Floquet spectrum, finite frequency cross-correlations and Berry phase in multiterminal Josephson junctions

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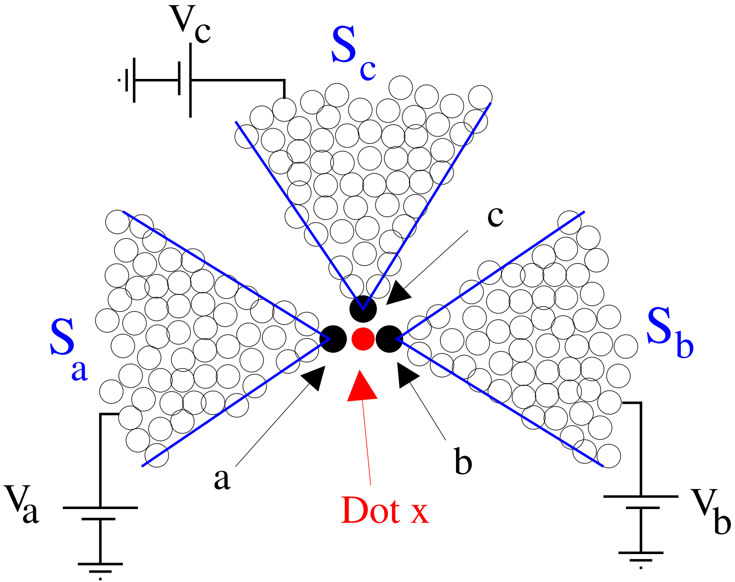
France

One of the Devices for (Numerical) Experiments

A. Freyn, B. Douçot, D. Feinberg, R. Mélin, PRL 2011



List of Collaborators

Laboratoire de Physique Théorique et des Hautes Energies (Jussieu, Paris, France)

- **Benoît Douçot**, Kang Yang

Karlsruhe Institute of Technology (KIT, Germany)

- Group of **Romain Danneau**

Mathematics Department of INSA (Rouen, France)

- **Jean-Guy Caputo**

Weizmann Institute (Rehovot, Israel)

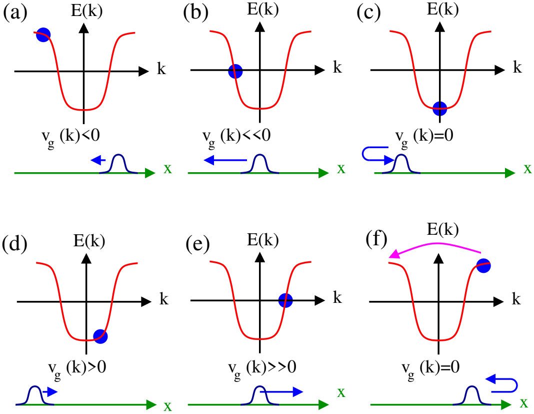
- Part of group of **Moty Heiblum**:
  Yuval Ronen (now post-doc at Harvard), Yonatan Cohen, Jung-Hyun Kang, Hadas Shtrikman

Bloch Oscillations

A Theory of the Electrical Breakdown of Solid Dielectrics.

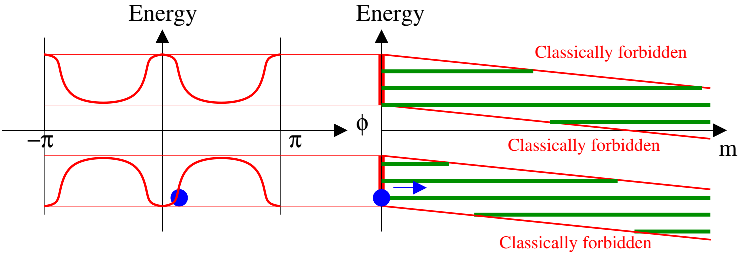
CLARENCE ZENER, H. H. Wills Physics Laboratory, Bristol.

(Communicated by R. H. Fowler, F.R.S.—Received December 27, 1933, Revised March 1, 1934.)



