Nonequilibrium generalization of Andreev bound states

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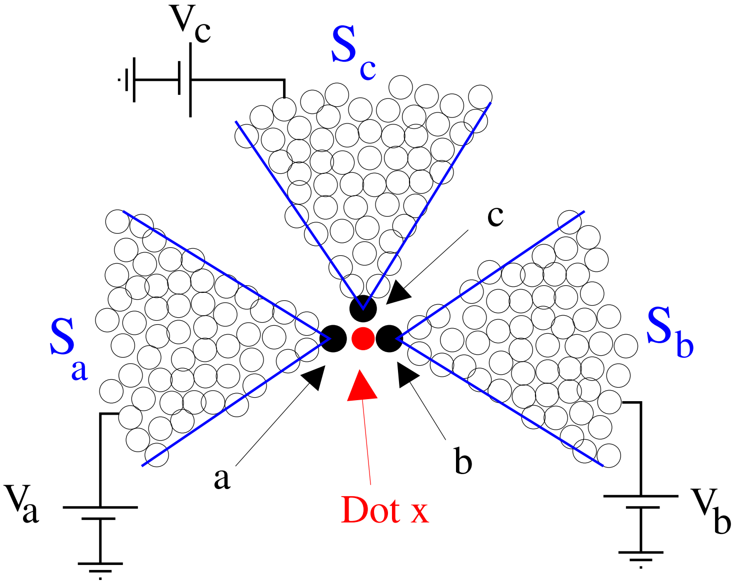
The Set-up for Numerical Experiments



Two Layers of Complexity

$\left\{ \begin{array}{l} \rightarrow \text{ Superconductivity} \\ \rightarrow \textbf{ Time-periodic Hamiltonian } (\varphi_n(t) = 2eV_n t/\hbar) \end{array} \right.$

- ⇒ **Floquet theory**
  - • *Wave-function more microscopic than Green's function*
- ⇒ Notions taken from **band theory in solids**
  - • *Tilted band picture*
- ⇒ Notion of **relaxation of Floquet states**
- ⇒ Experimental consequences for **spectroscopy**

Solid state physics	**Superconductivity**
Tight-binding model	Tight-binding model
Hopping term $n \rightarrow n \pm 1$	$N_b - N_a \rightarrow N_b - N_a \pm 2$ (2T)
Wave-vectors	Superconducting phases
Brillouin zone $-\pi < k_x, k_y < \pi$	Brillouin zone $-\pi < \varphi_a, \varphi_b < \pi$
Electric field $dk/dt \propto E$	Josephson relation $d\varphi_n/dt = 2eV_n/\hbar$
Bloch oscillations	Bloch oscillations
Topology	Topology

Collaborators on this Project

Jean-Guy Caputo:
Laboratoire de Mathématiques, INSA de Rouen
The one who optimized my codes and
provided access to the Rouen computing platform.

Kang Yang:
Laboratoire de Physique Théorique et des Hautes Energies, UPMC
Laboratoire de Physique des Solides, Orsay
The one who made his Master1 Internship at the time where everything
was unclear, and who is now making semi-classics.

Benoît Douçot:
Laboratoire de Physique Théorique et des Hautes Energies, UPMC
The one who found interpretation in terms of
Floquet-Wannier-Stark-Andreev ladders.

R. Mélin, J.G.- Caputo, K. Yang and B. Douçot, Phys. Rev. B '17

Rotating-Wave Approximation for a Periodically Driven Qu-Bit

$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'(t)$

- $\mathcal{H}_0 = \frac{\Delta}{2}\hat{\sigma}^z$
- $\mathcal{H}'(t) = \omega_R \cos(\Omega t)\hat{\sigma}^x$

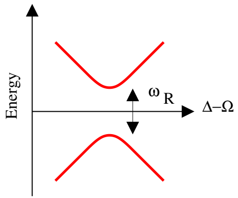
$$\mathcal{H}'_I(t) = \omega_R \cos(\Omega t)\left[\hat{\sigma}_I^+(t) + \hat{\sigma}_I^-(t)\right] \quad (1)$$

$$= \omega_R \cos(\Omega t)\left[e^{i\Delta t}\hat{\sigma}^+ + e^{-i\Delta t}\hat{\sigma}^-\right] \quad (2)$$

Rotating-wave approximation:

$\mathcal{H}'_I(t) \to \mathcal{H}_{RWA}$, with $\mathcal{H}_{RWA} = \frac{\omega_R}{2}\left[e^{i(\Delta-\Omega)t}\hat{\sigma}^+ + e^{i(\Omega-\Delta)t}\hat{\sigma}^-\right]$

At resonance ($\omega = \Delta$): $\mathcal{H}_{RWA} = \frac{\omega_R}{2}\left[\hat{\sigma}^+ + \hat{\sigma}^-\right] = \frac{\omega_R}{2}\hat{\sigma}^x$



Spin-1/2 in static magnetic field:

$\mathcal{H}_{RWA} = -\mathbf{h}_{eff}.\hat{\sigma}$, with

$$\mathbf{h}_{eff} = -\left(\frac{\Delta - \Omega}{2}\right)\hat{\mathbf{z}} - \frac{\omega_R}{2}\hat{\mathbf{x}} \quad (3)$$

Beyond the Rotating Wave Approximation

$2\pi/\Omega$ time-periodic Hamiltonian $\Rightarrow$ We apply time Bloch theorem

$$|\psi_\alpha(t)\rangle = e^{-iE_\alpha t/\hbar}|\chi_\alpha(t)\rangle \text{ (with } \alpha = 1, 2)$$

- $|\psi_\alpha(t)\rangle$ = two-component wave-function of the two-level system
- $|\chi_\alpha(t)\rangle$ = periodic with period $T = 2\pi/\Omega$

Then we have the following expansion:

$$|\chi_\alpha(t)\rangle = \sum_{m=-\infty}^{+\infty} e^{-im\Omega t}|\chi_m^{(\alpha)}\rangle$$

