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Observation of the $\frac{1}{2}$ power law in Rayleigh-Bénard convection

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The $\frac{1}{2}$ power law is reported in a Rayleigh-Bénard experiment: $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$, where Ra and Nu are the Rayleigh and Nusselt numbers. This observation is coherent with the predictions of the ultimate convection regime, characterized by fully turbulent heat transfers. Ordered rough boundaries are used to cancel the correction due to the thickness variation of the viscous sublayer, and the observation of the asymptotic regime is therefore possible. This result supports the interpretation of a laminar-turbulent boundary-layer transition to account for the observation of Chavanne *et al.* of a new regime [X. Chavanne *et al.*, Phys. Rev. Lett. **79**, 3648 (1997)].

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Turbulent convection theories predict an ultimate regime of convection, characterized by a fully turbulent and advective heat transfer in the boundary layers (see, for example, Refs. [1–3]). This new regime is expected to be triggered by a laminar-turbulent transition in the boundary layer, but the onset Rayleigh or Reynolds numbers remain difficult to predict. However, the heat transfer law can be derived with a few traditional assumptions on turbulent boundary layers. Asymptotically, it gives: $\mathrm{Nu} = \sigma \mathrm{Ra}^{1/2}$, where Nu and Ra are respectively the Nusselt and Rayleigh numbers, and σ a Prandtl number (Pr) dependent factor. Such a high-exponent power law has never been measured, and during the last decades, this prediction has been feeding an active experimental search for the ultimate regime.

A practical motivation is sustained by meteorology, oceanography, climatology, and engineering, where the ultimate regime is a stumbling block in the understanding of geophysical and industrial flows: oceans, atmosphere, power plants, storage tanks, etc. For example, a two-decade variation of the threshold Ra to the ultimate regime will result in a 150% variation in the heat flux.

Up to now, apparent discrepancies between experiments as well as diverging predictions (Pr dependence, onset of the laminar-turbulent transition, etc.) have highlighted a general lack of understanding. Experimental evidences for a transition to a new regime in mercury [4] and in cryogenics helium [5,6] are balanced by other experiments showing no transition [7,8]. The most convincing results showing a transition to the ultimate regime are due to Chavanne *et al.*, who fitted the raising of the new regime with a $\mathrm{Nu} \sim \mathrm{Ra}^{0.39}$ power law over more than two decades above $\mathrm{Ra} = 10^{12}$. They invoked logarithmic corrections to the $\mathrm{Nu} \sim \mathrm{Ra}^{0.5}$ power law to account for their 0.39 exponent These corrections correspond to the expected thickness variation of the viscous sublayer with the Rayleigh or Reynolds (Re) numbers.

The purpose of this Rapid Communication is to show that an appropriate periodic roughness can constrain the viscous sublayer thickness and allows us to measure the 0.5 exponent predicted by theories, 40 years ago. Our experiment covers 11 decades in Rayleigh numbers, from the pure diffusive regime ($\mathrm{Nu} = 1$) below the onset of convection, up to the new regime evidenced by Chavanne *et al.* [6] for $\mathrm{Ra} > 2 \times 10^{11}$. Figure 1 gathers the Nu and corresponding Pr as functions of the Ra (note that Pr varies essentially by step between two series of measurements). Apart from the roughness, our cell is similar to the one used in Ref. [6] and the same measuring apparatus is used. Details about the setup are presented in Refs. [9,10].

Figure 2 is a schematic view of the cell and the wall roughness. The cell is a cylinder 20 cm high and 10 cm in diameter (aspect ratio 0.5) hanging in a cryogenics vacuum. The measured stainless steel wall thermal conductance is 327 μW/K at 4.7 K. The corresponding heat flux is subtracted in the data presented. Heat leaks from the bottom plate to the calorimeter or to the top plate are negligible in such a setup, as shown in Ref. [10].

Top and bottom plates are made of 2.5 cm thick copper plates annealed during brazing. Its conductivity has been measured to be 880 $\mathrm{W\,K^{-1}\,m^{-1}}$ at 4.2 K [11]. The roughness is a 110 μm deep V-shape groove with top and bottom angle of 90°. The grooves cover the whole interior of the cell, plates and side walls.

