Observation of the Ultimate Regime in Rayleigh-Bénard Convection

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In a low temperature He gas Rayleigh-Bénard experiment, Rayleigh numbers from 10^3 to more than 10^{14} are explored. Local velocity is estimated through the time lag between two closeby temperature probes. This allows characterizing of the high Rayleigh regime ($\mathrm{Ra} > 10^{11}$) as a fully turbulent one, possibly corresponding to the asymptotic regime predicted by R. Kraichnan [Phys. Fluids **5**, 1374 (1962)]. [S0031-9007(97)04440-2]

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The Rayleigh-Bénard problem [1] (natural convection in a cell heated from its flat horizontal bottom) has raised constant interest in the last few decades. This is due both to its important implications in the atmospheric, oceanic, or industrial context, and to the intellectual challenge of understanding the interplay between the thermal boundary layer and the buoyancy flow.

In the Boussinesq approximation [1], fluid properties are assumed constant despite the temperature gradient between the top and bottom plates, except for the buoyancy force which linearly depends on temperature. In this context, the Nusselt number Nu (effective conductivity of the cell normalized to the purely conductive one) depends on a few nondimensional parameters: the cell geometry, through its aspect ratio; the Prandtl number $\mathrm{Pr} = \frac{\nu}{\kappa}$; and the Rayleigh number $\mathrm{Ra} = g\alpha\Delta h^3/\nu\kappa$, where g is the gravity acceleration, α is the constant pressure thermal expansion, $\Delta = T_b - T_t$ is the difference between the bottom and the top plate temperatures, h is the cell height, ν is the kinematic viscosity, and κ is the heat diffusivity, respectively.

The Nu behavior at large Rayleigh numbers has been recently reviewed by Siggia [2]. Experiments using low temperature helium gas as working fluid [3,4] allow a large range of Ra covering, due to the variation of ν and κ with the gas pressure up to the critical point. They revealed a power law regime $\mathrm{Nu} \propto \mathrm{Ra}^{\gamma}$ with γ close to $\frac{2}{7}$ [4]. The neat difference with $\gamma = \frac{1}{3}$ invalidates theories where top and bottom boundary layers are independent, corresponding to the heat flux Q being independent on h [2]. The result also differs from the expected asymptotic regime predicted by Kraichnan [5], $\gamma = \frac{1}{2}$, where Q does not depend on ν or κ but essentially on their ratio Pr. In this asymptotic regime, both the heat and momentum (velocity) transport would be turbulent in the boundary layers. Some measurements [6] of the velocity and thermal boundary layers width for $\mathrm{Pr} \geq 1$ up to $\mathrm{Ra} \simeq 10^{11}$ show that the velocity boundary layer is larger than the thermal one, but decreases more rapidly when Ra is increased, suggesting a transition (toward the asymptotic regime?) for $\mathrm{Ra} \simeq 10^{16}$. For a low Prandtl number fluid (in mercury $\mathrm{Pr} = 0.025$), an anomalous behavior of Nu for the highest Ra obtained ($\sim 10^9$) has been interpreted as a possible occurrence of the asymptotic regime [7].

The experiment we present here covers the whole range of Ra from the conductive regime ($\mathrm{Nu} = 1$) up to the largest Ra obtained in a laboratory experiment. The cell is a vacuum isolated cylinder whose height $h = 20$ cm is twice its diameter (aspect ratio $\Gamma = \frac{1}{2}$). Top and bottom plates are made of 2.5 cm thick copper. The top plate is cooled by a liquid helium bath and regulated via a high resolution proportional integrate derivative controller [8] allowing less than 0.1 mK variations. The bottom plate is Joule heated with a constant power. The cell is filled with helium at various average densities, both above and below the critical one ($\simeq 69.6$ kg/m^3). The quantity of helium introduced in the cell is measured at room temperature using a calibrated volume.

The temperature difference Δ is measured using a specially designed thermocouple AuFe/NbTi [9]. It allows evidencing and characterizing the adiabatic gradient effect (across our cell it corresponds to 0.4 mK at 7 g/m^3, the lowest density, up to 0.8 mK close to the critical point). Once this effect has been corrected, more than three decades of Ra can be covered with a single density.

Helium properties were determined using the Arp and McCarty [10] interpolation formulas far from the critical point, and the Kierstead formulas in its neighborhood [11]. Correct use of these formulas requires accurate thermometry measurements. The temperature we determined for the critical point was within 1 mK of the Ref. [11] value. More details about the experimental setup are given in Refs. [9] and [12].