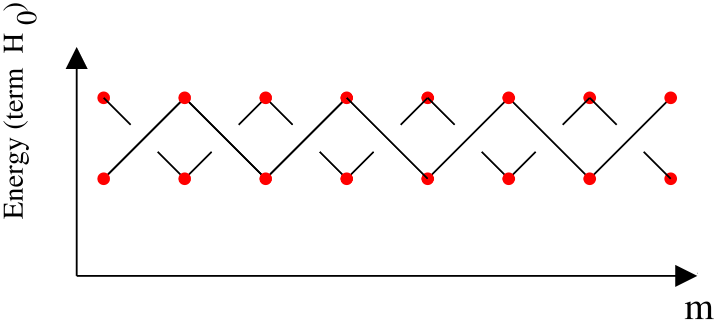
Schrödinger equation $i\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{\mathcal{H}}(t)|\psi(t)\rangle$

$\Rightarrow$ infinite number of equations (due to time periodicity)

$$(E_\alpha + m\Omega)\,|\chi_m^{(\alpha)}\rangle = \frac{\Delta}{2}\hat{\sigma}^z|\chi_m^{(\alpha)}\rangle + \frac{\omega_R}{2}\hat{\sigma}^x|\chi_{m+1}^{(\alpha)}\rangle + \frac{\omega_R}{2}\hat{\sigma}^x|\chi_{m-1}^{(\alpha)}\rangle,\ \forall\ m$$

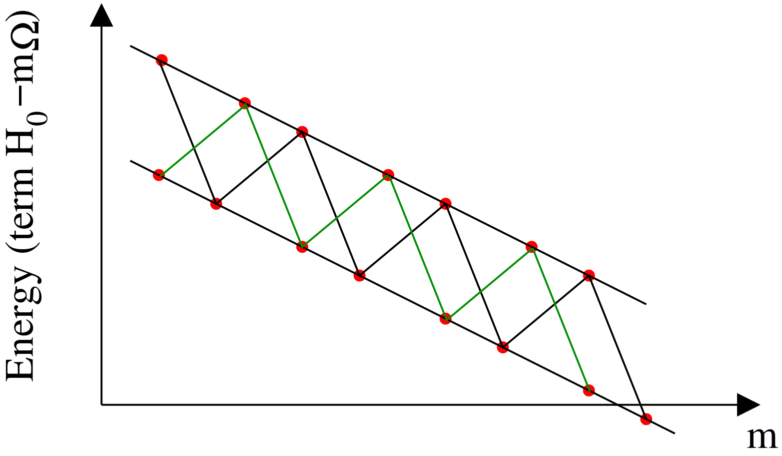
$\Rightarrow$ Tight-binding Hamiltonian on a **ladder**

Tight-Binding Hamiltonian in the m Representation



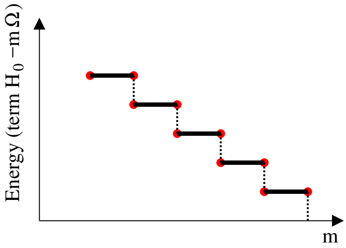
Tight-Binding Hamiltonian in the m Representation

"$m\Omega$" term $\Rightarrow$ Better representation of the Hamiltonian involves a tilt in the (m, E) plane:



Meaning of the Rotating Wave Approximation in the m Representation

$\Omega = \Delta \Rightarrow$ Horizontal steps. For a single ladder:



Vertical couplings are neglected in the rotating wave approximation
$\Rightarrow$ Infinity of horizontal dimers

$$\begin{cases} n \to n \Rightarrow \text{DC} \\ n \to n+1 \Rightarrow \text{Oscillations with fundamental frequency } \Omega \\ \text{No oscillations with frequencies } 2\Omega,\ 3\Omega,\ \ldots \\ \text{in the rotating wave approximation} \end{cases}$$

Limit $\Omega \to 0$ in the ϕ-Representation (No Tilt)

Plane waves: $|\chi_m^{(\phi)}\rangle = e^{im\phi}|\chi^{(\phi)}\rangle$

ϕ is conjugate to m

We recover the Hamiltonian with a frozen phase:

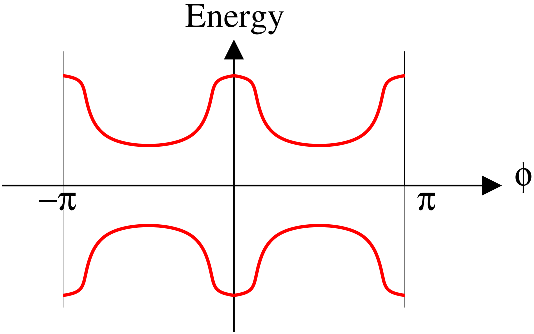
$$E_\phi|\chi^{(\phi)}\rangle = \left(\frac{\Delta}{2}\hat{\sigma}^z + \omega_R cos(\phi)\hat{\sigma}^x\right)|\chi^{(\phi)}\rangle$$

$$\left\{\begin{array}{l} k \text{ in band theory} \leftrightarrow \phi \text{ in this problem} \\ k \in [0, 2\pi] \text{ (Brillouin zone)} \leftrightarrow \phi \in [0, 2\pi] \text{ (phase modulo } 2\pi) \\ dk/dt \propto \text{electric field} \leftrightarrow \text{Brillouin zone swept at constant velocity} \end{array}\right.$$

Band structure:

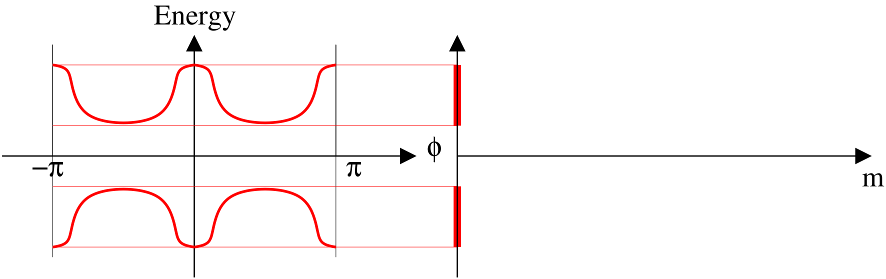
$$\left[\begin{array}{cc} \Delta/2 & \omega_R \cos\phi \\ \omega_R \cos\phi & -\Delta/2 \end{array}\right] = \hat{\mathcal{H}}(\phi) \Rightarrow E_\pm(\phi) = \pm\sqrt{\frac{\Delta^2}{4} + \omega_R^2 \cos^2\phi}$$