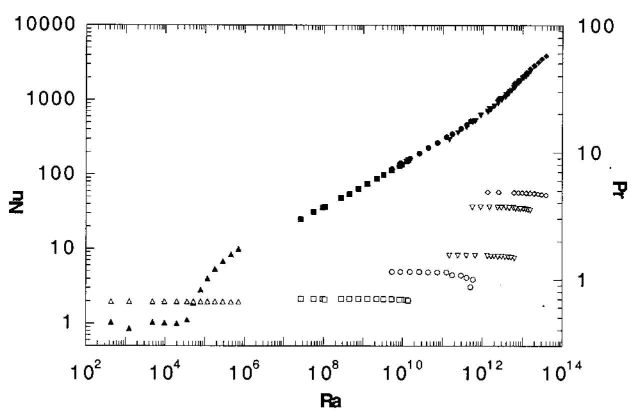

FIG. 1. Dependence of Nu (closed symbols) and Pr (open symbols) on Ra. The shape of each symbol is associated with the filling of the cell. The cell density in kg/m^3 is: ▲ △, 0.014; ■ □, 1.57; ● ○, 13.5; ▼ ▽, 39.8; and ♦ ◇, 66.3. Note that Ra varies from below the onset of convection up to the new regime.

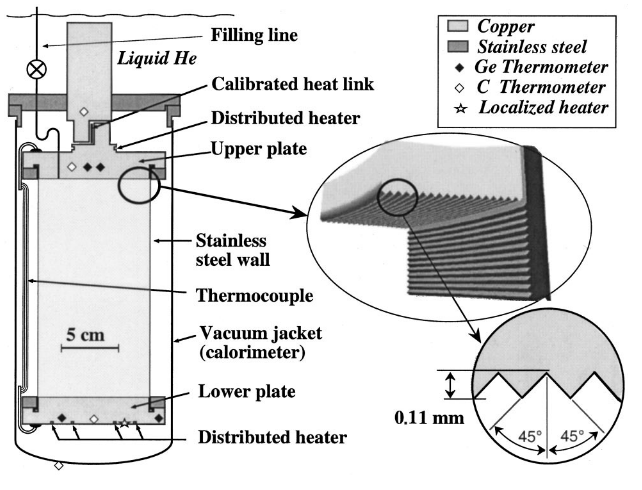

FIG. 2. Cross section of the cylindrical cell with zooms on the surface roughness. The distributed heater on each plate consists of a single 2 m long constantan wire, distributed axisymmetrically.

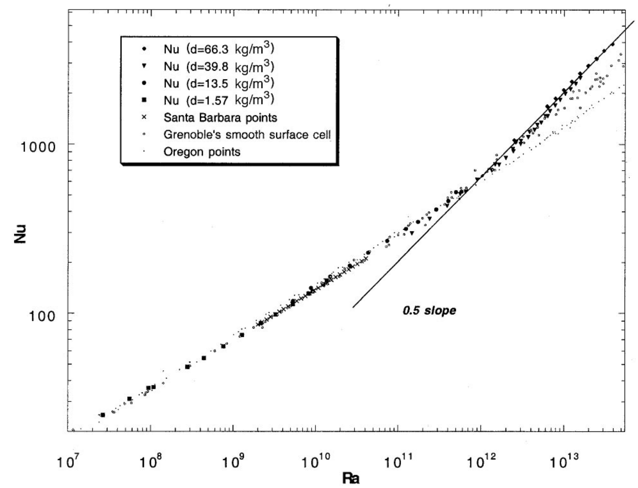

FIG. 3. Dependence of Nu on Ra in the hard turbulence and new regimes in cells of aspect ratio 0.5 and for 0.6<Pr<5. Closed symbols, our rough cell (see Fig. 1 for details of the symbols); ○ Grenoble smooth surface cell [6,9,10]; ● Oregon experiment [8] (smooth surface), and × Santa Barbara experiment [18] (smooth surface). The 0.5 slope corresponds to the predicted ultimate regime asymptotic dependence.

The top plate is in thermal contact with a liquid helium bath through a measured 7 K/W brass heat link, and its temperature is regulated by a PID controller (down to an uncertainty of few tens of μK). The bottom plate is Joule heated with a constant power P ranging from $P<2$ mW for $\mathrm{Ra}<3\times10^6$, 7 $\mu\mathrm{W}<P<100$ mW for $3\times10^6<\mathrm{Ra}<3\times10^{10}$ up to 500 $\mu\mathrm{W}<P<200$ mW for $\mathrm{Ra}>3\times10^{11}$. The cell is filled by turns with five different densities (see the caption of Fig. 1) known with $\pm2\%$ uncertainty. The quantity of helium introduced in the cell is measured at room temperature using a calibrated volume. For each density, the mean temperature and heat flux are adjusted in order to vary the control parameters (Ra and Pr). The procedure benefits from the smoother dependence of the He properties versus the density, compared to that with the pressure when the critical point is approached, which is more difficult to measure in this sort of cryogenics setup. The Boussinesq criterion applied is $\alpha\Delta T<20\%$, where α and ΔT are the He constant pressure thermal expansion coefficient and the temperature difference between the two plates.

