Stability of Non Fermi Liquid states in Kondo problems: a few surprises

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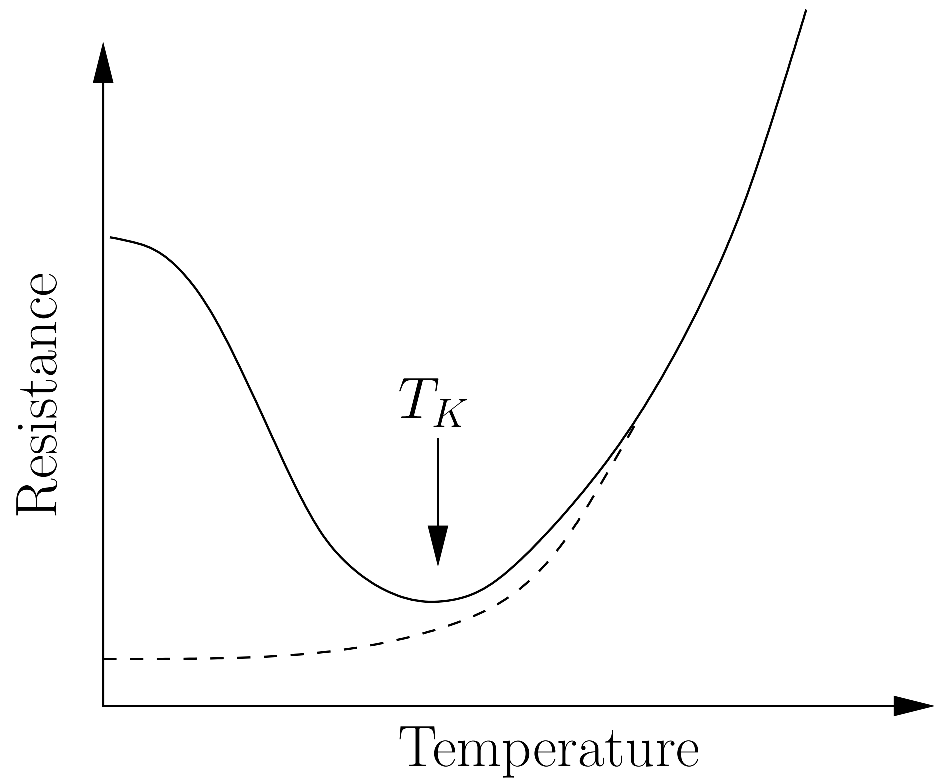
Summary

- The Kondo problem
- Overscreening: route to NFL
- Quantum dot with interactions in the leads: "uphill" entropy flow and two channel Kondo
- Conclusion

Diluted magnetic impurities in a metal:



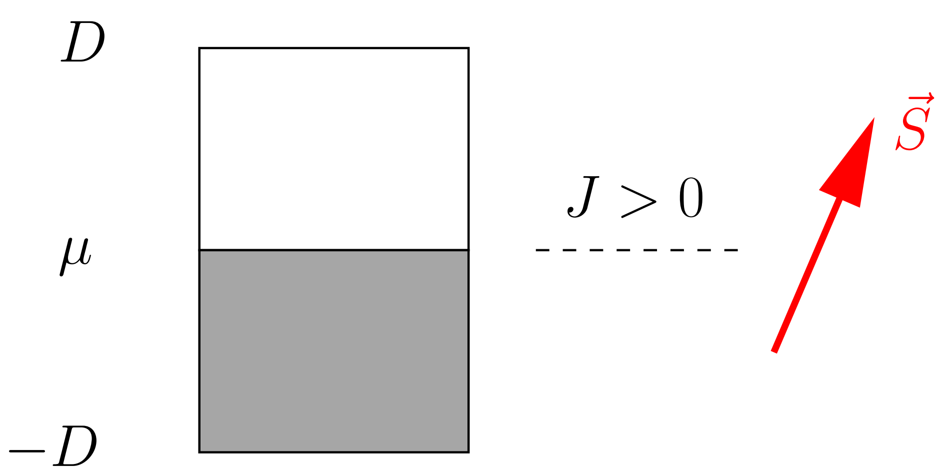
Resistivity in “pure” metals:

$$\rho(T) = a + bT^2 + cT^5$$

Resistivity in “dirty” metals:

- Temperature dependent impurity scattering

Impurity spin in metallic Fermi sea:



$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J\vec{S} \cdot \sum_{\sigma\sigma'} c^\dagger_\sigma(0) \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'}(0)$$

Limit $J = 0$: local moment

- $S(T) = \log 2$
- $\chi(T) = 1/(4T)$

Limit $J = \infty$: singlet state (screening)

