Microscopic structure of entanglement in the many-body environment of a qubit

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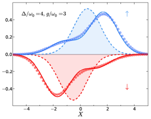
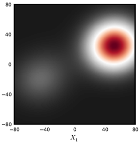
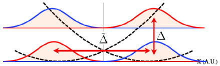
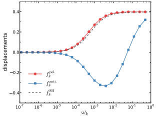





- Soumya Bera (Néel, Grenoble)

- Harold Baranger (Duke, USA)

- Nicolas Roch (ENS, Paris)

- Ahsan Nazir (Imperial, London)

- Alex Chin (Cavendish, Cambridge)

Outline

- Motivation : from cavity-QED to photonic Kondo effect
- Dissipative quantum mechanics in a nutshell
- Physical picture of environmental entanglement :
  quantum superpostions of polarons and antipolarons
- Many-body ground state Ansatz : coherent state expansion
- Wigner tomography & entanglement spectroscopy from NRG
- Perspectives

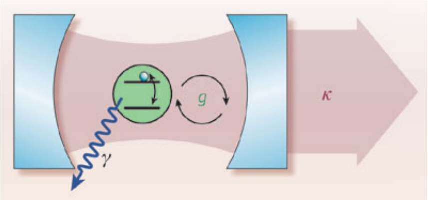
S. Bera *et al.*, arxiv :1307.5681

From cavity-QED to photonic Kondo

Cavity-QED

Pioneering experiment : Haroche, 2012 Nobel Prize

- Coupling of light and matter to manipulate and measure quantum states
- Original experiment very complex (15 years to build !)...
- ... simple conceptually (single cavity mode+two-level system)

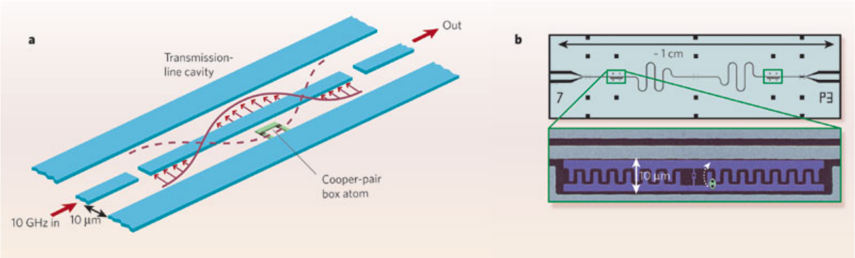


Rabi model : $H = \omega_0 a^\dagger a + \Delta\sigma_x + g(a^\dagger + a)\sigma_z + H_{\rm out}$

Circuit QED

Recent progresses : on-chip cavity QED Berkeley, Paris, Yale, Zurich...

- Atom $\rightarrow$ superconducting qubits (non-linear element)
  Macroscopic two-level system with large dipole moment
- Microwave circuits : design more complex electromagnetic environment
- New route to explore strong correlations ?

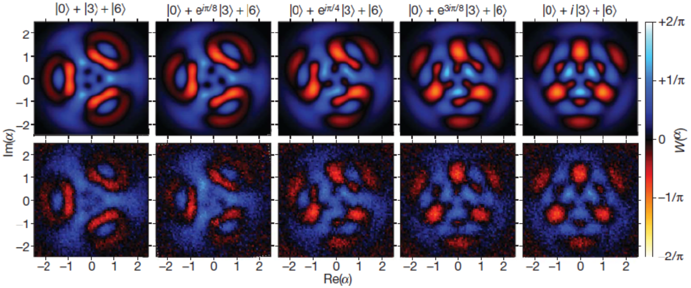


Generation and measurement of arbitrary quantum states

Setup :

- Single electromagnetic mode (cavity)
- Qubit-coupling used to generate complex photonic states
- Measurement : quantum tomography

Nice example : $|\Psi\rangle = |0\rangle + e^{i\phi}|3\rangle + |6\rangle$ Hofheinz *et al.*, Nature (2009)