The temperature difference ΔT is measured using a specially designed AuFe/NbTi thermocouple of 15 μK accuracy and absolute resolution [10]. Both adiabatic gradient measurements and operation with superfluid helium in the cell validated small ΔT operation and the absolute zeroing. For absolute calibration of the germanium resistance thermometers, the critical temperature T_c was approached down to 0.4 mK.

The helium properties that are used are compiled from many sources [12–17]. Some improvements since the work reported in Ref. [9] address transport properties [11]. The data from Ref. [9] presented below have been recalculated accordingly and they do not show significant differences. In this work, the critical region was never approached closer than $d=66.3$ kg/m^3 and $T=5.54$ K and thus the thermodynamics properties fit of Ref. [12] is used, without any critical point extra correction. An experimental validation of the fits is provided by redundant measurements of (Ra,Nu) pairs at a given Pr and for different temperatures and densities. We observe that identical Ra and Pr obtained with different densities in Ref. [9] result in the same Nu.

Below $\mathrm{Ra}=10^6$, the onset-of-convection region is used as a test regime for the sensitivity of the setup. Indeed, the cell is operated at a very low level of heating (a few hundreds of nanowatts) and for temperature difference between the plates down to 650 μK. This onset Ra number (4×10^4) is in agreement with the measured value in $\Gamma=0.5$ aspect ratio cells of low sidewall conductivity [9–11].

Figure 3 gathers our data and those from the Chavanne *et al.* smooth surface cell [9]. For comparison, data from the Oregon experiment [8] (Pr=0.7) and the recent measurements by Xu *et al.* [18] in acetone (Pr=4 and aspect ratio 0.5) are plotted. from $\mathrm{Ra}=3\times10^7$ up to 2×10^{11}, the Pr numbers remain close to 0.90 (±0.25) in both Grenoble experiments. Within the data uncertainty, there is no difference between the results. Defining the thermal boundary layer thickness as $\lambda_{\mathrm{th}}=h/2\,\mathrm{Nu}$, λ_{th} is always at least 1.5 times thicker than our roughness. Under these conditions, it was previously shown on smaller ranges of Ra numbers ($10^9<\mathrm{Ra}<10^{11}$) that the heat transfer is not affected by the sublayer roughness (see, for example, Ref. [19]).

From $\mathrm{Ra}=2\times10^{11}$ up to roughly 2×10^{12}, $275>\lambda_{\mathrm{th}}>110$ μm and no measurable difference appears between the Grenoble's data from the rough and the smooth surface cells (note that in this regime, the formula for λ_{th} should be considered as an estimate). The rising of the new regime measured in Ref. [6] is indeed not affected by the addition of our roughness.

Above roughly $\mathrm{Ra}=2\times10^{12}$, the Nu(Ra) dependence is explored for three different Pr numbers (1.5, 3.65, and 4.75). These constant Pr numbers series are obtained for densities of 39.8 kg/m^3 (Pr=1.5, 3.65) and 66.3 kg/m^3 (Pr=4.75). the data can be fitted by a $\mathrm{Nu}=\sigma\,\mathrm{Ra}^{\gamma}$ power law with $\gamma=0.51\pm0.015$. This γ exponent optimizes the compensated plots $\mathrm{Nu}/\mathrm{Ra}^{\gamma}$ versus Ra. $\mathrm{NuRa}^{-0.5}$ is plotted on Fig. 4. This uncer-

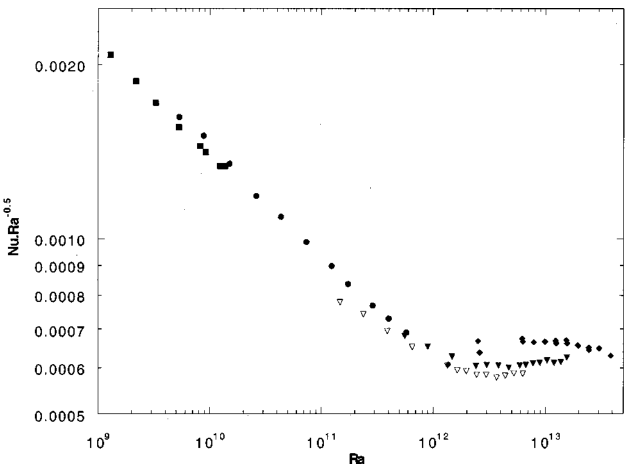

FIG. 4. Dependence of the compensated quantity $\mathrm{NuRa}^{-0.5}$ on Ra. ■, Pr<1; ●, 1<Pr<1.15; ▽, 1.4<Pr<1.55; ▼, 3.5<Pr<3.75; ♦, 4.6<Pr<4.9.