$\Omega \to 0$ **First step: Band structure**



$(\Omega \ll \omega_R)$ **Second step: Projection on the energy axis**

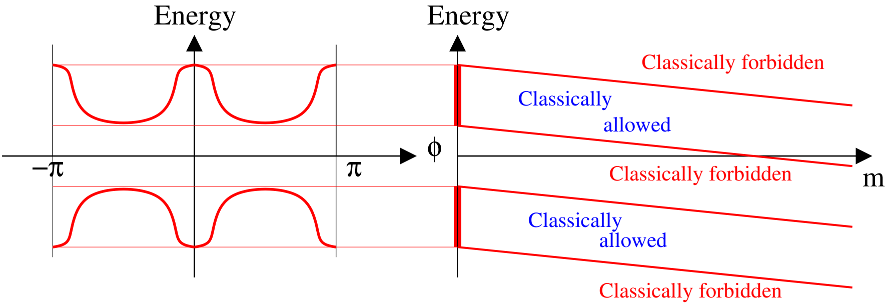
$[\hat{m}, \hat{\phi}] = 0$ (classical limit)



Combining the ϕ- and m-Representations

($\Omega \ll \omega_R$) **Third step: Introducing the tilt**

Still $[\hat{m}, \hat{\phi}] = 0$



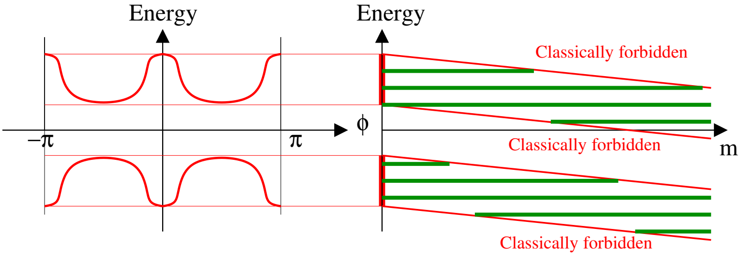
$(\Omega \ll \omega_R)$ **Fourth step: Re-quantization** (Ω play the role of usual $\hbar$)

Use $\hat{m} = -i\partial/\partial\phi$ (e.g. $[\hat{m}, \hat{\phi}] = 0$)

$\Rightarrow$ wave-function solution of a first-order differential equation

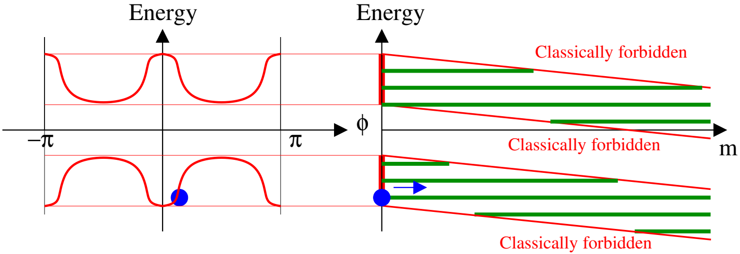
2π-periodicity on wave-function $\Rightarrow$ discrete energy levels

$\Rightarrow$ **Floquet-Wannier-Stark ladders**



If $\Omega \to 0$, extent of wave-functions along the m axis becomes very large $\Rightarrow$ Emergence of a large number of harmonics and strong deviations from rotating wave approximation

Bloch oscillations



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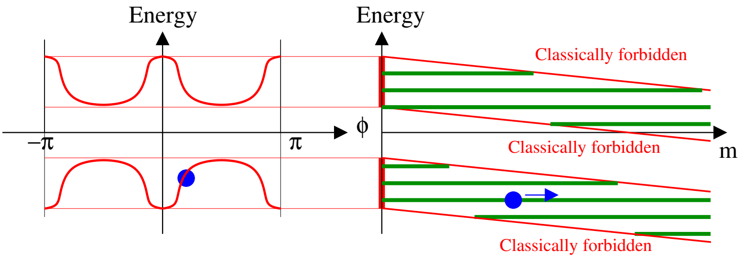

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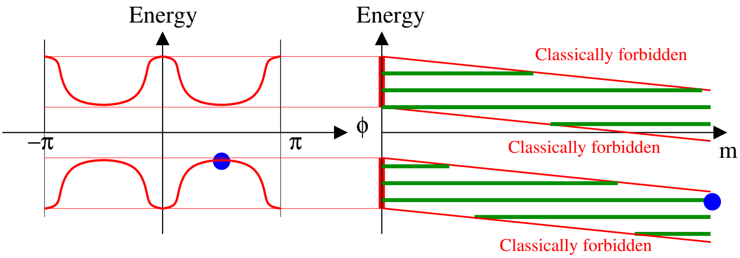

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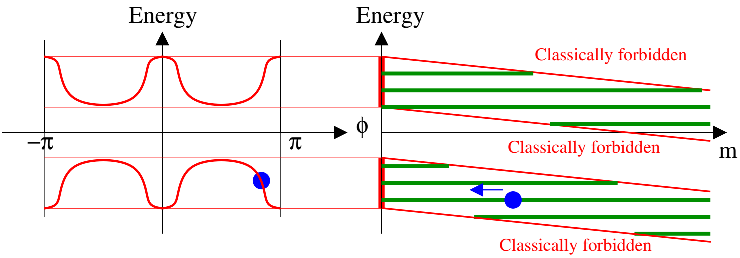