- $S(T) = 0$
- $\chi(T = 0)$ does not diverge

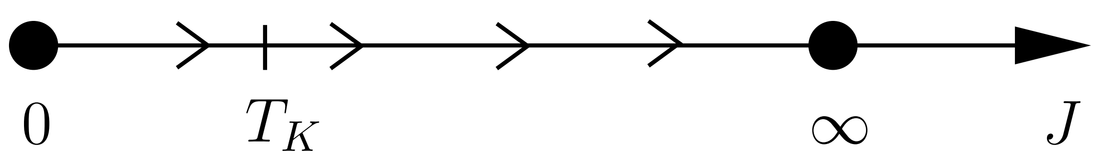
Kondo effect in a nutshell

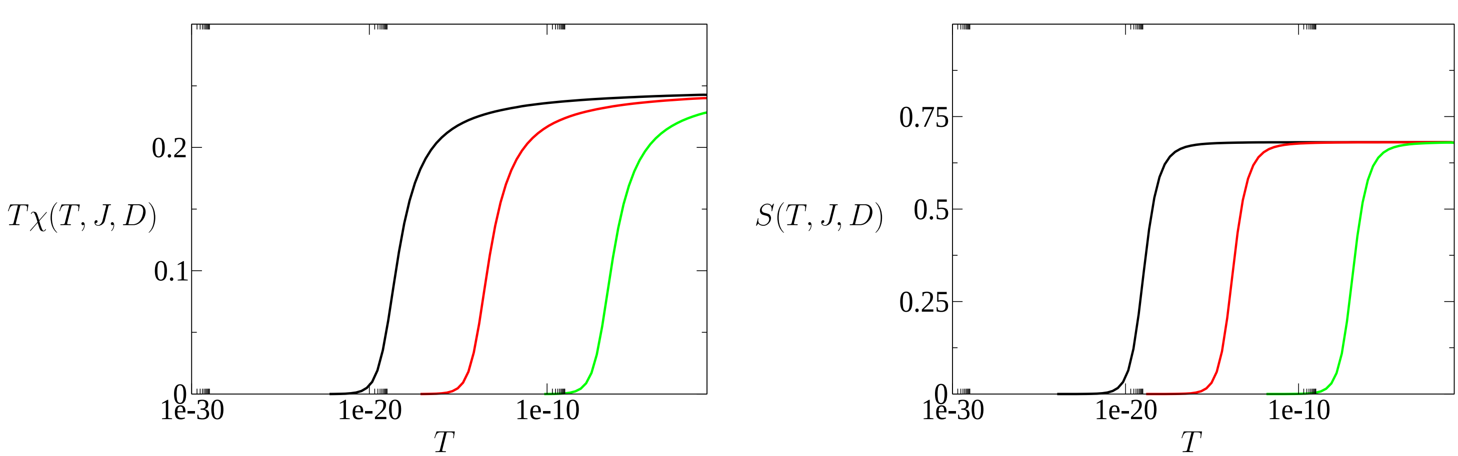
Weak coupling RG: $\log(T/D)$ divergent

$$\frac{\partial J(\Lambda)}{\partial \log \Lambda} = -J^2(\Lambda) \Rightarrow J(\Lambda) = \frac{1}{\log(\Lambda/T_K)}$$

Kondo scale: $T_K = D\exp(-D/J)$

Universal crossover:





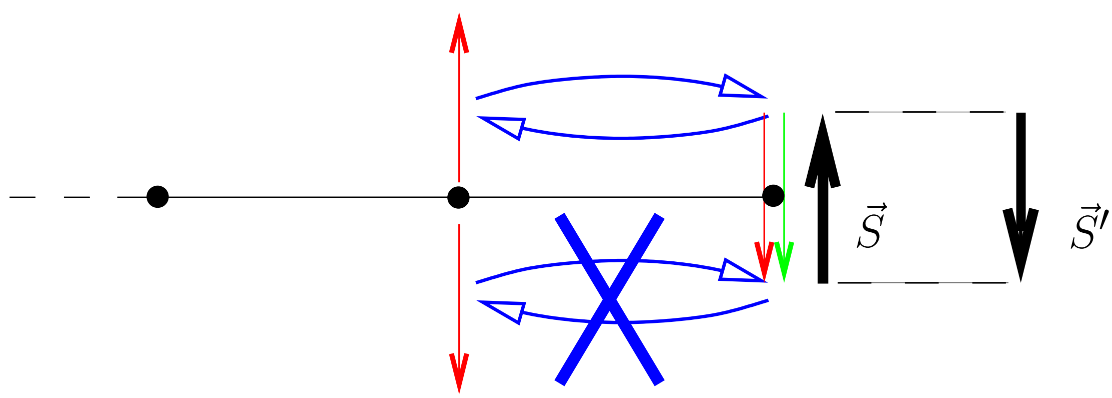
Universality: $T\chi(T,J,D) = \Phi(T/T_K)$

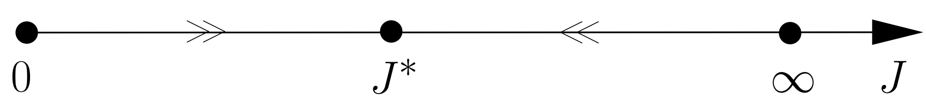
Two channel Kondo effect

Two independent Fermi seas (m=1,2):

$$H = H_1 + H_2 + \sum_{\sigma\sigma' m} J_m \, c^\dagger_{\sigma m} \vec{\tau}_{\sigma\sigma'} c_{\sigma' m} \cdot \vec{S}$$

Over-screening if $J_1 = J_2$ [Nozières-Blandin]:



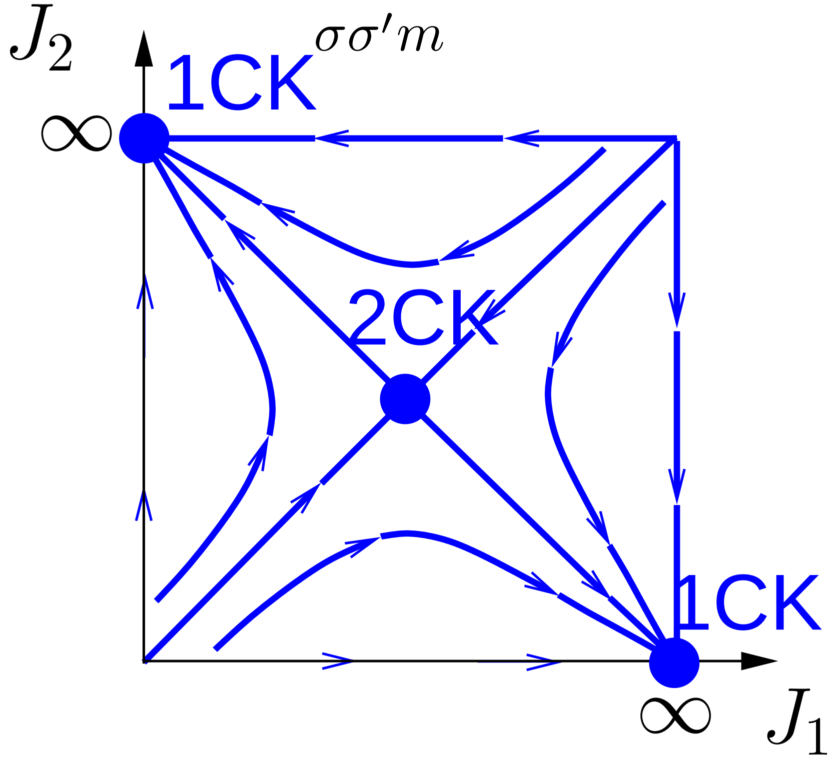


Weak coupling RG: $\beta(J) = -J^2 + KJ^3$

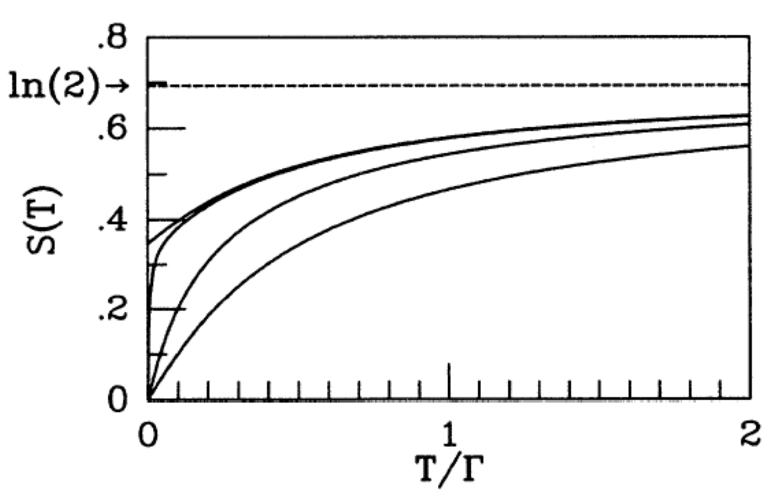
- $K = 2$ = # channels
- $J^* = 1/K$ NFL fixed point
- $S(T=0) = \log(\sqrt{2})$
- $\chi(T) \propto \log(T_K/T)$

Anisotropies destabilize 2CK

$$H = H_1 + H_2 + \sum_{\sigma\sigma' m} J_m \, c^\dagger_{\sigma m}\vec{\tau}_{\sigma\sigma'}c_{\sigma' m} \cdot \vec{S}$$



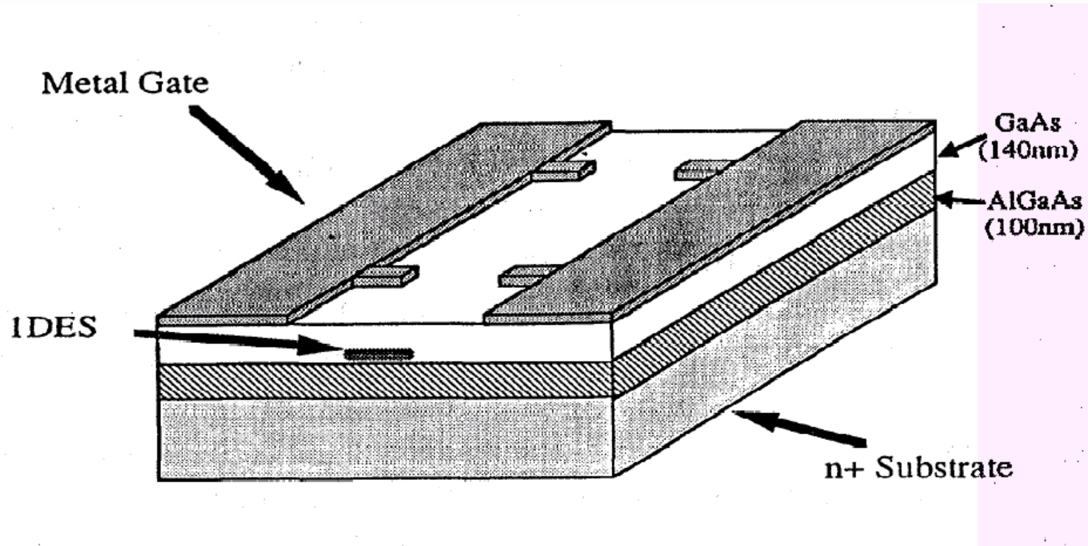
The strongest channel wins!



$S(T)$ decreases along the flow: g-theorem

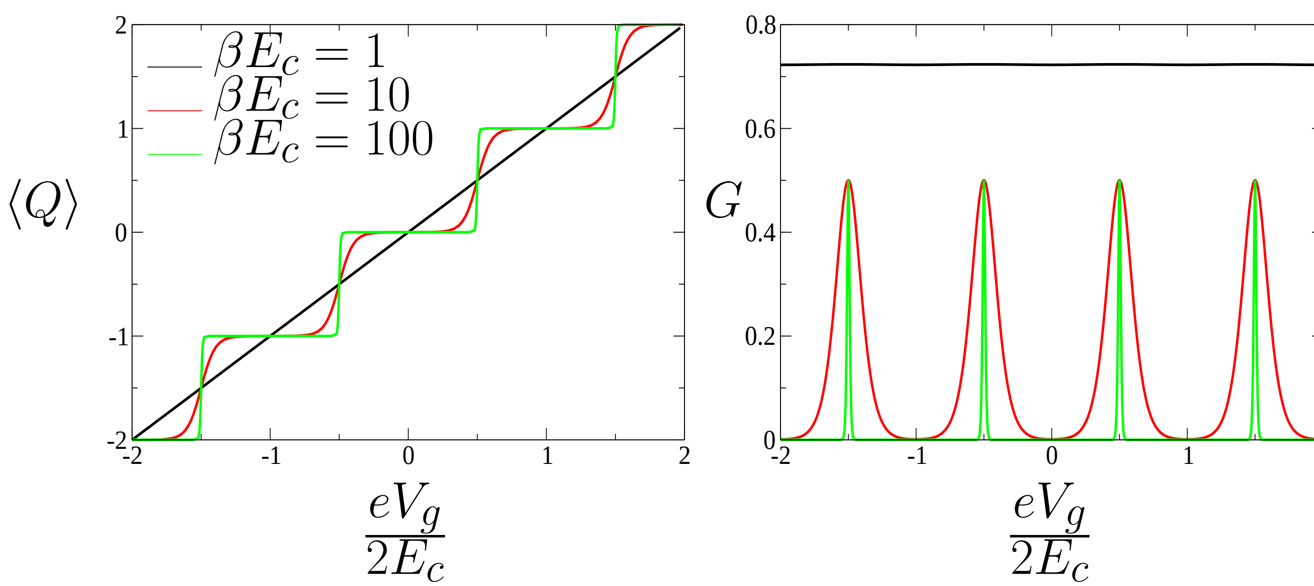
[Affleck-Ludwig]

