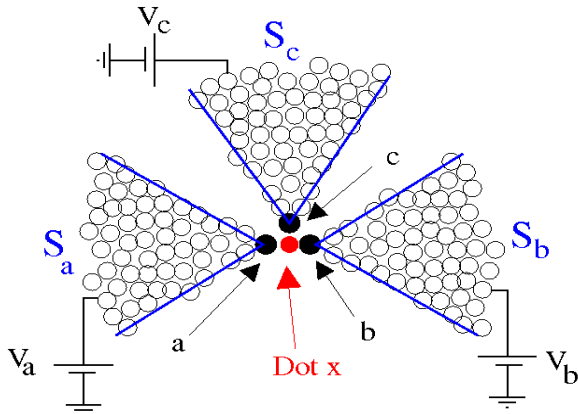# Wannier Resonances and Two-Frequency Problems in Three-Terminal Josephson Junctions

Master 2 Internship and PhD Thesis Projects

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A three-terminal Josephson junction, in which a quantum point contact x is connected to three superconductors.

Introduction : Since the beginning of the 1960s, the Josephson effect has focused interest of the condensed matter physics community. It lead to major applications in the field of quantum information and technologies. The Josephson effect appears when two superconductors are connected by a non-superconducting material, and the underlying mechanism is related to the formation of Andreev bound states. These Andreev doublets produced by the proximity effect appear in the gap, and they are sensitive to the superconducting phase. The Andreev bound states can be seen as two-level systems, and they are due microscopically to the formation of pairs of entangled electrons in the normal conductor. Many properties of the Josephson effect, such as the value of the current, depend on the energy-phase relation of the Andreev bound states at zero bias voltage.

A deep analogy appears between band theory in solid state physics and Josephson junctions. For instance, the superconducting phase between 0 and 2π plays the role of the wave-vector in the first Brillouin zone. On the other hand, the acceleration of the electronic wave-vector due to an electric field (which leads to Bloch oscillations [Zener1934]) is similar to the Josephson relation for the time evolution of the superconducting phase variable. Josephson junctions are thus expected to produce effects similar to those of solid state physic, like spectral properties [Wannier1960] of Bloch oscillations [Zener1934], or topology-related effects.

Context : Our goal since several years has been to determine the fate of the equilibrium Andreev bound states in the presence of bias voltage. We have developed recently [Mélin2017] numerical and analytical calculations for three-terminal superconductor – quantum point contact – superconductor junctions (see figure above). We have developed an analogy between the nonequilibrium dc Josephson effect and notions from band theory. The nonequilibrium Andreev bound states look like "ladders of Wannier-Stark resonances" in semi-conducting superlattices [Wannier1960], which were observed by optical spectroscopy in the end of the 1980s [Mendez1988].

We have discovered two new effects on the basis of numerical calculations for a time-periodic Hamiltonian: correlations between Cooper pairs [Freyn2011] which were already demonstrated experimentally [Cohen2017]. Second, Wannier-Stark ladders [Mélin2017] are the subject of an on-going experimental work in the group of Romain Danneau in Karlsruhe. Methods more general than those relying on time periodicity will have to be developed in order to address two-frequency problem, for instance to sweep the Brillouin zone associated to superconducting phases in order to calculate a Chern number [Riwar2016]. The PhD thesis project aims at exploring such problems with two independent frequencies, which can be in a commensurate or incommensurate ratio.

Work program for the internship : A simpler problem will be proposed for the internship, so that the student can become acquainted with recursive Green's functions, in connection with an on-going experiment in Grenoble on the role of nonequilibrium quasiparticle populations injected by the tip of a scanning probe microscope in a two-terminal Josephson junction. Results on this problem are expected within a short period of time.

Work program for the PhD thesis : The first project of the PhD thesis will be to justify the Markovian approximation by testing it against numerically exact calculations that we master well [Mélin2017]. Once the method will have been tested, it will be proposed to couple a two or three-terminal Josephson junction to ac voltages, in order to excite transitions between Wannier-Stark levels belonging to different sub-bands. Two approaches will be developed: calculations based on the density matrix in the Markovian approximation which will be compared to the rotating wave approximation. These calculations will pave the way to NMR-type manipulations of a Floquet two-level system.

It will be also proposed to calculate the finite frequency noise spectrum, in connection with the experiments in Romain Danneau's group at the Karlsruhe Institute of Technology in the framework of a Grenoble-Karlsruhe international laboratory The goal of these experiments is to provide evidence for Floquet-Wannier-Stark ladders.

Finally, as mentionned above, it will be proposed to address a two-frequency problem, allowing for sweeping the Brillouin zone of the two relevant phase variables in the presence of two slightly different bias voltages V_a and V_b in an incommensurate ratio. This type of problem with a "slow" and a "fast" frequency was treated with micro-local analysis in the mathematical community, and it was used in Grenoble in order to calculate the bottom of the Hofstadter spectrum for a network of superconducting wires (Wilkinson-Rammal approach, in connection with Pannetier's experiments [Wang1987]). It will be proposed to develop these calculations in close collaboration with Frédéric Faure and Alain Joye at Institut Fourier in the mathematics department of Grenoble-Alpes University. It will also be proposed to tackle this question in the framework of the Markovian approximation. In particular, special care will be devoted to the fate of topology in the presence of relaxation, in connection with heat flows.

Environment of the projects : This project is within a long-standing collaboration between Régis Mélin et Benoît Douçot. Numerical calculations will be carried out on the Rouen platform (CRIANN) through a collaboration with Jean-Guy Caputo (applied mathematics at INSA-Rouen). A French-German ANR international project was proposed with Romain Danneau, who fabricates devices based on bilayer graphene, and realizes finite frequency noise measurement of Josephson junctions. Finally, a collaboration has started with Frédéric Faure and Alain Joye in the mathematics department of the Grenoble-Alpes University, with regular meeting on a monthly basis.

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Possible extension as a PhD: The internship could be followed by a PhD
Required skills: The candidate should be familiar with advanced quantum mechanics
Starting date: Spring 2019
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