Connection between Bloch oscillations and Wannier-Stark ladders

Wannier, Phys. Rev. 1960



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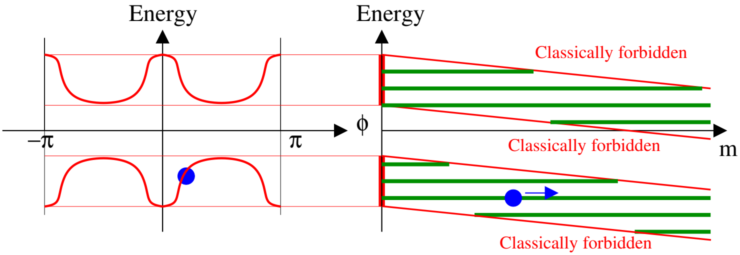

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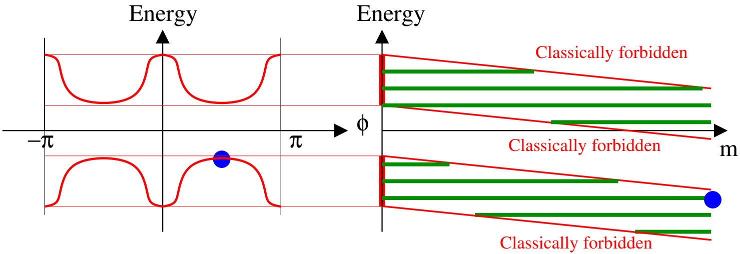

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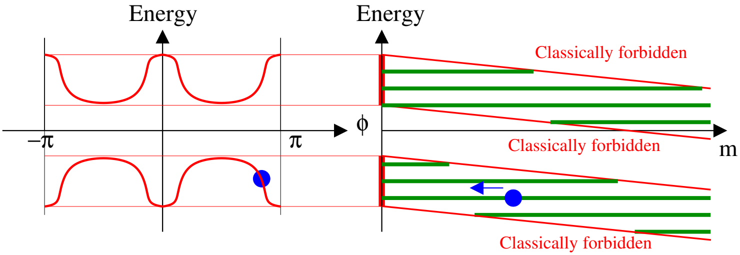

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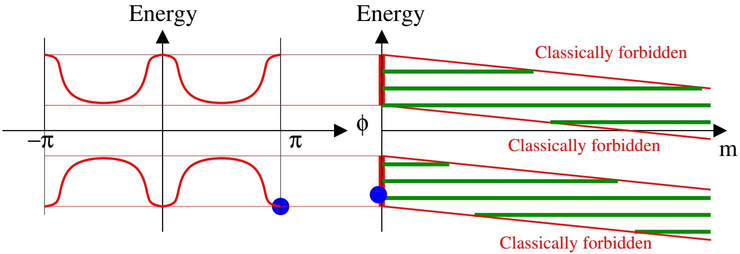


In practise: absence of Bloch oscillations in metals

- Bloch oscillations are not observed in metals because of inelastic collisions

$\Rightarrow$ **Semiconductor superlattices:**

- Brillouin zone $[-\pi/a, \pi/a]$, with a enhanced by about a factor 1000 compared to a metal

$\Rightarrow$ Period of oscillations much shorter than inelastic scattering time.

Wannier-Stark ladders in semiconducting superlattices

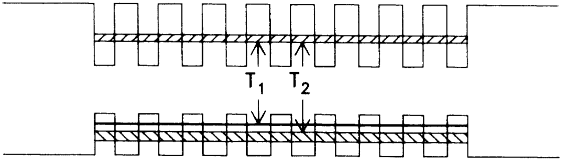
VOLUME 60, NUMBER 23 PHYSICAL REVIEW LETTERS 6 JUNE 1988

Stark Localization in GaAs-GaAlAs Superlattices under an Electric Field

E. E. Mendez, F. Agulló-Rueda, and J. M. Hong
IBM T. J. Watson Research Center, Yorktown Heights, New York 10598
(Received 21 January 1988)

We have observed that a strong electric field $\mathcal{E}$ shifts to higher energies the photoluminescence and photocurrent peaks of a GaAs-$Ga_{0.65}Al_{0.35}As$ superlattice of period D (=65 Å), which we explain by a field-induced localization of carriers to isolated quantum wells. Good agreement is found between observed and calculated shifts when the large field-induced increase of the exciton binding energy is taken into account. At moderate fields [$\simeq (2-3)\times 10^4$ V/cm], the coupling between adjacent wells is manifested by four additional peaks that shift at the rates $\pm e\mathcal{E}D$ and $\pm 2e\mathcal{E}D$ and correspond to transitions that involve different levels of the Stark ladder.