Coulomb blockade in a large dot

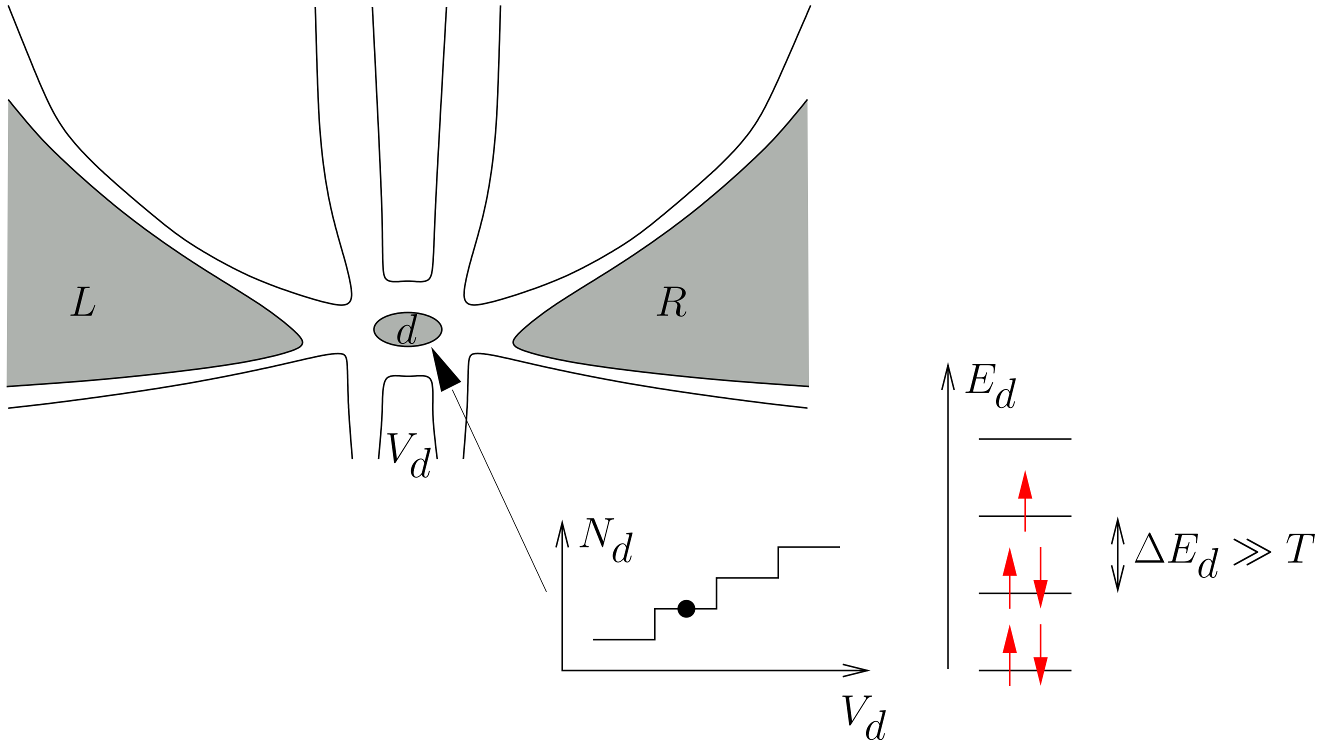


Charging energy:

$$
\begin{aligned}
H(Q) &= E_c Q^2 - eV_g Q \\
&= E_c\left(Q - \frac{eV_g}{2E_c}\right)^2 + \text{cst.}
\end{aligned}
$$



Can one realize 2CK in dots ?



Model: [Glazman-Raikh] $\hat{N}_d = \sum_\sigma d^\dagger_\sigma d_\sigma$

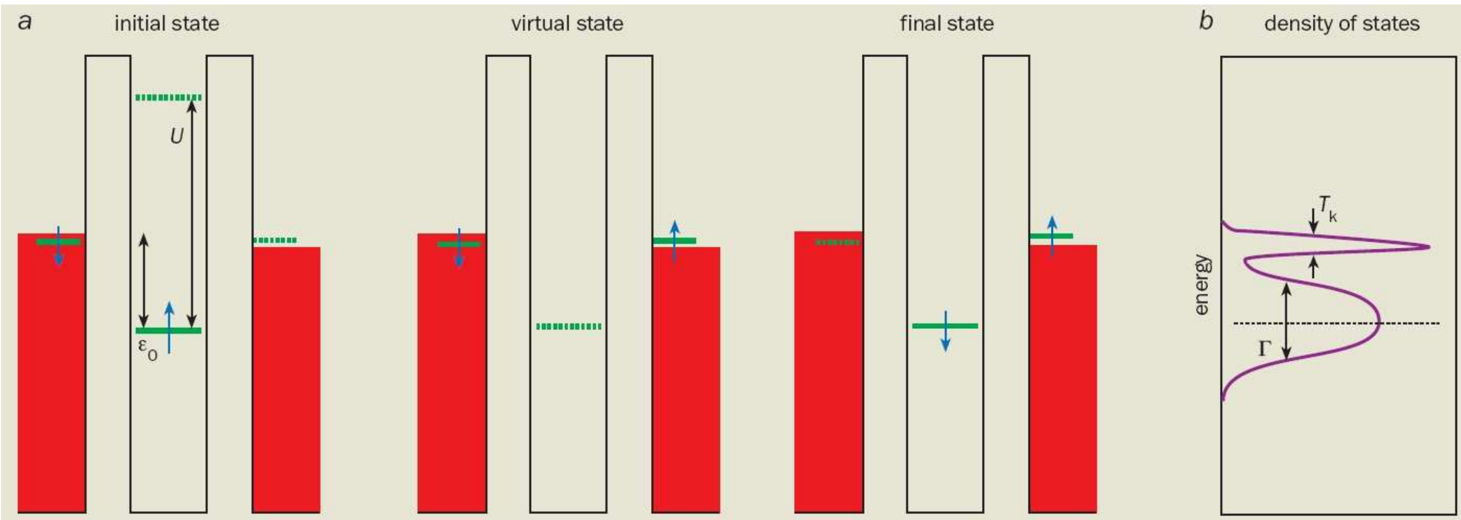
$$H = H_L + H_R + U_d \hat{N}_d^2 + E_d \hat{N}_d + \sum_{\sigma\alpha} t_\alpha d^\dagger_\sigma c_{\sigma\alpha} + h.c.$$