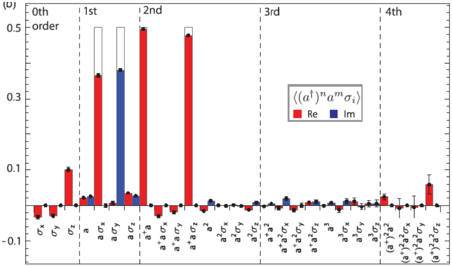


Generation and measurement of arbitrary quantum states

Setup :

- Single electromagnetic mode (cavity)
- Qubit-coupling used to generate complex photonic states
- Measurement : quantum tomography

Other example : $|\Psi\rangle = \Big[|1\rangle + |2\rangle\Big] \otimes |\downarrow\rangle + \Big[|1\rangle - |2\rangle\Big] \otimes |\uparrow\rangle$



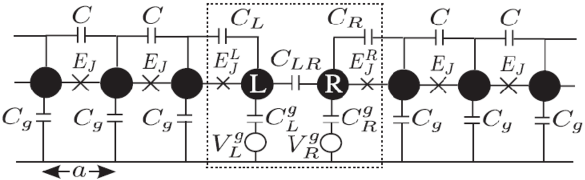
Qubit-conditioned photon moments Eichler *et al.*, PRL (2012)

Towards many-body photonic problems

Setup :

- Need many electromagnetic modes
- Taylored environment (Josephson junction arrays)

Goldstein, Devoret, Houzet, Glazman, PRL (2013)



Model :

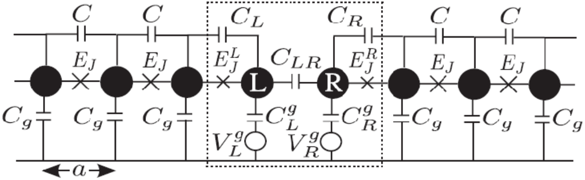
$$H = \sum_{i,j} 2e^2 n_i \left(C^{-1}\right)_{ij} n_j - E_J^{ij} \cos(\phi_i - \phi_j)$$

Limit $E_J \gg E_C$: harmonic environment (free bosonic bath !)

Towards many-body photonic problems

Quantum dot (qubit) : local "defect" with $E_J^{L/R} \ll E_C$

- Quantized $2e$ charge on the island (Cooper pair box)
- Tuned to charge degeneracy $N \leftrightarrow N+2$
  $\Rightarrow$ Effective two-level system :
  Pseudospin $\hat{\sigma}_z = [\hat{Q} - (N+1)]/2$



- Bosonic impurity model !

  Goldstein, Devoret, Houzet, Glazman, PRL (2013)

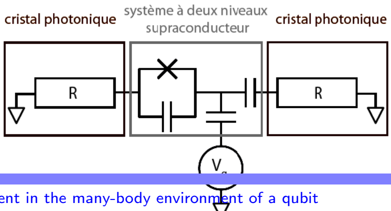
Standard model for dissipative qubit

Effective theory : spin boson hamiltonian Leggett *et al.* RMP (1987)

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Spectral density : $J(\omega) \equiv \sum_k g_k^2 \delta(\omega - \omega_k)$

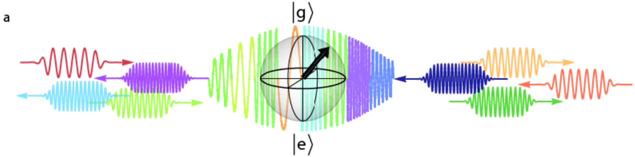
- Ohmic bath : $J(\omega) = 2\pi\alpha\omega$
- Dissipation strength : $\alpha = \sqrt{2E_{C_g}/E_J} \lesssim 1$
- Tunneling amplitude : $\Delta = E_J^{L/R}$



Aim of the talk

Physics at play :

- The qubit generates a non-linearity among photons
- Strong similarity with the electronic Kondo problem (even more than just an analogy...)