PACS numbers: 73.60.Br, 73.40.Lq, 78.55.Cr



(a)

Formation of minibands <u>in absence of electric field</u>

Wannier-Stark ladders in semiconducting superlattices

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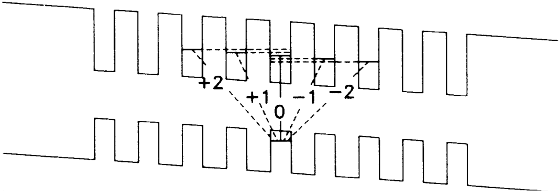

(b)

Intermediate electric field: Wannier-Stark ladders and possibility of transitions $m = 2, 1, 0, -1, -2$

Spatial extent of ladders $\propto$ (Electric field)$^{-1}$

Wannier-Stark ladders in semiconducting superlattices

VOLUME 60, NUMBER 23 PHYSICAL REVIEW LETTERS 6 JUNE 1988

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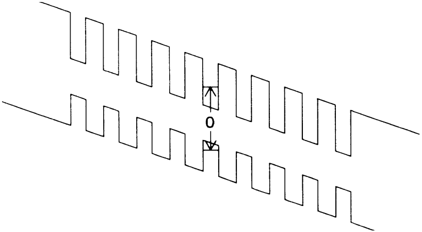


(c)

Localization in a single well in <u>strong electric field</u>

Spatial extent of ladders ≈ Size of a single quantum well
So-called "*Atomic limit*"

Spectroscopy

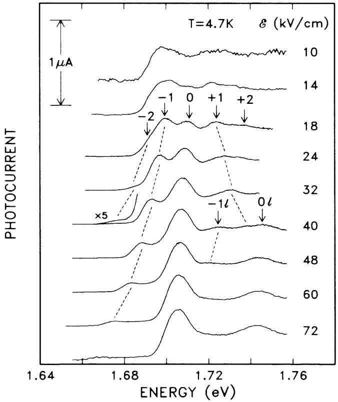


FIG. 3. Photocurrent (PC) spectra for the same superlattice of Fig. 2, at representative electric fields. The peaks labeled 0, ±1, and ±2 are for transitions involving heavy-hole states and electrons weakly delocalized, as illustrated in Fig. 1(b). Analogous transitions for light holes are denotes by $0l$ and $-1l$.