Average velocities in the cell are obtained from a couple of 200 μm cubic thermometers, made of silicon-doped arsenic [13]. They are aligned on a vertical axis, $d = 2.3$ mm away from each other. They are equidistant from the plates, 2 cm aside the symmetry axis of the cell. The phase φ of the cross correlation spectrum between the two temperatures they measure has a characteristic shape versus the frequency [14], going from zero to 180° when the frequency increases [Fig. 1]. The frequency ω_0 where

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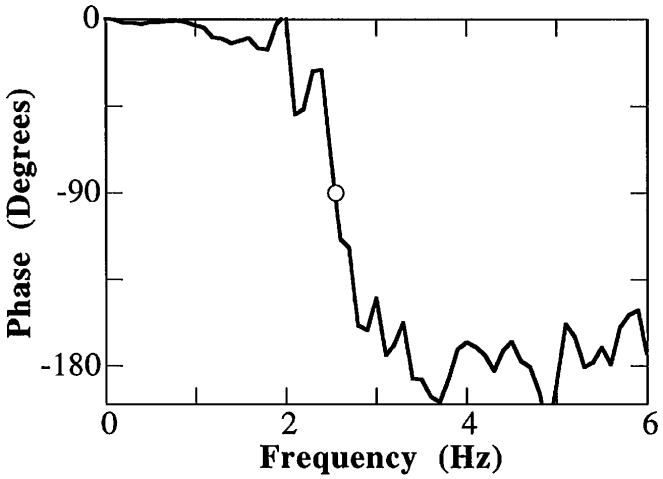

FIG. 1. Phase of the cross correlation spectrum between the two thermometers. Data are obtained by averaging 1600 samples of 2000 points each. For this point in liquid He, $\mathrm{Pr} = 0.6$, $\mathrm{Ra} = 10^{12}$, and $\Delta = 100$ mK.

$\varphi = 90°$ thus determines a characteristic time, which in turn measures a characteristic velocity $v_0 = \omega_0 d$. The Reynolds number we deduce is $\mathrm{Re} = v_0 h/\nu$. The phase of the cross correlation spectrum between the thermocouple signal and one thermometer has a similar evolution, the frequency it defines being 70 times smaller, in good agreement with our interpretation (the distance being 70 times larger).

Figure 2 presents the compensated quantity $\mathrm{Nu}/\mathrm{Ra}^{2/7}$. It shows that the range between $\mathrm{Ra} = 3 \times 10^7$ and $\mathrm{Ra} = 10^{11}$ is compatible with $\gamma = \frac{2}{7}$ as already claimed in several previous works [2]. At $\mathrm{Ra} = 10^{11}$ a transition occurs towards a new regime. Note that, for the same value of Ra in a similar cell (twice bigger), Wu *et al.* pointed out a transition they observed via the shape of the temperature fluctuation spectrum, but seemingly not in the Nu versus Ra dependence [15].

Figure 3 shows the dependence of $\mathrm{RePr}^{0.72}$ versus Ra. The exponent of Pr was empirically determined by minimizing the points scattering. Note, however, that it well corresponds to some predictions ($\frac{5}{7}$ in Ref. [2]). The transition at $\mathrm{Ra} = 10^{11}$ is not visible on this graph. It suggests that its origin is not in the bulk behavior and that a single mechanism is responsible for the Re dependence across the whole range.

According to Siggia [2], the product $\mathrm{PrRa}(\mathrm{Nu} - 1)$ is a measurement of the average viscous dissipation in the cell, in units where $\kappa = 1$, $h = 1$, and $\Delta = 1$ ($\nu = \mathrm{Pr}$). If the whole fluid is turbulent, this average dissipation would be proportional to v_0^3/h and thus $\mathrm{Re}^3\mathrm{Pr}^3$ in the previous units. On the other hand, if dissipation mainly occurs in laminar boundary layers, it concerns a fraction $\mathrm{Re}^{-1/2}$ of the volume. The average viscous dissipation $\langle \nu(\delta v/\delta x)^2 \rangle$ is then in the Siggia units proportional to $\mathrm{Pr}(\mathrm{RePr}/\mathrm{Re}^{-1/2})^2\mathrm{Re}^{-1/2} = \mathrm{Pr}^3\mathrm{Re}^{5/2}$.

In Fig. 4 we show the ratio $\mathrm{Re}^3\mathrm{Pr}^2/\mathrm{Ra}(\mathrm{Nu} - 1)$ versus Re in log-log scale (open symbols correspond to a constant Prandtl number $\mathrm{Pr} \simeq 0.7$). Note that Re up to 2×10^5 have been obtained. The neat transition visible in this diagram well corresponds to $\mathrm{Ra} = 10^{11}$. It shows that in the last regime the fluid should be fully turbulent. Before the transition, the $\frac{1}{2}$ slope is in good agreement with laminar velocity boundary layers.