Questions to be answered :

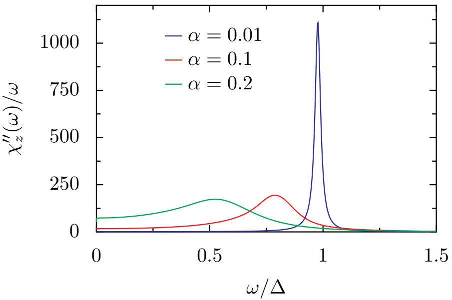
- What kind of correlations are created in the environment due to its coupling to the qubit ?
- What controls the coherence of the qubit ?

Dissipative quantum mechanics in a nutshell

Weak dissipation regime : $\alpha < 0.4$

Spin dynamics : underdamped Rabi oscillations

- Bosonic NRG "solves" the model [Bulla *et al.* PRL (2003)]
- Spin-spin dynamical correlation functions for arbitrary dissipation strength [Florens *et al.* PRB (2011)]

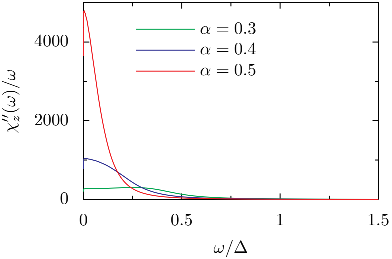


- Peak at renormalized scale $\Delta_R < \Delta$
- Non-lorentzian lineshape for $\alpha > 0.1$

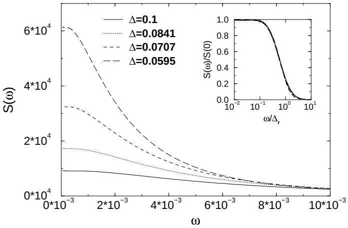
Strong dissipation regime : $\alpha > 0.4$

Spin dynamics : overdamped Rabi oscillations

- Linewidth $\Gamma > \Delta_R$: incoherent qubit
- Boring ? No ! : universal (Kondo) regime for $\alpha \lesssim 1$
  $\rightarrow$ strongly correlated many-body photonic state



$\alpha = 0.9$



Exact mapping to Kondo model

Spin boson Hamiltonian :

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

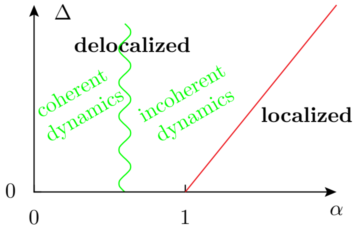
Unitary transformation : $U_\gamma = \exp\{-\gamma\sigma_z \sum_k \frac{g_k}{2\omega_k}(a_k^\dagger - a_k)\}$

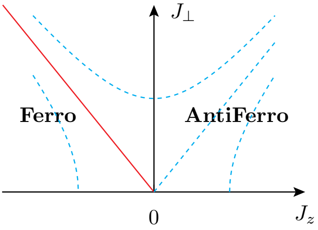
$$U_\gamma H U_\gamma^\dagger = \frac{\Delta}{2}\sigma^+ e^{-\gamma\sum_k \frac{g_k}{\omega_k}(a_k^\dagger - a_k)} + h.c. + (\gamma-1)\sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Bosonization (ohmic bath only) : [Guinea, Hakim, Muramatsu PRB (1985)]

$$\begin{aligned} U_\gamma H U_\gamma^\dagger &= \Delta\sigma^+ \sum_{kk'} c_{k\downarrow}^\dagger c_{k'\uparrow} + h.c. & \rightarrow J_\perp = \Delta \\ &+(1-\sqrt{\alpha})\omega_c \sigma^z \sum_{kk'} [c_{k\uparrow}^\dagger c_{k'\uparrow} - c_{k\downarrow}^\dagger c_{k'\downarrow}] & \rightarrow J_z \propto 1 - \sqrt{\alpha} \\ &+ \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} \end{aligned}$$

Summary : phase diagram





That's not all, folks !

Question : what kind of correlations/entanglement does the qubit generate into its environment ?