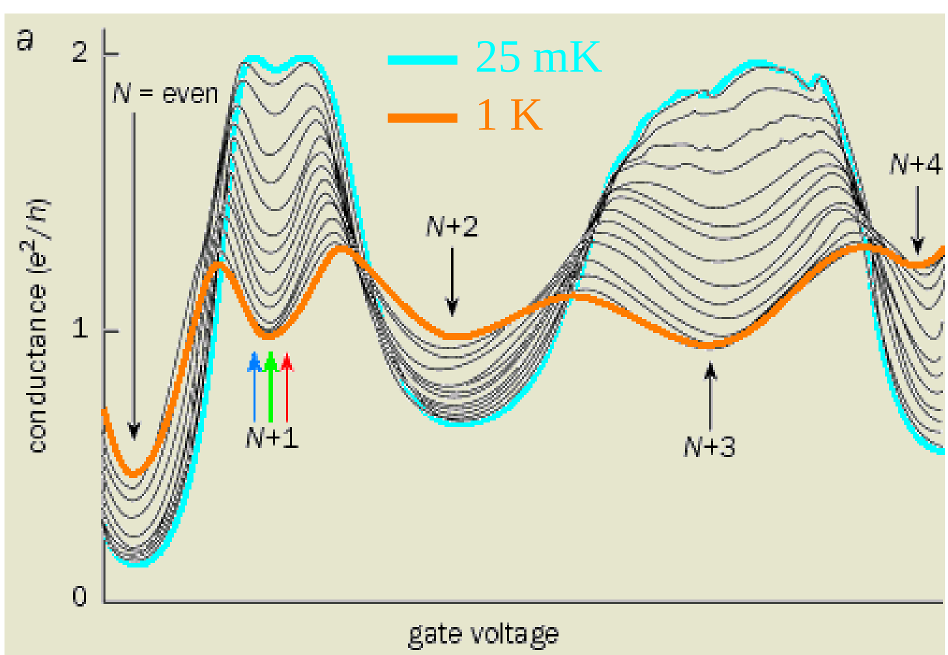
Magnetic coupling: $J_{\alpha\alpha'} = t_\alpha t_{\alpha'} / U_d \quad (\alpha = L, R)$

$$\begin{aligned} H &= H_L + H_R + \sum_{\alpha\alpha'} J_{\alpha\alpha'} c^\dagger_{\sigma\alpha} \vec{\tau}_{\sigma\sigma'} c_{\sigma'\alpha'} \cdot \vec{S} \\ &= H_+ + H_- + \frac{t_L^2 + t_R^2}{U_d} c^\dagger_{\sigma+} \vec{\tau}_{\sigma\sigma'} c_{\sigma'+} \cdot \vec{S} \end{aligned}$$

$\Rightarrow$ Mixing of lead indices: 1CK!

Observed 1CK in quantum dots

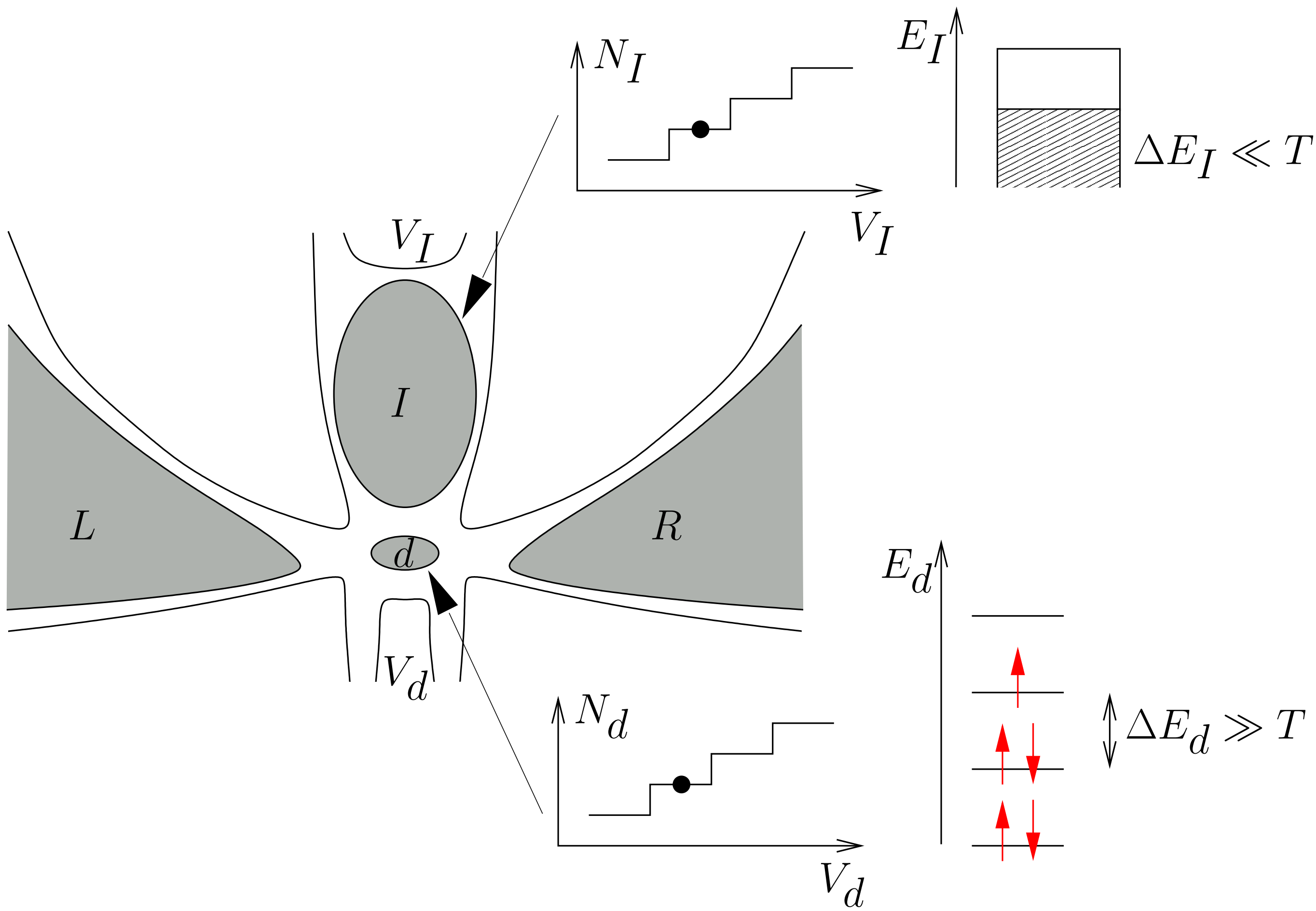




Exp. [Goldhaber-Gordon, Kouwenhoven...]