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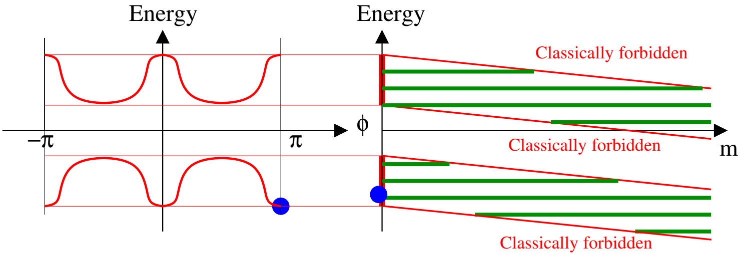


Bloch oscillations

- Bloch oscillations are not observed in metals because of inelastic collisions

$\Rightarrow$ **Semiconductor superlattices:**

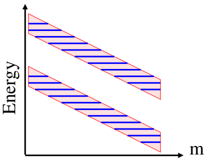
- Brillouin zone $[-\pi/a, \pi/a]$, with a enhanced by about a factor 1000 compared to a metal

$\Rightarrow$ Period of oscillations much shorter than inelastic scattering time.

- "*Self-diffracted four-wave mixing experiment*" '92

$\Rightarrow$ Bloch oscillations also observed in cold atoms experiments

Combining the ϕ- and m-Representations

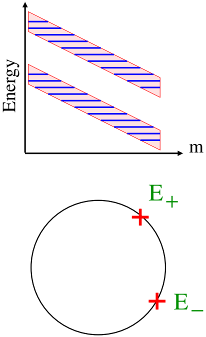


Energies on the circle (1/2)

- Let $(|\chi_m\rangle)_m$ be a solution for energy E_α
- Let us define $|\chi'_m\rangle = |\chi_{m-1}\rangle$
- Then $|\chi'_m\rangle$ is solution for $E'_\alpha = E_\alpha - \Omega$
- $e^{-i(E_\alpha-\Omega)t} e^{-i(m-1)\Omega t} = e^{-iE_\alpha t} e^{-im\Omega t}$

$\Rightarrow$ Same global wave-function in spite of different levels on the ladder

$\Rightarrow$ Energies E_α and $E_\alpha - \Omega$ should be identified



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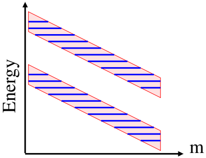
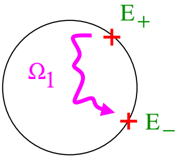
Energies on the circle (2/2)

Meaningless to say that $E_+ > E_-$ or $E_- < E_+$

Combining the ϕ- and m-Representations





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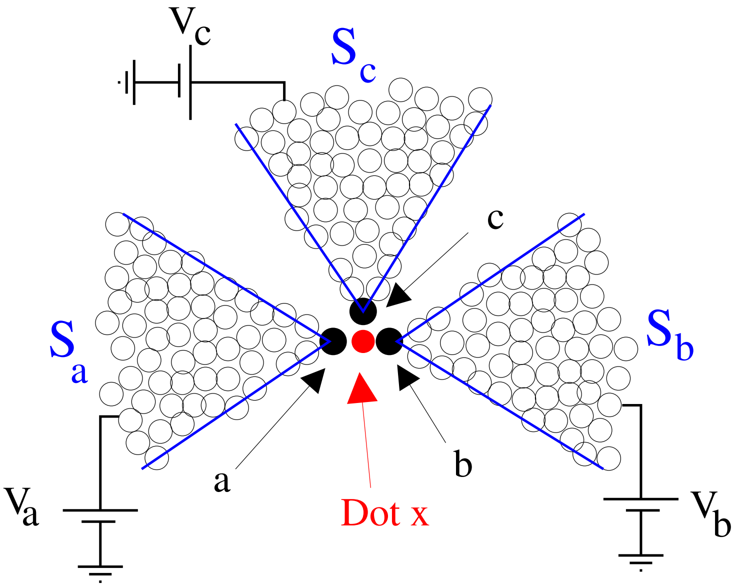
$\Rightarrow$ Energies E_α and $E_\alpha - \Omega$ should be identified

$\Rightarrow$ Spectroscopy experiments

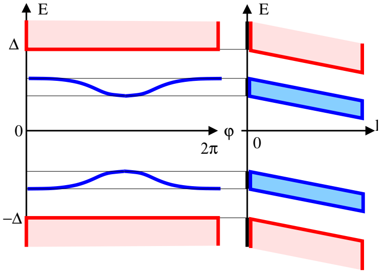
$\Omega_1 = \pm(E_+ - E_-) + n\Omega \Rightarrow$ Future developments with two independent frequencies for forthcoming three-terminal Josephson junctions. Goal: To motivate experiments in the group of Caglar Girit (Collège de de France) and Romain Danneau (Karlsruhe)

The Set-up for Numerical Experiments



Floquet-Wannier-Stark-Andreev Resonances (2 Terminals)

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_a + \hat{\mathcal{H}}_b + \hat{\mathcal{H}}_{a-b} - eV\left(\hat{N}_a - \hat{N}_b\right) \;,\; \left[\hat{N}_a - \hat{N}_b, \frac{\hat{\varphi}_a - \hat{\varphi}_b}{2}\right] = i$$



Two uncoupled FWS-Andreev bands:
$\hat{H}_\pm = E_\pm(\hat{\varphi}) - 2eV\hat{I}$, with
$\hat{I} = (\hat{N}_a - \hat{N}_b)/2$ (auxiliary variable)

Steady state $\Rightarrow$
$\hat{H}_\pm|\psi_\pm\rangle = E_\pm|\psi_\pm\rangle \Rightarrow$

$$\left[E_\pm(\hat{\varphi}) - 2eV\hat{I}\right]|\psi_\pm\rangle = E_\pm|\psi_\pm\rangle$$

with $I = i\partial/\partial\varphi$ (e.g. $[\hat{I},\hat{\varphi}] = i$)