Evolution of the peak energies as a function of electric field

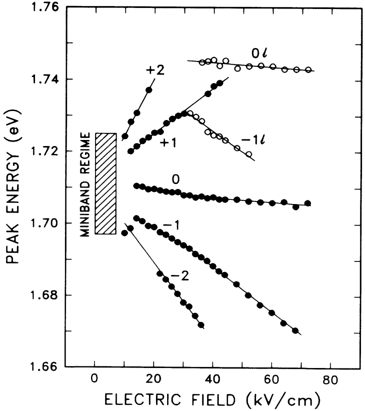


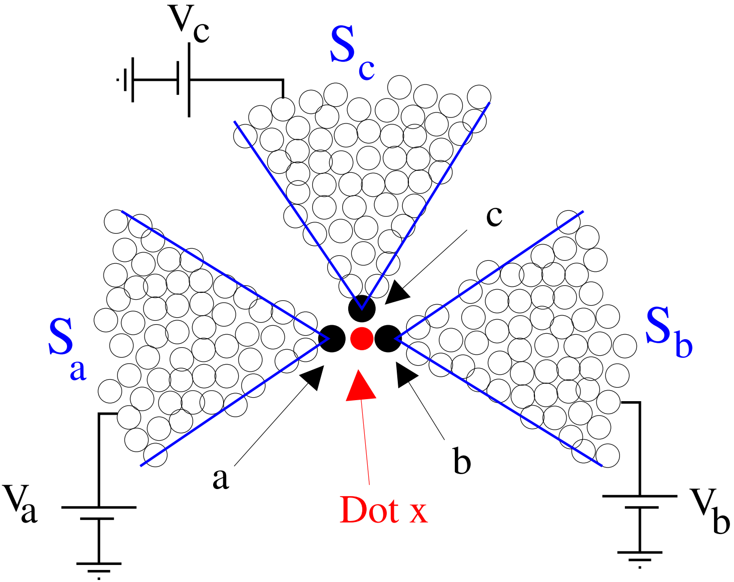
FIG. 4. Transition energies for the PC structures of Fig. 1(a) vs electric field. The filled circles correspond to heavy-hole transitions, whereas the open circles refer to light holes.

Analogy between solid state physics and superconductivity

R. Mélin, R. Danneau, K. Yang, J.-G. Caputo, B. Douçot,
arXiv:1903.04889, submitted to PRB

Band Theory	**Superconductivity**
Wave-vectors	Superconducting phases
Position x_n on the lattice in real space	Number N of transmitted Cooper pairs
Wannier functions labelled by sites on a periodic lattice	Periodicity in phases implies N integer
Plane waves in Bloch theory $\vert k\rangle = \sum_x \exp(ikx)\vert x\rangle$	States with fixed superconducting phase $\vert\varphi\rangle = \sum_N \exp(iN\varphi)\vert N\rangle$
Hopping between neighboring tight-binding sites	Transferring pairs between leads by Andreev reflection
Electric field $dk/dt = -eE$	Josephson relation $d\varphi_n/dt = 2eV_n/\hbar$
Wannier-Stark ladders	**Floquet-Wannier-Stark ladders**

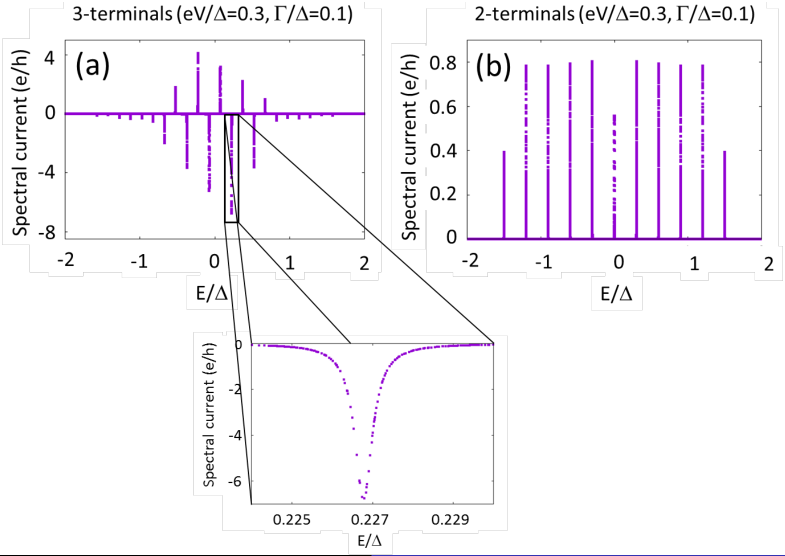
One of the Devices for (Numerical) Experiments



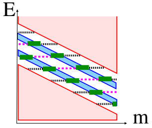
Spectral current:

Evidence for two Floquet-Wannier-Stark ladders

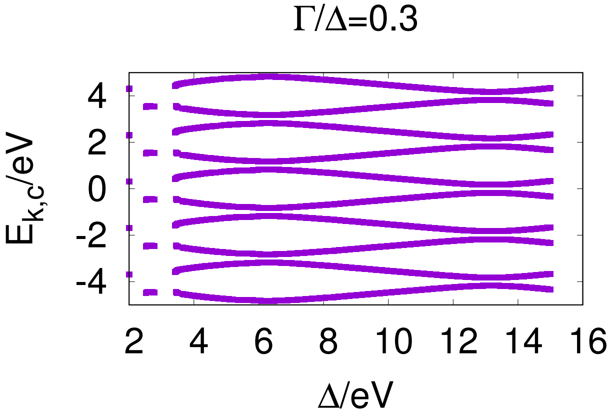
R. Mélin, J.-G. Caputo, K. Yang and B. Douçot, PRB '17