Energetics : polarons vs. antipolarons

“Classical” limit

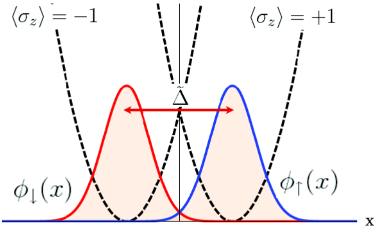
Spin boson Hamiltonian :

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Oscillators subject to a spin-dependent potential

$\Delta = 0$ limit : frozen potential $\Rightarrow$ doubly-degenerate ground state

$|\Psi_{\uparrow, f^{\mathrm{cl.}}}\rangle = |\uparrow\rangle \otimes |f^{\mathrm{cl.}}\rangle$
$|\Psi_{\downarrow, -f^{\mathrm{cl.}}}\rangle = |\downarrow\rangle \otimes |-f^{\mathrm{cl.}}\rangle$
where $f_k^{\mathrm{cl.}} = g_k/(2\omega_k)$
and $|\pm f\rangle \equiv e^{\pm \sum_k f_k (a_k^\dagger - a_k)}|0\rangle$



Zero tunneling $\Rightarrow$ localized coherent states !

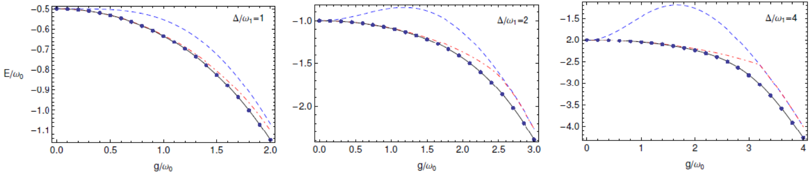
Bare polaron theory for $\Delta \neq 0$

Ground state Ansatz : antisymmetric combination (polaron)

$$|GS\rangle \simeq |\uparrow\rangle \otimes |+f^{\mathrm{cl.}}\rangle - |\downarrow\rangle \otimes |-f^{\mathrm{cl.}}\rangle$$

... but no quantum fluctuations among the oscillators !

Testing on Rabi model : consider single mode ω_1 below



Exact solution (blue dots) vs. bare polaron (blue dashed line)

$\Rightarrow$ Failure at increasing Δ/ω_1 : problematic for many modes $\forall\Delta$!

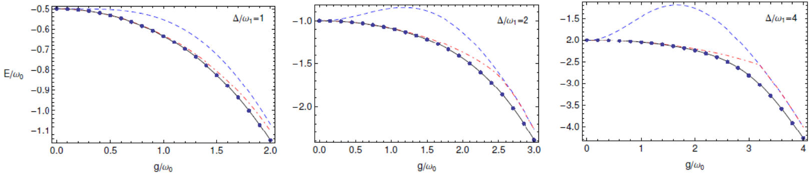
Variational polaron theory : Silbey-Harris state

Ground state Ansatz : polaron with optimized displacement

$$|GS\rangle \simeq |\uparrow\rangle \otimes |+f_k^{\text{pol.}}\rangle - |\downarrow\rangle \otimes |-f_k^{\text{pol.}}\rangle \Rightarrow f_k^{\text{pol.}} = \frac{g_k/2}{\omega_k + \Delta_R}$$

Renormalized tunnel rate Δ_R reduces the displacement
$\Rightarrow$ Quantum fluctuations within the well

Testing on Rabi model : consider single mode ω_1 below



Exact solution (blue dots) vs. SH polaron (red dot-dashed line)

Energy improves, but something is still missing

Energetics : competition localization vs. tunneling

Elastic cost of displacement f_k :

- $\delta E = \sum_k \omega_k [f_k - g_k/2\omega_k]^2$
- Favors positive displacement $f_k = +g_k/2\omega_k$
- Applies for high frequency modes : adiabatic response