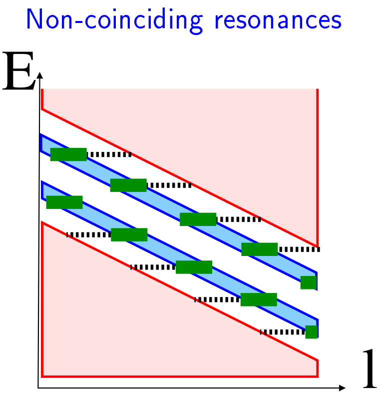
$\Rightarrow$ **First order differential equation for wave-function**

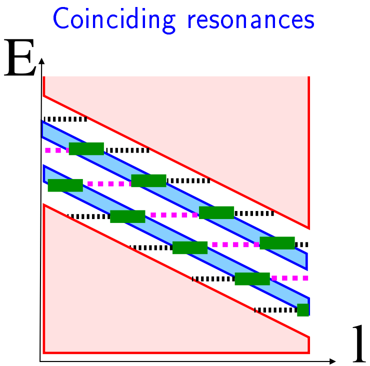
Imposing 2π-periodicity in φ leads to quantized energy levels:

$$\left\{ \begin{array}{l} E_j = 2eVj + \langle E\rangle \\ E'_{j'} = 2eVj' - \langle E\rangle \end{array} \right. \;,\; \text{with } \langle E\rangle = \frac{1}{2\pi}\int_0^{2\pi} E_+(\varphi)d\varphi$$

$\Rightarrow$ **Two Floquet-Wannier-Stark-Andreev ladders**

Floquet-Wannier-Stark-Andreev Ladders





- Tunneling between ladders and continua

$\Rightarrow$ Finite width of FWS-Andreev resonances

- Tunneling between ladders and continua
- Inter-ladder tunneling

$\Rightarrow$ Landau-Zener-Stückelberg transitions

Differences Between 2 and 3 Terminals

- Ladders parameterized by the quartet phase φ_Q
  $\Rightarrow$ Level crossings as a function of φ_Q
- Multiple Andreev Reflections become Phase-sensitive Multiple Andreev reflections $\Rightarrow$ Interference process in the tunnel effect

A single picture for four phenomena:

- Width of resonances
- Landau-Zener-Stückelberg transitions
- Phase-sensitive Multiple Andreev Reflections
- Houzet-Samuelsson thresholds

Experimental Consequences / Numerical Experiments

We want to suggest new experiments on the spectroscopy of those ladders:

$\Rightarrow$ Variation of the resonance energies with voltage

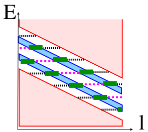
$\Rightarrow$ Variation of the width with voltage

We want to understand mechanisms for the width of the resonances:

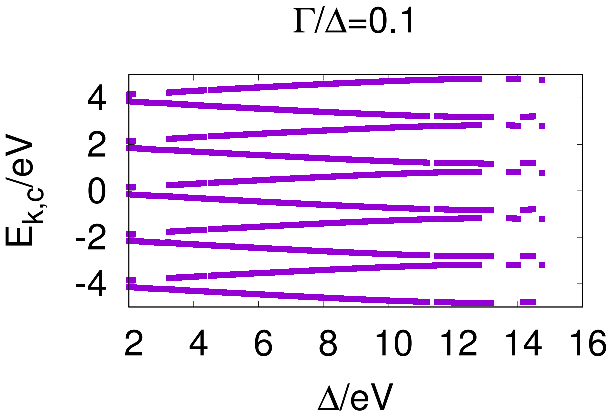
- Equilibration with quasiparticle semi-infinite continua
- Electron-phonon scattering

We want to determine connections between spectrum and DC-transport and noise:

- Same energy scales in spectrum and DC-transport ?
- Connection with DC-current
- Connection with noise

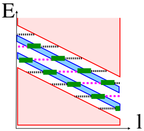


Inter-ladder tunneling for $\Delta/eV \simeq 14$
$\Rightarrow$ Landau-Zener-Stückelberg transitions



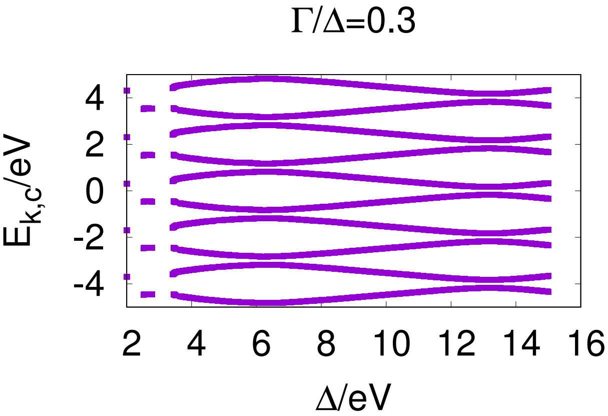
Floquet-Wannier-Stark-Andreev Resonances (2/2)

$\Gamma/\Delta = 0.3$



Inter-ladder tunneling for $\Delta/eV \simeq 6, 13$

$\Rightarrow$ Landau-Zener-Stückelberg transitions



"Kronig-Penny model" $\mathcal{H}(N,f) = \frac{d^2}{dx^2} - \sum_{n=1}^{N} \delta(x-na) + fa^{-1}x$

Non-hermitian effective Hamiltonian at level crossing:

$$\mathcal{H}_{eff} = \begin{pmatrix} E+i\epsilon & X \\ X & E'+i\epsilon' \end{pmatrix} \text{ with } \epsilon \ll \epsilon'$$

One resonance strongly coupled to continum (ϵ'),
the other weakly coupled (ϵ)

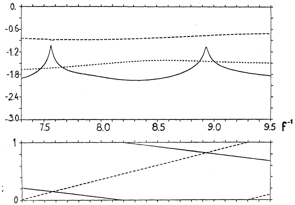


FIG. 1. f^{-1} behavior of three resonances followed by ontinuity: (short-dashed curve) $E_{1,J}(f)$ is a resonance first ladder and first rung (or first region); (solid irve) $E_{1,J'}(f)$ is a resonance in first ladder second ing (or second region); (long-dashed curve) $E_{2,J''}(f)$ a resonance in second ladder first rung (or second egion).