$\Gamma/\Delta = 0.3$

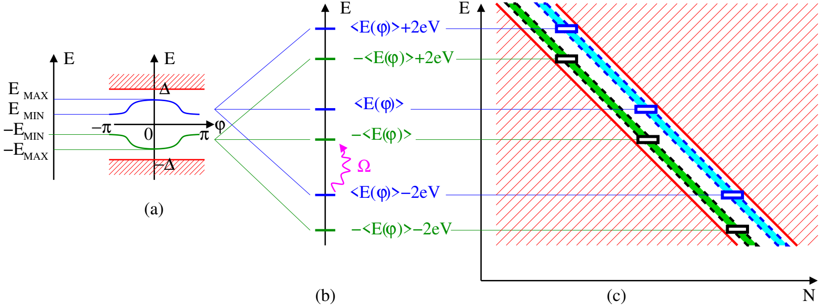


Inter-ladder tunneling for $\Delta/eV \simeq 6, 13$
$\Rightarrow$ Landau-Zener-Stückelberg transitions



Going one Step Further: Spectroscopy of Floquet-Wannier-Stark ladders

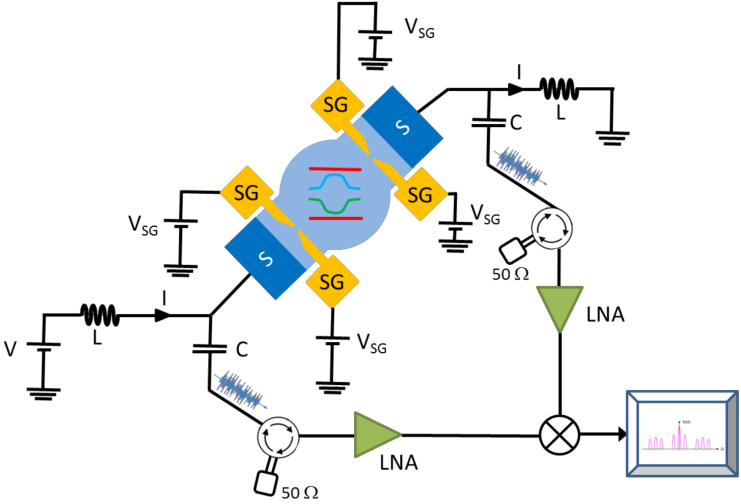
R. Mélin, R. Danneau, K. Yang, J.-G. Caputo, B. Douçot,
arXiv:1903.04889, submitted to PRB



- On-chip spectroscopy
- Finite frequency cross-correlations
  (On-going project with **Romain Danneau**, Karlsruhe)

Spectroscopy of FWS ladders with finite frequency cross-correlations

Submitted French-German ANR-DFG project
with **Romain Danneau** (KIT)



PHYSICAL REVIEW B **93**, 115436 (2016)

Gate-tunable zero-frequency current cross correlations of the quartet state in a voltage-biased three-terminal Josephson junction

Régis Mélin, Moïse Sotto, and Denis Feinberg
Université Grenoble-Alpes, Institut Néel, BP 166, F-38042 Grenoble Cedex 9, France
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(Received 26 November 2015; revised manuscript received 3 March 2016; published 25 March 2016)

A three-terminal Josephson junction biased at opposite voltages can sustain a phase-sensitive dc current carrying three-body static phase coherence, known as the "quartet current." We calculate the zero-frequency current noise cross correlations and answer the question of whether this current is noisy (like a normal current in response to a voltage drop) or noiseless (like an equilibrium supercurrent in response to a phase drop). A quantum dot with a level at energy ϵ_0 is connected to three superconductors S_a, S_b, and S_c with gap Δ, biased at $V_a = V$,