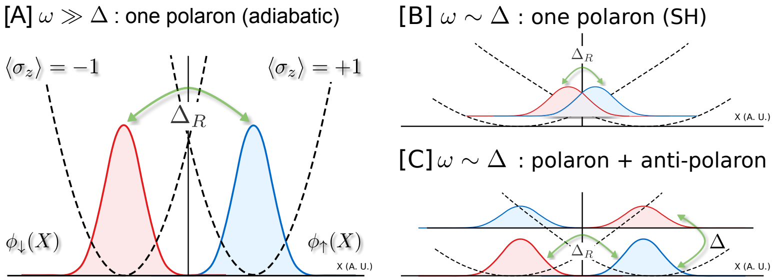
Tunnel process : tends to delocalize
Polaron state couples to a displaced state f_k with matrix element :

$$\langle \Psi_{\uparrow,\widetilde{f}_k} | K_+ | \widetilde{\Psi}_{\downarrow, g_k/2\omega_k} \rangle = \Delta e^{-\frac{1}{2}\sum_k (f_k + g_k/2\omega_k)^2}$$

- Favors negative displacement $f_k = -g_k/2\omega_k$
- Energy gain $= \Delta$ (bare !)
- Small ω_k modes can take advantage : anti-adiabatic response

Strong energetic constraints :
These reduce the phase space volume of allowed displacements

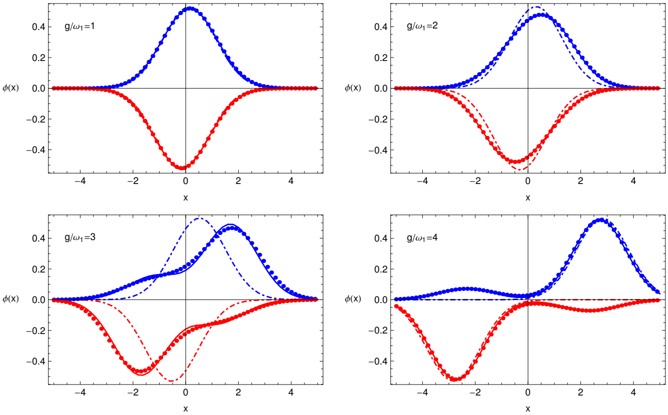
Physical picture



<u>Idea :</u> quantum mechanics allows for state superposition

- The best compromise is to allow positive and negative displacements

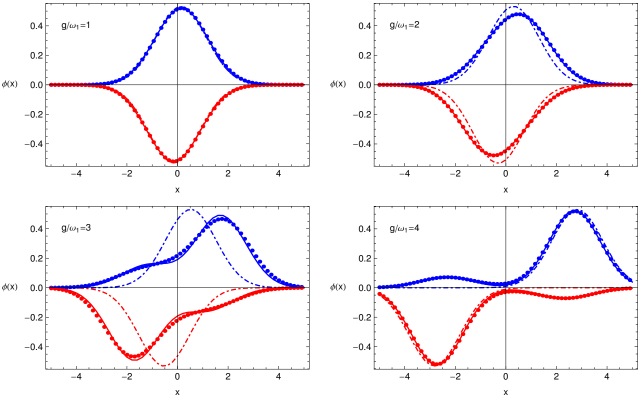
Checking the wavefunctions (one mode, $\Delta = 4\omega_1$)



Exact wavefunction (dots) vs SH single polaron state (dashed line)

Polaron+antipolaron state (one mode, $\Delta = 4\omega_1$)

$$|GS\rangle = |\uparrow\rangle \left[|+f_1^{\mathrm{pol.}}\rangle + p|+f_1^{\mathrm{anti.}}\rangle \right] - |\downarrow\rangle \left[|-f_1^{\mathrm{pol.}}\rangle + p|-f_1^{\mathrm{anti.}}\rangle \right]$$

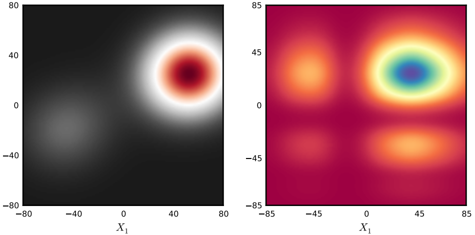


Trial wavefunction and energy (solid line) are nearly perfect !