Weak coupling between resonances: $|X| \ll \epsilon'$

System dominated by dissipation:
no level repulsion

$$\begin{cases} \ln|\mathrm{Im} E_i| & \text{flat for level strongly coupled to continuum} \\ \ln|\mathrm{Im} E_i| & \text{has peak for level weakly coupled to continuum} \end{cases}$$

Wannier resonances (2/2)

Bentosela, Grecchi and Zironi, PRL '83

"Kronig-Penny model" $\mathcal{H}(N, f) = \frac{d^2}{dx^2} - \sum_{n=1}^{N} \delta(x - na) + fa^{-1}x$

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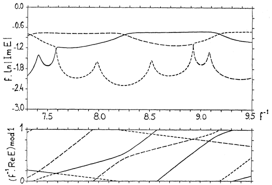

FIG. 2. f^{-1} behavior of three resonances in the third region followed by continuity. Each one goes through all the first three ladders by type (2) crossings.

85

Strong coupling between resonances

$|X| \gg \epsilon'$

Quasi-hermitian system

- Level repulsion for real parts
- Equality for imaginary parts

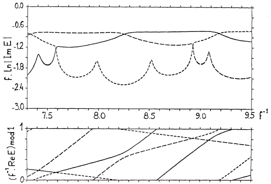

FIG. 2. f^{-1} behavior of three resonances in the third region followed by continuity. Each one goes through all the first three ladders by type (2) crossings.

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Strong coupling between resonances

$$|X| \gg \epsilon'$$

Quasi-hermitian system

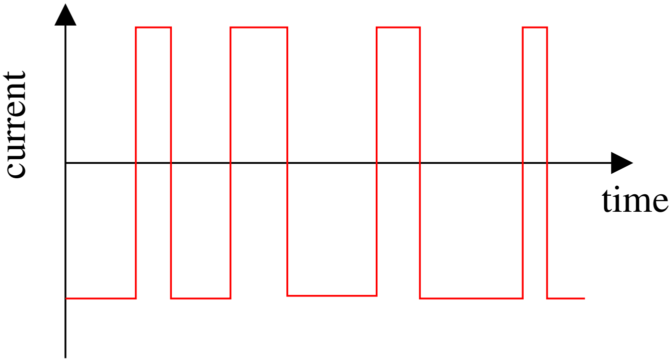
- Level repulsion for real parts
- Equality for imaginary parts

- Realizing numerically (and also experimentally) a "Floquet-qu-bit" on this principle
- Manipulations with NMR pulses
- Density matrix theory from first principle Green's function calculations
- Quantum trajectories

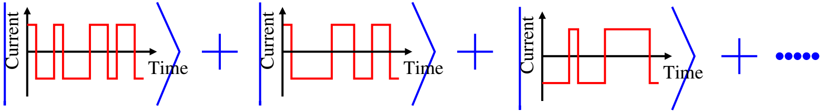
Still a lot of open questions on current-current cross-correlations ...

Relation with Noise Cross-Correlation Experiments

Thermal noise in a two-terminal point contact at equilibrium:



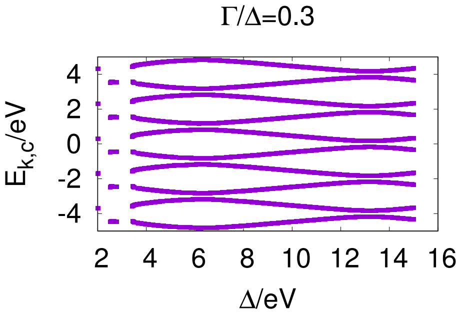
Possible Emergence of Schrödinger cats of Cooper pairs

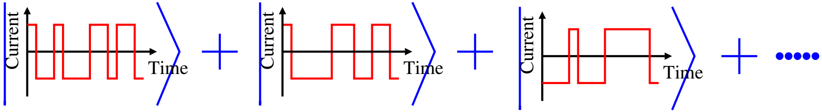


Correlation time for sign of current$=\hbar/\delta_0$

δ_0=Level splitting at avoided crossings

FWS-Andreev spectrum as function of Δ/eV:

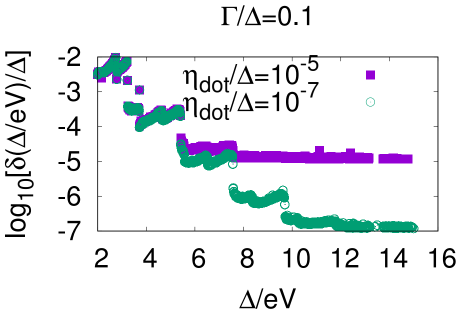




Correlation time for absolute value of current$=\hbar/\delta$

δ=width of Floquet-Wannier-Stark-Andreev resonances

log[**line-width broadening**]**(Δ/eV):**



Meaning of the Dynes parameter η_{dot}

Emergence of exponentially small energy scales

$\Rightarrow$ Those should be compared to lots of things
$\Rightarrow$ Dynes parameter as a source of extrinsic relaxation

Dynes parameter

Dynes, Narayanamurti, Garno, Phys. Rev. Lett. **41**, 1509 (1978)

Strong-coupling superconductor $Pb_{0.9}Bi_{0.1}$

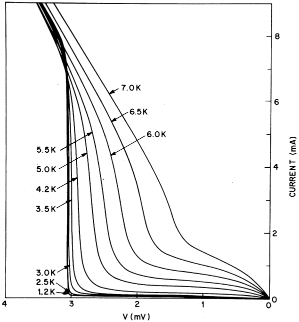


FIG. 1. I-V characteristic for a $Pb_{0.9}Bi_{0.1}$ tunnel junc-

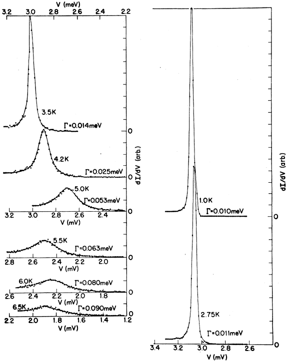


FIG. 2. dI/dV vs V determined from the data using Fig. 1. The solid curves are fits to the data using Eq. (2) with Γ an adjustable parameter.