Fermi Liquid: $G(T) = 2e^2/h[1 - (\pi T/T_K^{1C})^2]$

2CK: Oreg & Goldhaber-Gordon idea



$$H = H_+ + H_- + H_I + \sum_{\sigma\sigma' m m'} J_{mm'} c^\dagger_{\sigma m} \vec{\tau}_{\sigma\sigma'} c_{\sigma' m'} \cdot \vec{S}$$
$$+ E_c \Big[\sum_\sigma c^\dagger_{\sigma I} c_{\sigma I} - N_I^0 \Big]^2 \quad (m = +, I)$$

<u>If $E_c \gg T_K^{1C}$</u>: two decoupled channels appear

$$\Rightarrow G(T) \sim 1 e^2/h [1 - \sqrt{\pi T / T_K^{2C}}]$$

Interesting limit: $E_c \sim T_K$

Question: consider the extreme case $E_c \ll T_K^{1C}$

What happens at $T \to 0$?

Naïvely: $S(T) \ll \log(\sqrt{2})$ if $E_c \ll T \ll T_K^{1C}$

$\Rightarrow$ One channel Kondo ?!?

Counter argument: C.B. wins at $T \ll E_c$

$\Rightarrow$ Two channel Kondo ?!?

Formalization: phase conjugate to island charge

$$H = H_+ + H_- + H_I + E_c\left[\frac{1}{i}\frac{\partial}{\partial\theta} - N_I^0\right]^2 + \sum_{\sigma\sigma' mm'} J_{mm'}(\theta) c^\dagger_{\sigma m}\vec{\tau}_{\sigma\sigma'} c_{\sigma' m'} \cdot \vec{S}$$

with:

$$\begin{aligned} J_{++}(\theta) &= J_{++} \\ J_{II}(\theta) &= J_{II} \\ J_{+I}(\theta) &= J_{+I}e^{i\theta} = J^*_{I+}(\theta) \end{aligned}$$

Perturbative analysis

Limit $E_c = 0$: θ locked $\Rightarrow H = H_{1\mathrm{CK}}$
in the symmetric combination of $m = +, I$ channels

Perturbative calculation at small E_c:

$$H = H_{1\mathrm{CK}} + J_{+I} \sum_{\sigma\sigma'} \vec{S}.c^{\dagger}_{\sigma+} \vec{\tau}_{\sigma\sigma'} c_{\sigma' I} (i\theta - \theta^2/2) + h.c.$$

$$\begin{aligned} \Rightarrow \Delta F &\propto \int_0^{\beta} \mathrm{d}\tau \left\langle [\theta(\tau) - \theta(0)]^2 \right\rangle T(\tau) G_0(\tau) \\ &\propto \int_T^{T_K^{1C}} \frac{2E_c}{\omega^2} \omega \propto E_c \log\left[\frac{T_K^{1C}}{T}\right] \end{aligned}$$

using $S_\theta = \int_0^\beta \mathrm{d}\tau \frac{(\partial_\tau \theta)^2}{4E_c} \Rightarrow \left\langle \theta(i\omega)\theta(-i\omega) \right\rangle = 2E_c/\omega^2$
and $T(\tau)G_0(\tau) = 1/(\pi^2\tau^2)$ (Friedel)

$$\Rightarrow \boxed{S(T) = -\frac{\partial F}{\partial T} \simeq \frac{T}{T_K^{1C}} + \frac{E_c}{T}}$$

- Crossover scale in S: $T_S = \sqrt{E_c T_K^{1C}}$
- Breakdown of expansion at $T \lesssim E_c$