In the following we stress that the last regime, after the transition, is compatible with logarithmic corrections to an asymptoticlike relation $\mathrm{Nu} \propto \mathrm{Ra}^{1/2}$. Indeed, a turbulent

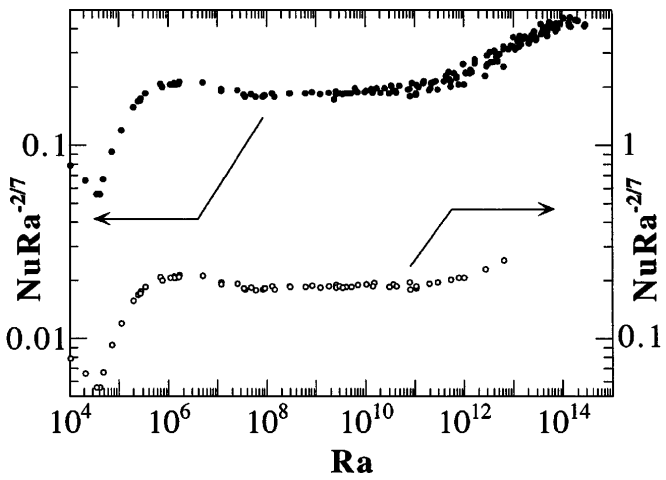

FIG. 2. Full symbol: compensated quantity $\mathrm{Nu}/\mathrm{Ra}^{2/7}$ for the whole data, showing the $\gamma = \frac{2}{7}$ range ($3 \times 10^7 \leq \mathrm{Ra} \leq 10^{11}$) and the transition. The dispersion at high Ra could be due to a Prandtl number dependence. Open symbols: same for $0.6 < \mathrm{Pr} < 0.73$. The transition is clearly visible. (Note the shifted right hand scale.)

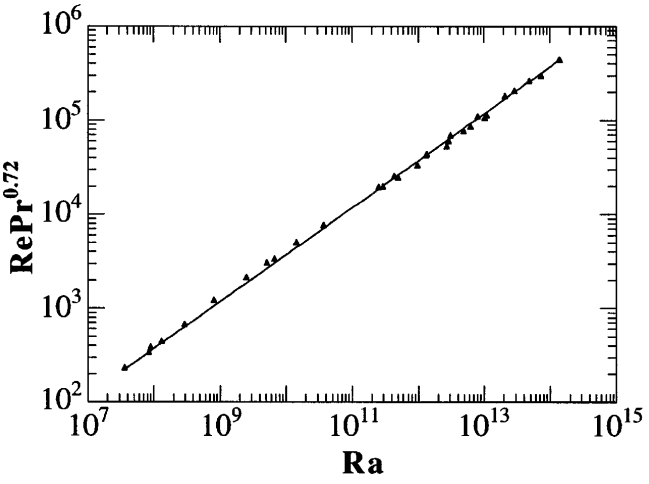

FIG. 3. $\mathrm{RePr}^{0.72}$ versus Ra. The 0.72 exponent of the Prandtl number has been determined by minimizing the points scattering (for $\mathrm{Ra} > 10^{10}$, Pr values can differ up to a factor of 5 at a given Ra). No transition is visible on this more than 7 orders of magnitude range. The straight line has the $\frac{1}{2}$ slope ($\mathrm{RePr}^{0.72} = 3.74 \times 10^{-2}\mathrm{Ra}^{1/2}$). The best fit gives a 0.49 slope.

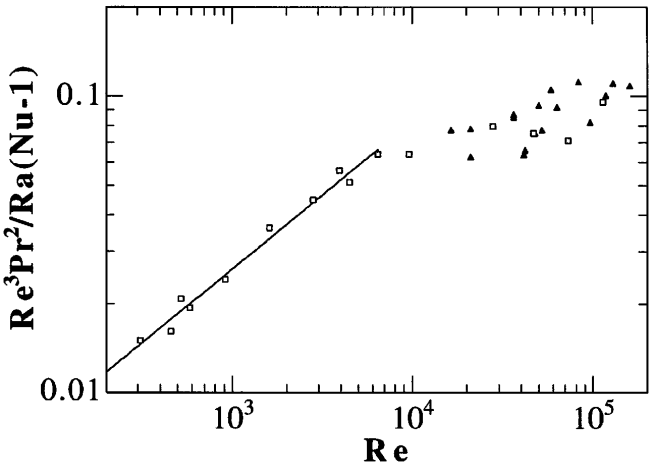

FIG. 4. Compensated quantity $\mathrm{Re}^3\mathrm{Pr}^2/\mathrm{Ra}(\mathrm{Nu}-1)$. Open symbols correspond to $0.6 < \mathrm{Pr} < 0.73$. The highest Re plateau suggests a fully turbulent state. The straight line has a $\frac{1}{2}$ slope, showing that the low Re points are coherent with a laminar boundary layer.

