh-Bénard convection. When the wall material is more conductive than the fluid, this wall effect is controlled by the wall number W defined as the ratio of the empty cell conduction to that of the quiescent fluid. We find good agreement between the predictions of the model and the controlled experiments we made. It appears that the spurious effect of the wall can explain some surprising accepted results. It can also appear as a crossover between two scaling laws, spoiling the experimental check of recent theories. Finally, it suggests that the logarithmic slope of Nu *versus Ra* is closer to 1/3 than often admitted.

We acknowledge correspondence with G. Ahlers and K. Sreenivasan, and interesting discussions with R. Verzicco and J. Niemela.

References

1. E.D. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
2. S. Grossmann, D. Lohse, J. Fluid Mech. **407**, 27 (2000).
3. X. Xu, K.M.S. Bajaj, G. Ahlers, Phys. Rev. Lett. **84**, 4357 (2000).
4. G. Ahlers, Phys. Rev. E **63**, 15303 (2001).
5. S. Ciliberto, S. Cioni, C. Laroche, Phys. Rev. E **54**, 5901 (1996).
6. A. Belmonte, A. Tilgner, A. Libchaber, Phys. Rev. E **51**, 5681 (1995).
7. P.-E. Roche, Ph.D. thesis (unpublished), Grenoble University (2001).
8. X.Z. Wu, A. Libchaber, Phys. Rev. A **45**, 842 (1992).
9. J.J. Niemela, L. Skrbek, K.R. Sreenivasan, R.J. Donnelly, Nature **404**, 837 (2000).
10. Y.B. Xin, K.Q. Xia, P. Tong, Phys. Rev. Lett. **77**, 1266 (1996).
11. S. Cioni, S. Ciliberto, J. Sommeria, Dyn. Atmos. Oceans **24**, 117 (1996).
12. A.M. Garon, R.J. Goldstein, Phys. Fluids **16**, 1818 (1973).
13. R.J. Goldstein, S. Tokuda, Int. J. Heat Transfer **23**, 738 (1979).
14. X. Chavanne, B. Castaing, B. Chabaud, F. Chillà, B. Hébral, Cryogenics **38**, 1191 (1998).
15. X.Z. Wu, Ph.D. thesis (unpublished), Chicago University (1991).
16. R. Verzicco, R. Camussi, *Proc. of the 8th Eur. Turbulence Conf. (Barcelona, 2000)*, edited by C. Dopazo, CIMNE, Advances in Turbulence VIII, p. 611.
17. F. Chillà, S. Ciliberto, C. Innocenti, E. Pampaloni, Il Nuovo Cimento D **15**, 1229 (1993).
18. A. Tilgner, A. Belmonte, A. Libchaber, Phys. Rev. E **50**, 269 (1994).