Introduction to the Kondo effect

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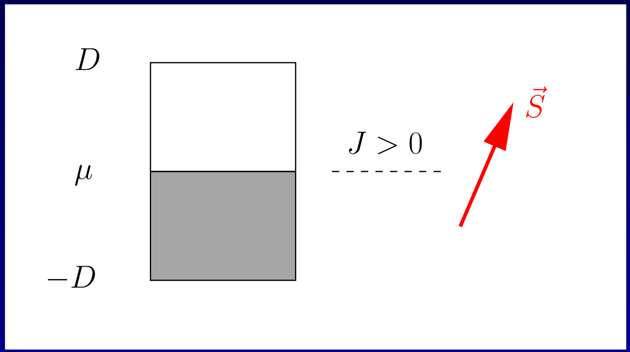
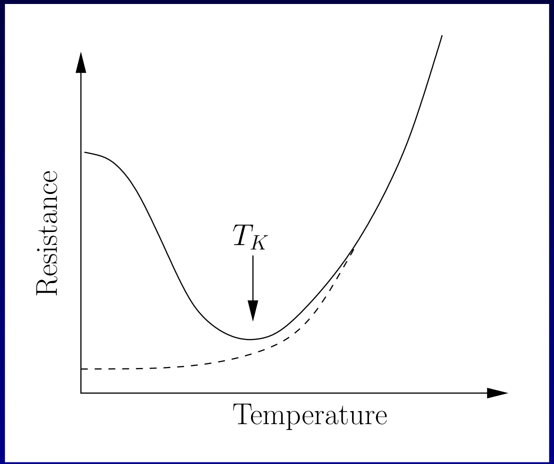
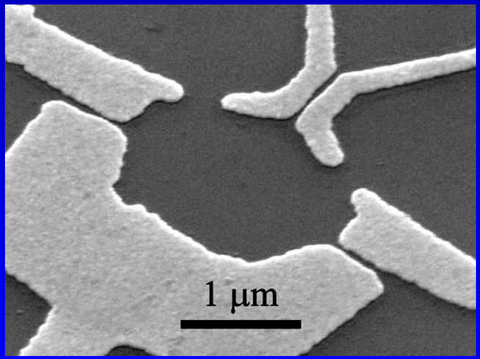
Why Kondo today?

- Basic system: magnetic impurity in metal



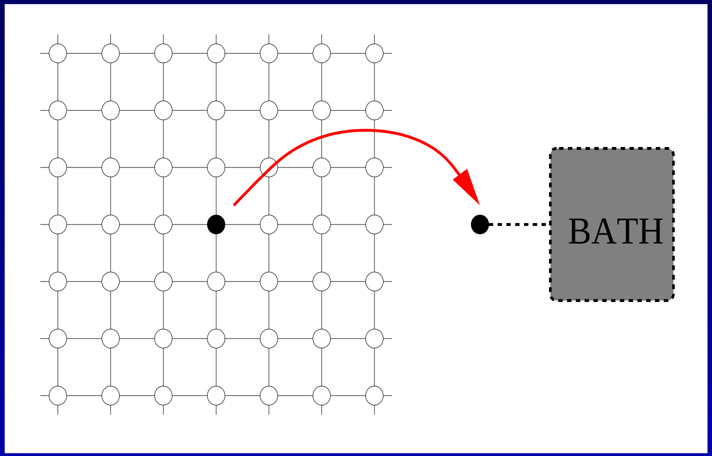


- Exp 30’s, Theory 60-70’s
- New way of doing experiments: quantum dots



Fruitful theory playground

- Simplest model of strongly correlated fermions
- Description of lattice models: DMFT



- Exotic extensions:
  - NFL (2 channel Kondo, $S = 1$ Kondo)
  - Quantum criticality (2 impurity Kondo...)
  - Mesoscopic Kondo
  - Kondo in superconductors, in graphene...

Summary and concepts

- What is a magnetic impurity? (Friedel, Anderson)
  Local Moment Formation [very high T]

- Fermi Liquid (Nozières, Yamada)
  Renormalized quasiparticles [very low T]

- Kondo logs (Kondo, Anderson)
  Universality, Kondo temperature [high T]

- How to bridge the "gap"?
  - Numerics: NRG (Wilson), QMC
  - Analytical: bosonization, Bethe ansatz
  - Approx: slave boson, LMA,...