tainty on γ is compatible with the helium properties fit uncertainty. Indeed, the $\mathrm{Ra}/\Delta T$ factor, which only depends on the He properties, varies monotonously by less than 15% within each Pr series. Assuming a 30% uncertainty on the variation of this factor would give a ± 0.02 maximum uncertainty of γ. The Pr dependence of the σ factor is weak: around a 15% increase for nearly 220% increase in Pr and could result from the uncertainty in the He properties fits. Within experimental uncertainty, the γ exponent is independent to a tilt of the cell (5°) and to nonsymmetrical bottom plate heating (for the location of the heater, see Fig. 2).

This $\mathrm{Ra}^{0.5}$ dependence is observed on a wide range (a factor of 20 in Ra, almost the total range of an experiment with classical fluids). This allows to discriminate from a simple crossover where the effective surface felt by the fluid would increase as the boundary layer gets thinner than the roughness. The effect of such a crossover with a similar V-shape type of roughness has already been observed at lower Ra numbers [19]. When the boundary layer λ_{th} equals 80% of the roughness height, the Nu increases abruptly over less than 0.2 decades in Ra. In the same experiment, the γ exponent has shown to be the same before and after the crossover (see also Ref. [20]). These observations are at variance with ours. They are also at variance with the observed effect of a wide distribution of scales for the roughness which widens the crossover and mimics a different exponent [21]. This is why we choose this monodisperse, already tested, V-shape roughness, and this confirms us in the following interpretation.

Most theories [2] for the ultimate convection regime rely on three keypoints:

(i) accepted results for passive scalar heat transport in turbulent boundary layers,
(ii) an exact relation between $(\mathrm{Nu}-1)\mathrm{Ra}$ and the dissipation within the cell,
(iii) an estimation of the viscous dissipation based on the logarithmic turbulent velocity profile.

The resulting prediction for the Nu (Ra) dependence is up to a numerical factor (see, for example, Ref. [2]): $\mathrm{Nu}=\mathrm{Ra}^{0.5}/(\ln \mathrm{Ra})^{1.5}$. In this relation, the logarithmic factor results from the variation of the viscous sublayer thickness with Ra. In our experiment, the roughness imposes a new length scale to the boundary layers when the thermal boundary layer gets thinner than typically 110 μm that is for Ra $\approx 2\times 10^{12}$. In this case, the sublayer thickness is fixed by the roughness and the logarithmic correction becomes irrelevant. Consequently, for Ra as low as $\mathrm{Ra}\approx 2\times 10^{12}$, the Nu (Ra) dependence has to be asymptotic.

This can be seen as the thermal transport equivalence of a well known result [22] for velocity turbulence on a plate. Indeed the logarithmic velocity profile on flat plate goes down to the viscous sublayer whose depth depends on the Reynolds number. This results in a logarithmical dependence of the friction coefficient versus Re. On a rough plate it goes down to the roughness scale, which makes the friction coefficient independent of Re.

The observation we report of this wall roughness effect supports the interpretation of a laminar-turbulent transition in the boundary layer to explain the occurrence of a transition in the smooth cell of Chavanne *et al.* [6]. The reason why this transition is not seen in the Oregon experiment [8] is another problem, which remains unexplained. Studies are under progress [23] to understand the mechanism occurring in the boundary conditions which would favor, delay, or prevent the occurrence of this transition.

The main result reported in this paper is the observation of a $\mathrm{Nu}\sim\mathrm{Ra}^{0.5}$ power law. This dependence is coherent with the asymptotic prediction for the ultimate convection regime. An analogy with the turbulent friction coefficient over flat and rough plates suggests that roughness cancels the corrections introduced in ultimate regime theories and makes possible the observation of the asymptotic regime. A comparison with the Chavanne *et al.* experiment [6] (smooth surface cell) supports the interpretation of a boundary layer laminar-turbulent transition of the ultimate regime for Ra as low as 2×10^{11}.

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