Many-body ground state Ansatz : coherent state expansion

How do we build the many-body wavefunction ?

Insight from two modes :



Left panel : exact wavefunction $\Psi_\uparrow(X_1, X_2)$ compatible with

$$\langle \uparrow |\Psi\rangle = |f^{\text{pol.}}\rangle_1 \otimes |f^{\text{pol.}}\rangle_2 + p|f^{\text{anti.}}\rangle_1 \otimes |f^{\text{anti.}}\rangle_2$$

Right panel : hypothetical product state (no entanglement)

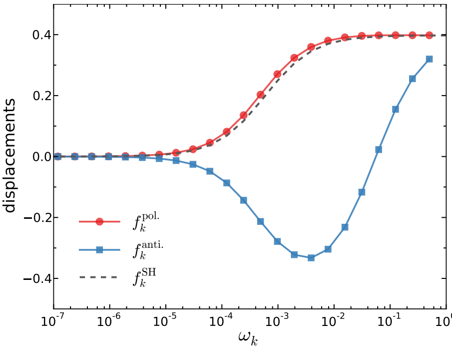
$$\langle \uparrow |\Psi\rangle = \left\{|f^{\text{pol.}}\rangle_1 + p|f^{\text{anti.}}\rangle_1\right\} \otimes \left\{|f^{\text{pol.}}\rangle_2 + p|f^{\text{anti.}}\rangle_2\right\}$$

Many-modes : two-polaron variational Ansatz

Proposed wavefunction : built with $|f\rangle \equiv e^{\sum_k f_k(a_k^\dagger - a_k)}|0\rangle$

$$\big|GS^{2\text{pol.}}\big\rangle = |\uparrow\rangle \otimes \Big[\big|+f^{\text{pol.}}\big\rangle + p\big|+f^{\text{anti.}}\big\rangle\Big] - |\downarrow\rangle \otimes \Big[\big|-f^{\text{pol.}}\big\rangle + p\big|-f^{\text{anti.}}\big\rangle\Big]$$

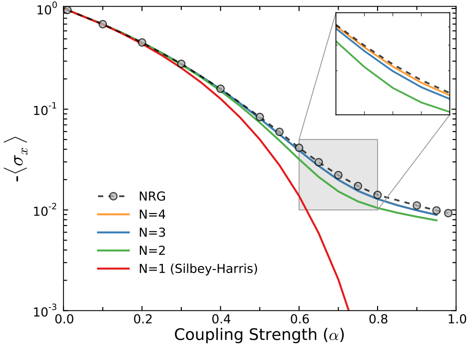
- $\alpha = 0.5$
- $\Delta/\omega_c = 0.01$



Adiabatic/anti-adiabatic crossover captured !

Many modes : ground state properties

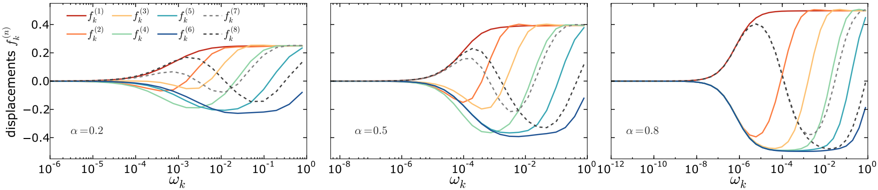
Ground state "coherence" $\langle \sigma_x \rangle$:



- The one-polaron (SH) breaks down at large dissipation
- Antipolarons (entanglement) helps in "preserving" $\langle \sigma_x \rangle$
- Add more polarons : convergent expansion !