PNAS

Nonlocal supercurrent of quartets in a three-terminal Josephson junction

Yonatan Cohen[a,1], Yuval Ronen[a,1], Jung-Hyun Kang[a], Moty Heiblum[a,2], Denis Feinberg[b], Régis Mélin[b], and Hadas Shtrikman[a]

[a]Department of Condensed Matter Physics, Braun Center for Submicron Research, Weizmann Institute of Science, 76100 Rehovot, Israel; and [b]Institut Néel, CNRS, Université Grenoble-Alpes, Institute of Engineering (INP), 38000 Grenoble, France

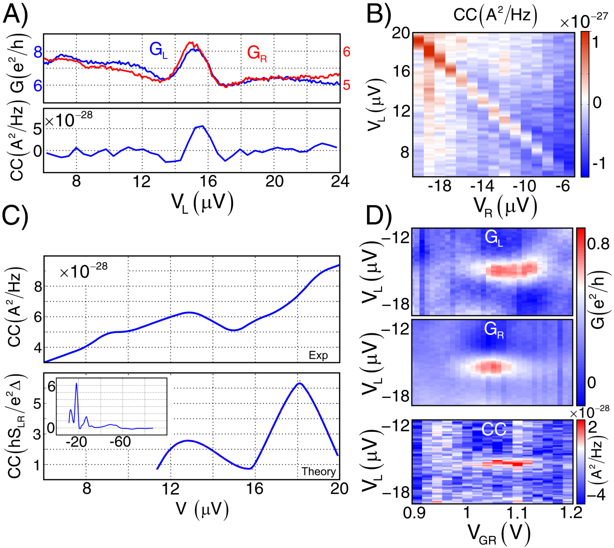
Edited by Eduardo Fradkin, University of Illinois at Urbana–Champaign, Urbana, IL, and approved May 21, 2018 (received for review January 2, 2018)

A novel nonlocal supercurrent, carried by quartets, each consisting of four electrons, is expected to appear in a voltage-biased three-terminal Josephson junction. This supercurrent results from a non-

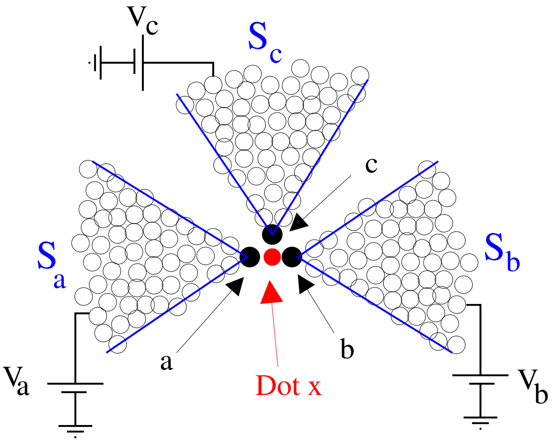
several alternative models for that current could not be ruled out. Here, we verify an emergent coherent quartet supercurrent in a 3TJ, which is formed in a proximitized semiconducting

Zero-Frequency Cross-Correlations

Y. Cohen, Y. Ronen, J.-H. Kang, M. Heiblum, D. Feinberg, R. Mélin and H. Shtrikman, PNAS 2018



One of the Devices for (Numerical) Experiments

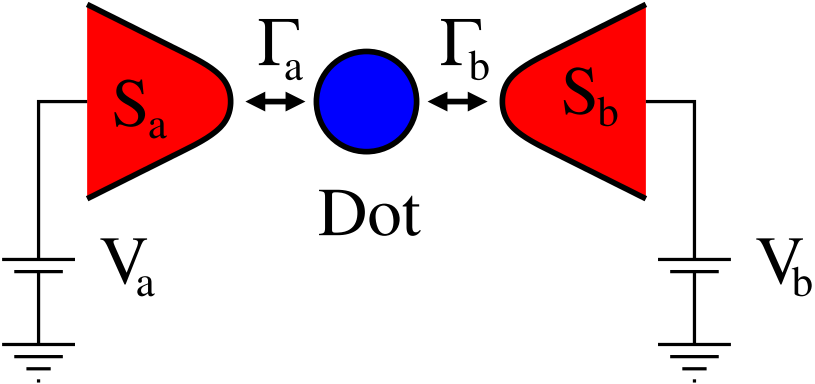


Argument showing possibility of success of Romain Danneau's experiment:

For this type of "0D" device, we have solved recently the connection between:

1. The spectrum of FWS resonances at energies E_n in the resolvant
2. The spectrum of peaks at frequency Ω in cross-correlations
3. Expected result: $\Omega_{n,m} = E_n - E_m$.