Basic description

Question: what becomes a magnetic atom in a metal?
Exp. fact: mag. impurities not always show a moment

Simplified atom: $H_U = \epsilon_d(n_{d\uparrow} + n_{d\downarrow}) + U n_{d\uparrow} n_{d\downarrow}$
$\Rightarrow$ Moment stabilized by U, high energy excitations

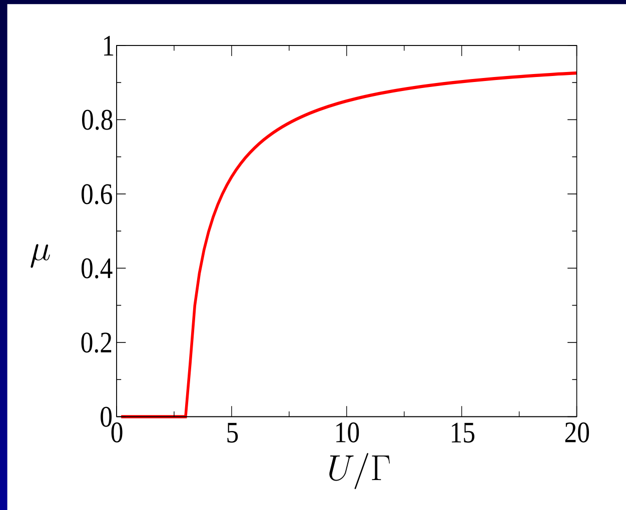
Resonant level: $H_V = \sum_{k\sigma} [\epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + V d^\dagger_\sigma c_{k\sigma} + h.c]$
$\Rightarrow$ No moment, low energy features

Anderson model: $H = H_U + H_V$
$\Rightarrow$ Competition localized/delocalized

What in between the two above limits?

Local moment formation

Mean field theory: frozen moment $\mu = \langle n_{d\uparrow} - n_{d\downarrow} \rangle$



Results:

- Local moment $\mu \neq 0$ for $U > \pi\Gamma = \pi V^2 \rho_0$ $\Rightarrow$ need narrow orbitals and low metallic DoS
- Hubbard bands $\Rightarrow$ high energy features remain

But: this fails at low energy!

Fermi liquid

Small U: perturbation theory well-behaved

$\Rightarrow$ Self-energy: $\Sigma_d(\omega) = (1 - Z^{-1})\omega + A\omega^2$

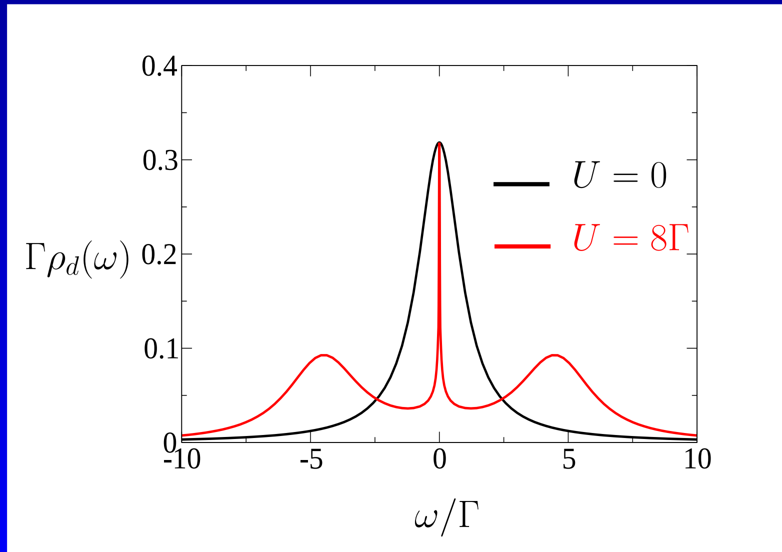
Effective width: $\Gamma_R = Z\Gamma$ (Kondo resonance)
$\Rightarrow$ Renormalized low energy quasiparticles
e.g. $\chi_{imp}(T = 0, U) \propto Z^{-1}\chi_{imp}(T = 0, U = 0)$

Sum rule: $\rho_d(\omega = 0) = 1/(\pi\Gamma)$ for all U at $T = 0$

Interpretation:
Screening at $T < Z\Gamma$

Problem:

- tough at $U \gg \Gamma$
- is that all?



??

Kondo model

S-W transformation: keep spin states at $U \gg \Gamma$

$\Rightarrow$ AF coupling $J = 8V^2/U$ to the Fermi sea

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J\vec{S} \cdot \sum_{\sigma\sigma'} c^\dagger_\sigma(0) \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'}(0)$$

Limit $J = 0$: local moment

- $S(T) = \log 2$ and $\chi_{imp}(T) = 1/(4T)$

Limit $J = \infty$: singlet state (screening)

- $S(T) = 0$ and $\chi_{imp}(T)$ finite

Kondo logs

Perturb in $j = J/D$: badly convergent at $T \ll D$!!