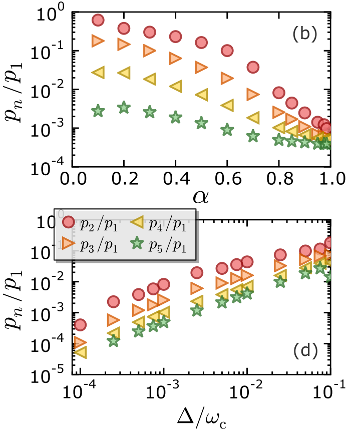
General multi-polaron (coherent state) expansion

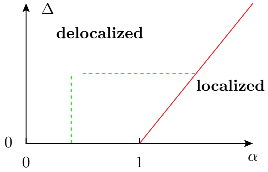
$$\left|GS\right\rangle = \sum_{n=1}^{N_{\rm pols}} p_n \Big[\left| +f^{(n)} \right\rangle \otimes \left| \uparrow \right\rangle - \left| -f^{(n)} \right\rangle \otimes \left| \downarrow \right\rangle \Big]$$



- Adiabatic/antiadiabatic crossover obeyed...
- ... but new displacements with “double kinks” for $n > 6$
- Approach to the scaling limit $\alpha \to 1$: data ”collapse“

Nature of the strong dissipation state





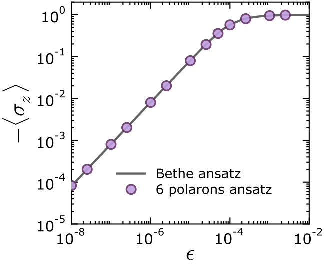
- The displacements vanish for $\Delta \to 0$: bare polarons
- The strong dissipation state at $\alpha \to 1$ is non-trivial

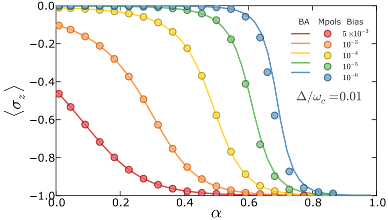
Including a "bias"

Tuning the charge asymmetry : Leggett *et al.* RMP (1987)

$$H = \epsilon\sigma_z + \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

$\Rightarrow$ finite magnetization $\langle \sigma_z \rangle \neq 0$





- Perfect agreement with (fermionic) Bethe Ansatz
- Deviations at $\alpha > 0.9$: more polarons needed

How do we check the form of the wavefunction ?
Many-body Wigner tomography

Wigner spectroscopy of the polaron component

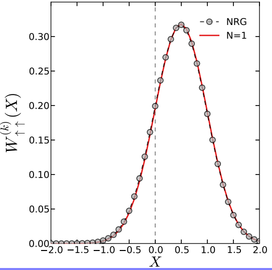
Definition :

$$W^{(k)}_{|\uparrow\rangle\langle\uparrow|}(X) = \int \frac{\mathrm{d}^2\lambda}{\pi^2}\, e^{X(\bar{\lambda}-\lambda)} \langle GS| \left[e^{\lambda a_k^\dagger - \bar{\lambda} a_k} |\uparrow\rangle\langle\uparrow| \right] |GS\rangle$$

From the two-polaron Ansatz :

$$W^{(k)}_{|\uparrow\rangle\langle\uparrow|}(X) = \frac{1}{\pi} e^{-2(X - f_k^{\mathrm{pol.}})^2} + (\text{small terms})$$

NRG computation : use moments $\langle GS|[a_k^\dagger]^m [a_k]^n |GS\rangle$



The SH state captures the complete density matrix reduced to a given but arbitrary high energy oscillator

Wigner spectroscopy of the antipolaron component

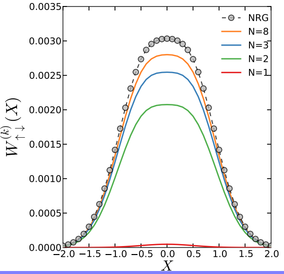
Definition :

$$W_{\sigma^+}^{(k)}(X) = \int \frac{\mathrm{d}^2\lambda}{\pi^2}\; e^{X(\bar{\lambda}-\lambda)} \langle GS| \left[e^{\lambda a_k^\dagger - \bar{\lambda} a_k} \sigma^+ \right] |GS\rangle$$

From the two polaron Ansatz :

$$W_{\sigma^+}^{(k)}(X) \simeq \frac{p}{\pi}\left[e^{-2\left(X+\frac{f_k^{\mathrm{pol.}}-f_k^{\mathrm{anti.}}}{2}\right)^2} + e^{-2\left(X-\frac{f_k^{\mathrm{pol.}}-f_k^{\mathrm{anti.}}}{2}\right)^2} \right]$$