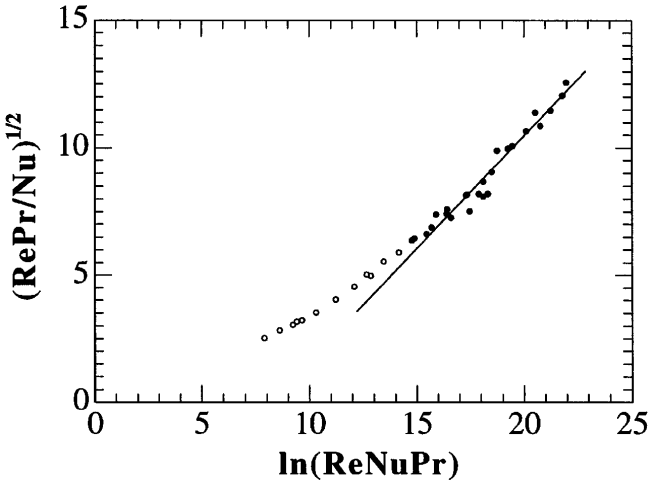

FIG. 5. Logarithmic dependence of the ratio $(\mathrm{RePr}/\mathrm{Nu})^{1/2}$. Open symbols correspond to the $\frac{2}{7}$ regime ($\mathrm{Ra} < 10^{11}$). Solid line is a linear fit for the $\mathrm{Ra} > 10^{11}$ points ($0.6 < \mathrm{Pr} < 6$).

boundary layer flow on a hot plate results in a logarithmic temperature profile [16]:

$$T_b - T(y) = \frac{\kappa Q}{\lambda v_*}\left[\ln\frac{y v_*}{\kappa} + f(\mathrm{Pr})\right],$$

where y is the distance to the plate, ρv_*^2 is the momentum flux (stress) from it, and ρ and λ are the fluid density and heat conductivity, respectively. For this simplified discussion we take Prandtl number of order 1 and we neglect the variations of the additional term $f(\mathrm{Pr})$. Using the definition of $\mathrm{Nu} = Qh/\lambda\Delta$, we obtain, up to numerical factors:

$$\frac{h v_*}{\kappa} = \mathrm{Nu}\ln\frac{h v_*}{\kappa}. \tag{1}$$

On the other hand,

$$\frac{h v_*}{\kappa} = \frac{\mathrm{RePr}}{\ln(h v_*/\kappa)}. \tag{2}$$

Using the dissipation behavior in the turbulent regime, $\mathrm{PrRaNu} = \mathrm{Re}^3\mathrm{Pr}^3$ and relations (1) and (2), we obtain

$$\left(\frac{\mathrm{RePr}}{\mathrm{Nu}}\right)^{1/2} \propto \ln(\mathrm{ReNuPr}), \tag{3}$$

$$\mathrm{Nu} \propto \frac{(\mathrm{RaPr})^{1/2}}{\ln^3(\mathrm{ReNuPr})}. \tag{4}$$

Relation (4) has the announced asymptotic behavior $\mathrm{Nu} \propto \mathrm{Ra}^{1/2}$, with logarithmic corrections. In Fig. 5 we test the third relation, which has the advantage to be independent of additional constant for the logarithm. In the last regime [above $\ln(\mathrm{ReNuPr}) = 16$, covering more than 3 orders of magnitude in Ra] the observed behavior is compatible with a linear dependence.

A complete discussion is needed for separate consideration of the $\mathrm{Pr} < 1$ and $\mathrm{Pr} > 1$ cases. The way the convective transport supersedes the diffusive one in the boundary layers has to be elucidated in each case. The proposed dependence of coordinates versus Pr in Fig. 5 have thus to be taken as empirical, minimizing the points scattering [17].

To conclude, the behavior of a typical velocity in our Rayleigh-Bénard cell allows us to characterize the transition we evidenced at $\mathrm{Ra} = 10^{11}$. The average viscous dissipation goes from a laminar to a turbulent boundary layer type. The dependence of Nu versus Ra in the high Rayleigh regime is the most rapid ever observed on such a 3 decades range, yielding to the highest Nusselt numbers in a laboratory experiment (close to 10^4). This behavior is coherent with the long ago announced asymptotic regime [5].

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[13] We are indebted to P. Tabeling, H. Willaime, and A. Libchaber who furnished us with thermometers. They are of the same type as those used by Wu *et al.* [4] (manufactured by the Goddard Space Flight Center).
[14] In a simple model, where velocity and shape of structures are uncorrelated, the cross spectrum is the Fourier transform of the time lag distribution, weighted by the common spectrum of both temperature signals. The phase φ of the cross spectrum is then roughly given by $\tan\varphi = (p_+ - p_-)\tan\omega\tau$, where p_+ (respectively, p_-) is the probability for positive (respectively, negative) time lag, and τ is an average time lag.
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