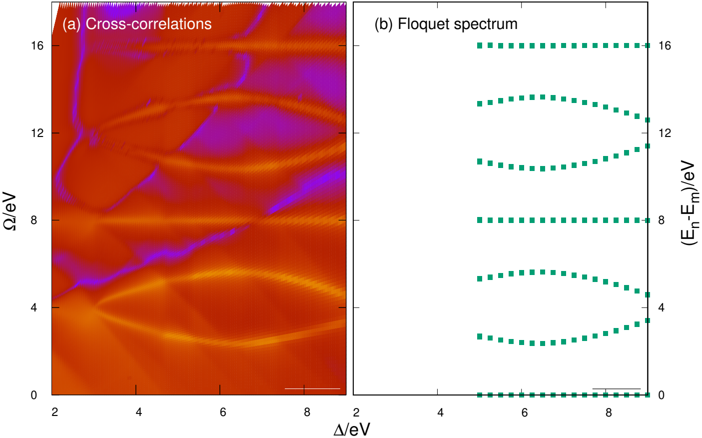
R. Mélin, R. Danneau, K. Yang, J.-G. Caputo, B. Douçot,
arXiv:1903.04889, submitted to PRB



$$\Gamma_a/\Delta = 0.4 \text{ and } \Gamma_b/\Delta = 0.2$$

Asymmetric two-terminal device (2/2)

R. Mélin, R. Danneau, K. Yang, J.-G. Caputo, B. Douçot,
arXiv:1903.04889, submitted to PRB



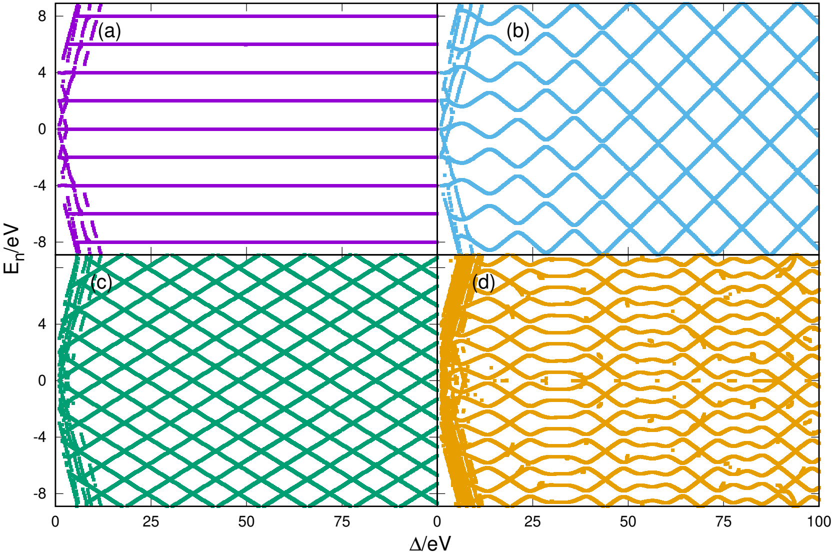
Repulsion between Floquet levels

⇒ Quantum coherence between the two FWS ladders

⇒ Towards a Floquet qu-bit

A zoo of Floquet spectra

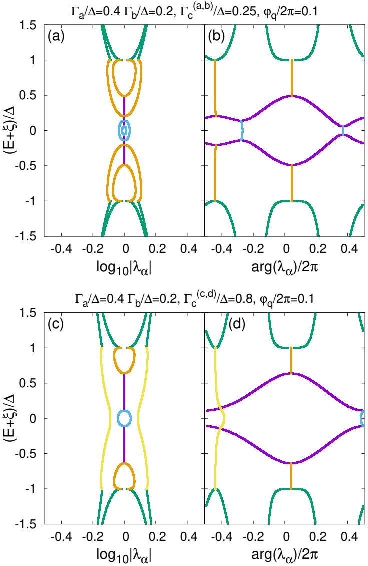
R. Mélin, R. Danneau, K. Yang, J.-G. Caputo, B. Douçot, arXiv:1903.04889, submitted to PRB