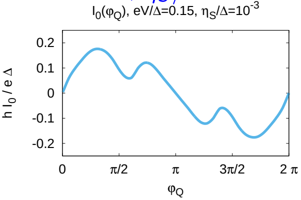
1511

$$I = G_N \int_{-\infty}^{+\infty} \rho(E)\rho(E+V)\left[f(E) - f(E+V)\right] dE$$

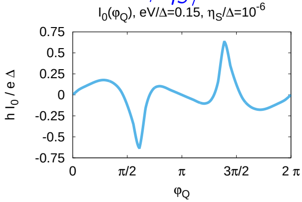
$$\rho(E,\Gamma) = (E - i\Gamma)/\left[(E-i\Gamma)^2 - \Delta^2\right]^{1/2}$$

Double quantum dot: Four Floquet-Wannier-Stark-Andreev ladders

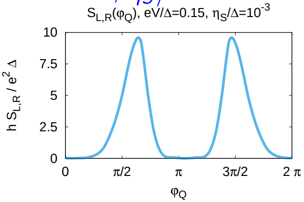
Current, $\eta_S/\Delta = 10^{-3}$



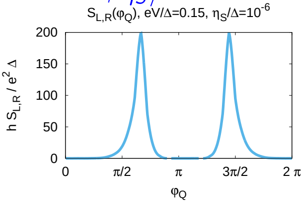
Current, $\eta_S/\Delta = 10^{-6}$



Noise, $\eta_S/\Delta = 10^{-3}$



Noise, $\eta_S/\Delta = 10^{-6}$



Interpretation: Avoided crossings between Floquet-Wannier-Stark-Andreev ladders tuned by quartet phase

Emergence of a tiny η_S^*/Δ

Connection With RF-Irradiated Josephson Junctions (2/2)

Bergeret, Virtanen, Ozaeta, Heikilä, Cuevas,
Phys. Rev. B **84**, 054504 (2011)

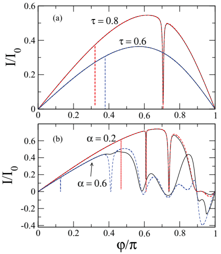


FIG. 5. (Color online) (a) The CPR for $\alpha = 0.1, \hbar\omega = 0.6\Delta$, and two values of the transmission coefficient, $\tau = 0.8$ and $\tau = 0.6$. (b) The CPR for $\hbar\omega = 0.3\Delta$, $\tau = 0.95$, and two values of α, 0.2 and 0.6. In both panels the solid lines correspond to the microscopic theory and the dashed lines to the two-level model.

Recall also following paper:
Voltage-induced Shapiro steps
in a superconducting multiterminal structure
J.C. Cuevas and H. Pothier, Phys. Rev. B **75**, 174513 (2007)

Motivations for Introducing η_{dot}

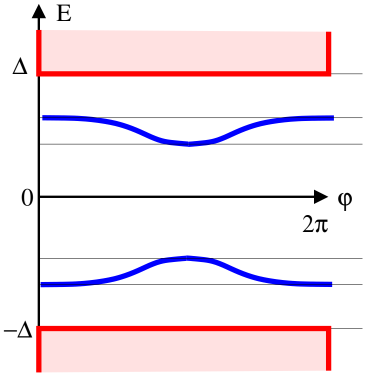
- Dynes parameter η_{dot} on the quantum dot
- Dynes parameter η_S in superconductors

η_{dot} has much stronger influence on current than η_S

Considered scenario for η_{dot}: Electron-phonon scattering

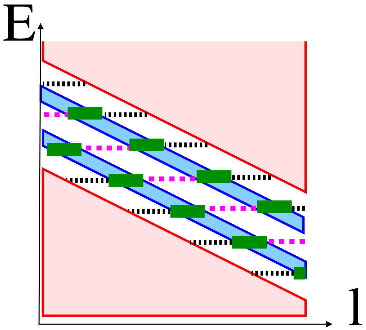
Spectrum ↔ Transport

Equilibrium
Andreev bound states



$$I = -\frac{2e}{\hbar}\,\frac{d}{d\varphi}E(\varphi)$$

Nonequilibrium
Resonances



$I = ???$

Spectrum and Self-Induced Rabi Resonances

1) Two Floquet-Wannier-Stark-Andreev ladders:

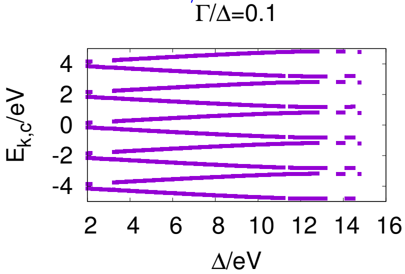
$$\left\{ \begin{array}{l} E_j = 2eVj + \langle E \rangle \\ E'_{j'} = 2eVj' - \langle E \rangle \end{array} \right. , \text{ with } \langle E \rangle = \frac{1}{2\pi} \int_0^{2\pi} E(\varphi) d\varphi.$$

2) Self-induced Rabi resonance whenever $E_j = E'_{j'} \Rightarrow$

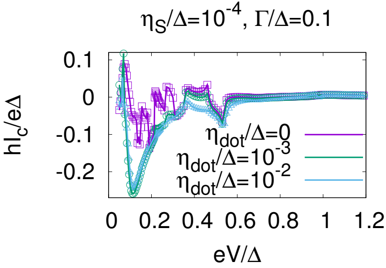
$$2eVj + \langle E \rangle = 2eVj' - \langle E \rangle \Rightarrow eV_k = \frac{\langle E \rangle}{k} , \text{ with } k = j' - j$$