NRG computation : $\alpha = 0.8$ here

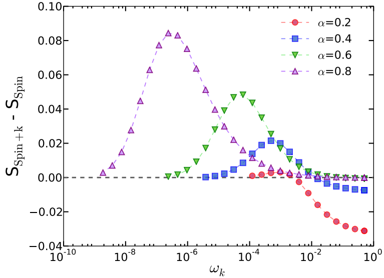


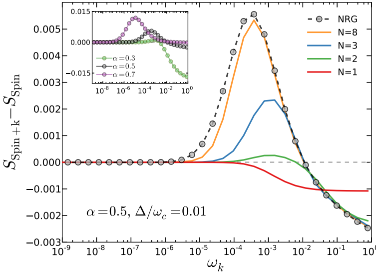
These correlations are only captured by the antipolaron part of the wavefunction

Entanglement properties

Entropy spectroscopy : $S_{\text{spin+k}} = -\text{Tr}_{\text{spin}+k}\left[\rho_{\text{spin}+k} \log \rho_{\text{spin}+k}\right]$

$$S_{\text{spin}} = -\text{Tr}_{\text{spin}}\left[\rho_{\text{spin}} \log \rho_{\text{spin}}\right]$$





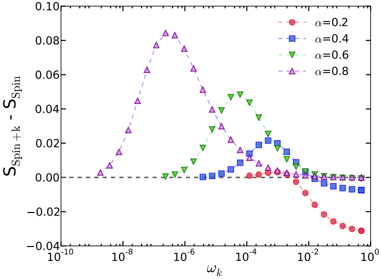
Negative part in $S_{\text{spin+k}} - S_{\text{spin}}$:

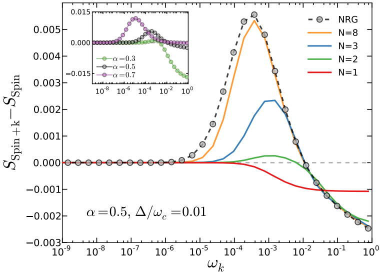
- Fully accounted by SH
- Origin : entanglement of spin with modes

Entanglement properties

Entropy spectroscopy : $S_{\mathrm{spin+k}} = -\mathrm{Tr}_{\mathrm{spin}+k}\left[\rho_{\mathrm{spin}+k} \log \rho_{\mathrm{spin}+k}\right]$

$$S_{\mathrm{spin}} = -\mathrm{Tr}_{\mathrm{spin}}\left[\rho_{\mathrm{spin}} \log \rho_{\mathrm{spin}}\right]$$





Positive part in $S_{\mathrm{spin+k}} - S_{\mathrm{spin}}$:

- Positive peak is not accounted by SH
- Origin : entanglement of modes with modes (antipolarons)
- Kondo lineshape (wide entanglement) at $\alpha \rightarrow 1$

$\Rightarrow$ Massively entangled photonic Kondo cloud

Conclusion

- Quantum tunneling of a qubit subsystem can drive strong correlations in its environment
- Environmental entanglement builds from superposition of polarons and antipolarons
- These ideas can be rationalized quantitatively from a coherent state expansion of the many-body ground state
- Strong phase space constraints make the expansion quickly convergent

Open questions :

- Quantum certification : can we really "prove" convergence of the polaron expansion for the complete many-modes wavefunction (for all many-body density matrices) ?

- Can we reliably simulate quantum quenches and non-linear photon transport experiments using a coherent-state based time-dependent variational framework ? [Related works in quantum chemistry, e.g. Burghardt *et al.* J. Chem. Phys. (2003)]

- Can future superconducting circuit experiments give evidence for the wide-entanglement that characterizes the strongly dissipative (Kondo-like) photonic states ?

- Can we learn something on the structure of the wavefunction for strongly correlated fermions via the Kondo analogy ?