Dispersion relations

B. Douçot, R. Danneau, K. Yang, J.-G. Caputo, R. Mélin

arXiv:1904.03132, submitted to PRL



$\lambda_\alpha = \exp(ik_\alpha)$ (complexified fast superconducting phase variable)

ABS: $E + \xi = \pm E_A(k)$, $\xi = meV$
Parallel transport of BdG wave-functions along a cycle
+ periodic orbits ⇒ **Berry phase** when matching the wave-function over one period

Landau-Zener-Stückelberg tunneling paths

Leakage from dot to quasiparticle continua (transient process)

Multiple Andreev reflections to the continua

Solution of semiclassical theory for a multiterminal superconducting-quantum dot (due to Benoît Douçot)

B. Douçot, R. Danneau, K. Yang, J.-G. Caputo, R. Mélin

arXiv:1904.03132, submitted to PRL

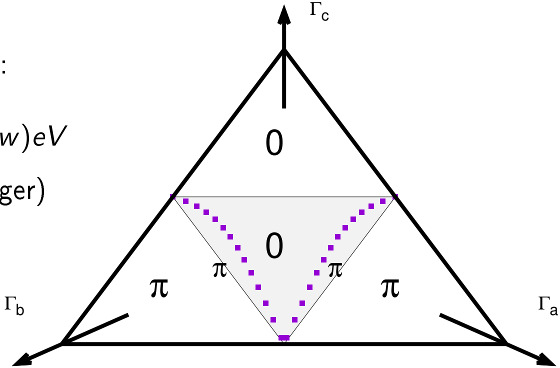
For decoupled FWS ladders:

$$E = \sigma \langle E_A \rangle - (2n + w) eV$$

winding number w (an integer)

Berry phase: $\varphi_B = \pi w$

$\varphi_B = 0$ or π

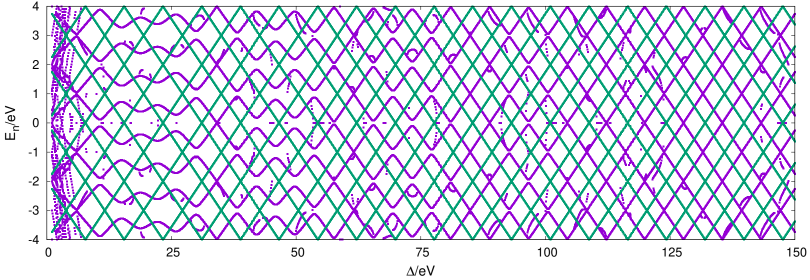


$\varphi_q / 2\pi = 0.2$

Floquet-Wannier-Stark ladders with Berry phase $\varphi_B = \pi$

B. Douçot, R. Danneau, K. Yang, J.-G. Caputo, R. Mélin

arXiv:1904.03132, submitted to PRL



- In Green: $E_n/eV = \sigma\langle E_A\rangle/eV + 2n$
- In Magenta: Maxima of the resolvent

$\Rightarrow$ Spectroscopic evidence for $\varphi_B = \pi$

$\Rightarrow$ SQUID interferometry or tunnel spectroscopy

Summary

1. Analogy with solid-state physics
   (*i.e.* Bloch oscillations and Wannier-Stark ladders)

2. Finite frequency noise spectroscopy of
   the Floquet-Wannier-Stark ladders
   $\Rightarrow$ Possible experiments are considered

3. Semiclassical theory of the Berry phase
   $\Rightarrow$ Emergence of nontrivial Berry phase $\varphi_B = \pi$

4. Effect of Berry phase on Floquet-Wannier-Stark ladders
   $\Rightarrow$ Shift of half a period of the Floquet spectra if $\varphi_B = \pi$
   $\Rightarrow$ SQUID interferometry or tunneling spectroscopy