Spectrum $\leftrightarrow$ Transport: Current $I_c(eV/\Delta)$

Spectrum
x-axis=Δ/eV
Avoided crossing
at $\Delta/eV \simeq 14$

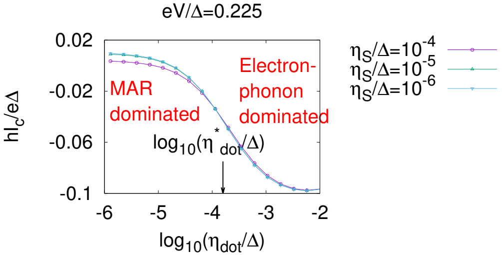


Transport, current I_c
x-axis=eV/Δ
Maximum in $|I_c|$
at $eV/\Delta \simeq 0.1$



- Possible explanations for difference between the two:
  - Transport couples also to Floquet wave-function and populations
  - Two cross-over values evaluated with different methods
- Ultra-sensitivity on tiny η_{dot}/Δ
  $\Rightarrow$ Additional energy scale η^*_{dot}/Δ

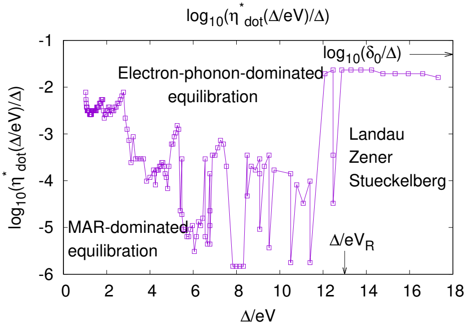
Energy Scale η^*_{dot}/Δ in Current (1/3)



- $\log_{10}(\eta^*_{dot}/\Delta)$ defined as inflection point on those curves
- $\hbar/\eta^*_{dot}$ is intrinsic characteristic time
- Important effect of η_{dot}/Δ (change of sign in current I_c)

- Much stronger effect of η_{dot}/Δ than η_S/Δ
- Possible experimental relevance of new regime $\eta_{dot} \gg \eta^*_{dot}$ in which quantum dot degrees of freedom are nonequilibrated with quasiparticle continua

$\log[\eta^*_{dot}/\Delta]$ as function of Δ/eV



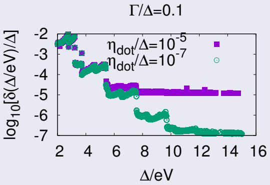
Remarkably:

Spectrum ↔ current relation holds qualitatively (but not exactly).

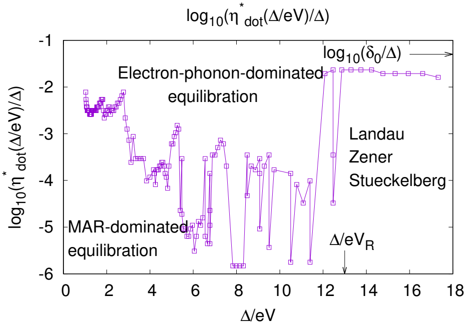
Namely:

Exponentially small energy scales in current and in line-width broadening

log[line-width broadening]



$\log[\eta^*_{dot}/\Delta]$ as function of Δ/eV



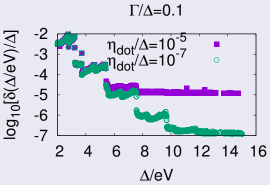
Interpretation:
Relaxation due to resonant coupling to the gap edges at the thresholds of multiple Andreev reflections (like Houzet-Samuelsson thresholds)

Remarkably:

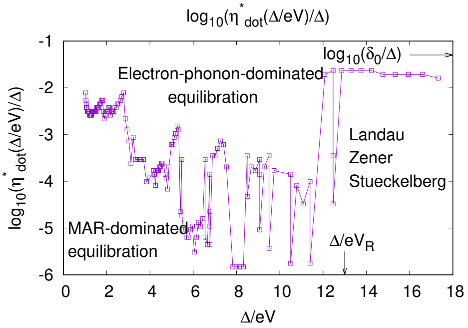
Spectrum ↔ current relation
holds qualitatively
(but not exactly).

Namely:

log[line-width broadening]



$\log[\eta_{dot}/\Delta]$ as function of Δ/eV



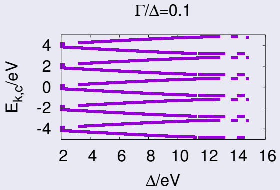
Remarkably:

Spectrum $\leftrightarrow$ current relation holds qualitatively (but not exactly).

Namely:

Compatible with Landau-Zener-Stückelberg resonance splitting δ_0

FWS-Andreev spectrum



Conclusions (1/2)

Two complementary points of view:

1) *From the point of view of superconductivity:*

- Quartets current
- Positive current cross-correlations
- Interpretation of experiments
  with Green's function calculations

2) *From the point of view of time-periodic Hamiltonians:*

- Floquet theory
- First generalization of Andreev states to nonequilibrium
- Avoided crossings between
  Floquet-Wannier-Stark-Andreev resonances
- Continua of quasiparticles for a three-terminal Josephson junction, but not for a driven qu-bit.

Conclusions (2/2)

Three relevant low-energy scales:

1) Line-width broadening of Floquet-Wannier-Stark-Andreev resonances
2) Resonance level splitting at avoided crossings of Floquet-Wannier-Stark-Andreev resonances
3) Cross-over Dynes parameter η_S^*/Δ or η_{dot}^*/Δ

New predictions for spectroscopy experiments

Qualitative connection between spectrum and transport:
even in presence of strong effect of weak relaxation

Interesting perspective on quantum thermodynamics:
In infinite-gap limit, no entropy flows from dot to superconducting leads $\Rightarrow$ Interest of investigating heat transport, and, maybe, in connection with entanglement of quartet state

Interesting perspective on semi-classics:
Kang Yang and Benoît Douçot are now developing semi-classical theory on the basis of the Floquet-Wannier-Stark-Andreev viewpoint $\Rightarrow$ Analytical results