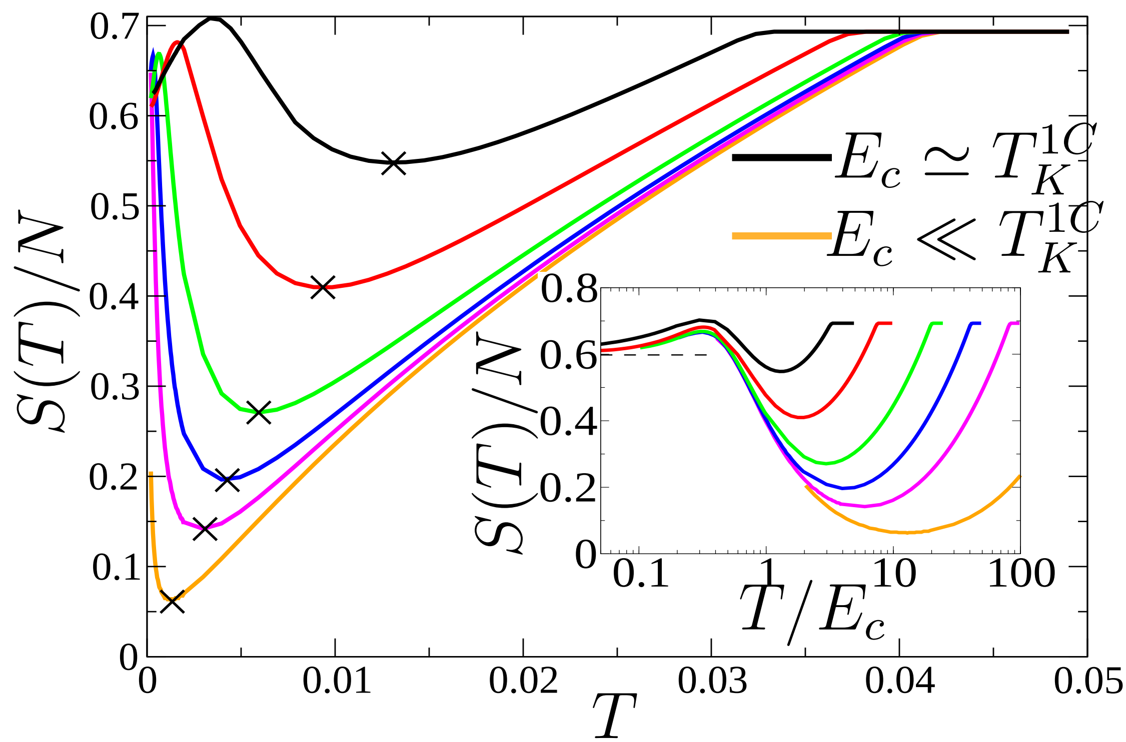
Non perturbative solution

New large N limit: SU(N) spin and K channels

$\Rightarrow$ Condense slave boson $\Rightarrow$ screening

$\Rightarrow$ Selection of Non Crossing diagrams $\Rightarrow$ NFL

Crossover 1CK $\rightarrow$ 2CK in the entropy:

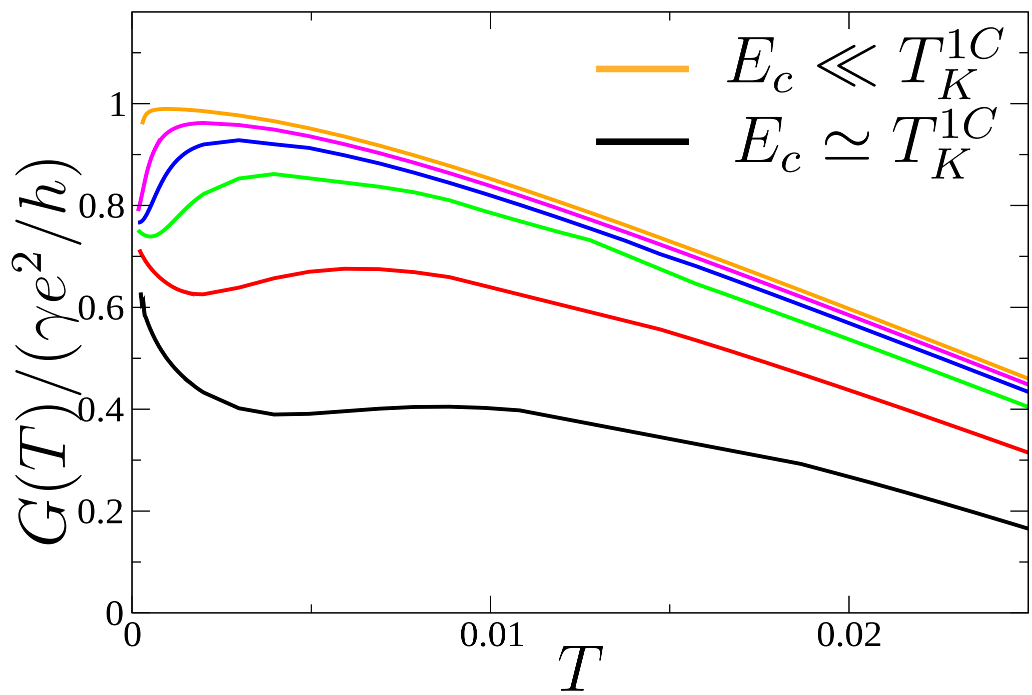


$\Rightarrow S(T)$ rises below $T_s = \sqrt{E_c T_K^{1C}}$!!

$\Rightarrow T_K^{2C} = E_c$ at $Ec \ll T_K^{1C} \Rightarrow T \ll E_c$ is enough!

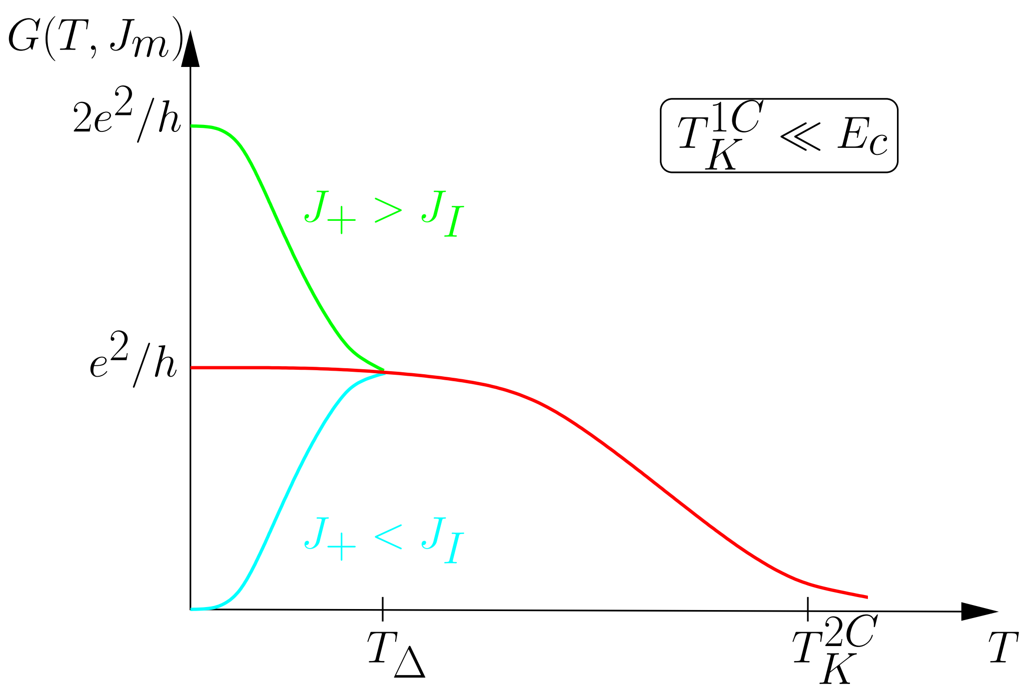
1CK → 2CK in conductance

Apply interacting Landauer formula:



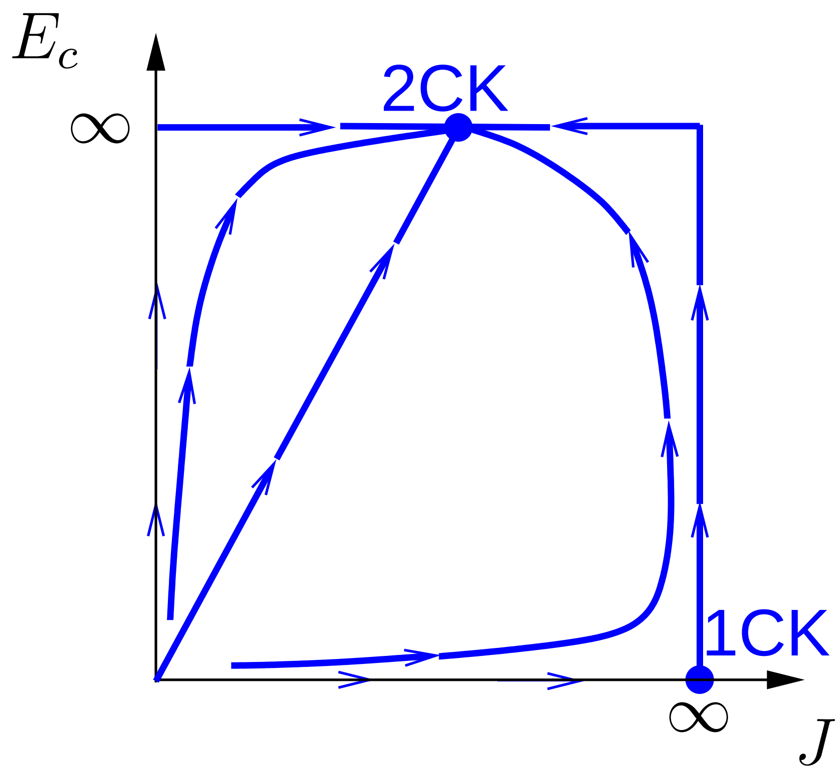
The smoking gun: play with channel anisotropy!

$T_\Delta \propto (J_+ - J_I)^2$ [Pustilnik et al.]



Smallness of T_K ?

Flow diagram:



$T_K^{2C} = De^{-2/j}$ at $E_c > D$ as $\hat{J} = \begin{bmatrix} J & 0 \\ 0 & J \end{bmatrix}$

$T_K^{2C} = E_c$ at $E_c < T_K^{1C} = De^{-1/j}$ as $\hat{J} = \begin{bmatrix} J & J \\ J & J \end{bmatrix}$

What sets T_K^{2C} at $T_K^{1C} < E_c < D$?

2-step 1 loop RG for $T_K^{1C} < E_c < D$

- At $E_c < \Lambda < D$, single channel Kondo RG:

$$\frac{\partial j}{\partial \log \Lambda} = -j^2$$

Flow starts at $j(D) = j$:

$$\Rightarrow j(\Lambda) = 1/\log(\Lambda/T_K^{1C})$$

- At $\Lambda < E_c$, multichannel Kondo RG:

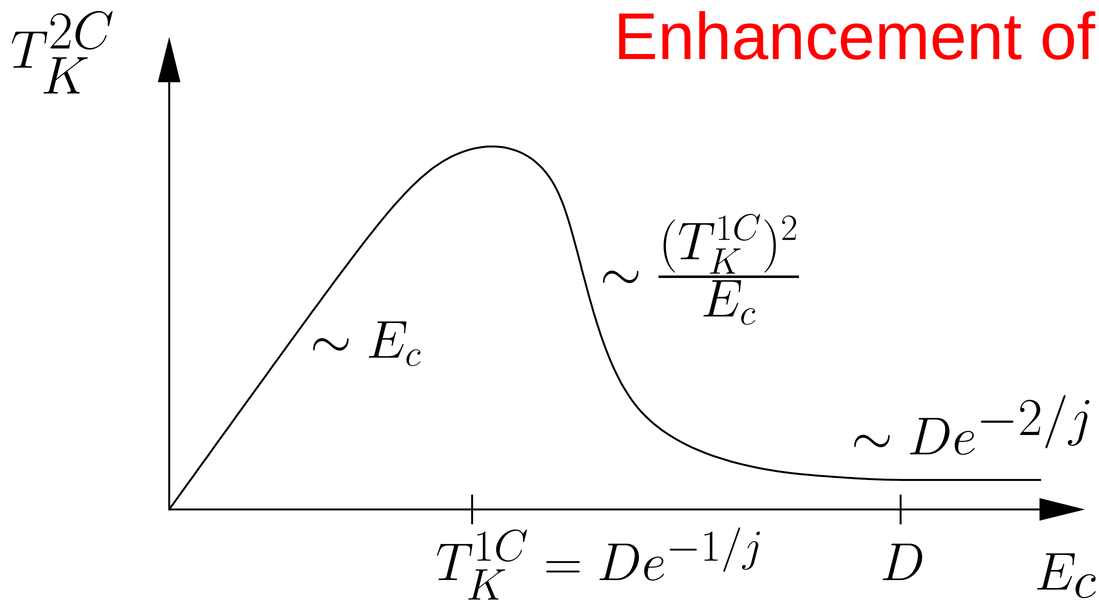
$$\frac{\partial j}{\partial \log \Lambda} = -\frac{j^2}{2}$$

Flow starts at $j(E_c) = 1/\log(E_c/T_K^{1C})$:

$$\Rightarrow j(\Lambda) = \frac{1}{\log Ec/T_K^{1C} + 1/2\log(\Lambda/E_c)}$$

Pole at $\boxed{T_K^{2C} = (T_K^{1C})^2/E_c} \gg De^{-2/j}$

Enhancement of T_K^{2C}!



Conclusion

- NFL states in Kondo problem have a finite entropy
- Even small charging energies do stabilize 2CK
- Novel "uphill" entropy flow generic
- New enhancement of Kondo scale
- Experiments on two channel Kondo...
- Extension to the lattice?