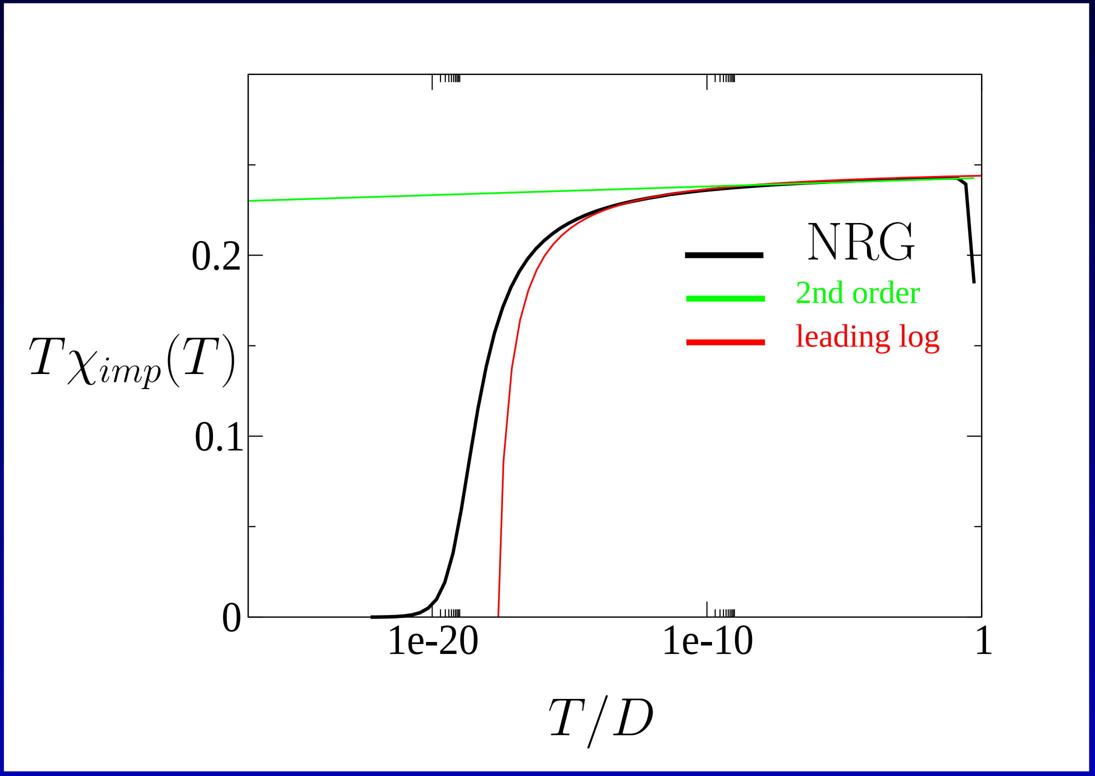
$$\chi_{imp}(T) = \frac{1}{4T}\left[1 - j - j^2 \log\frac{D}{T} - j^3 \log^2\frac{D}{T} + \ldots\right]$$
$$= \frac{1}{4T}\left[1 - j_R(T) + O(j_R^2)\right]$$

Renormalized coupling:

$$j_R(T) = \frac{1}{\log(T/T_K)}$$

Kondo temperature: $T_K = De^{-D/J} = De^{-\pi U/8\Gamma}$

What below T_K?



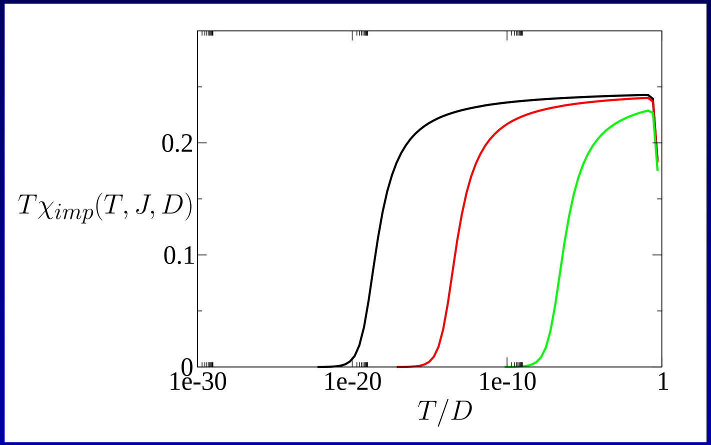
The spin is screened at $T = 0$ in agreement with FL

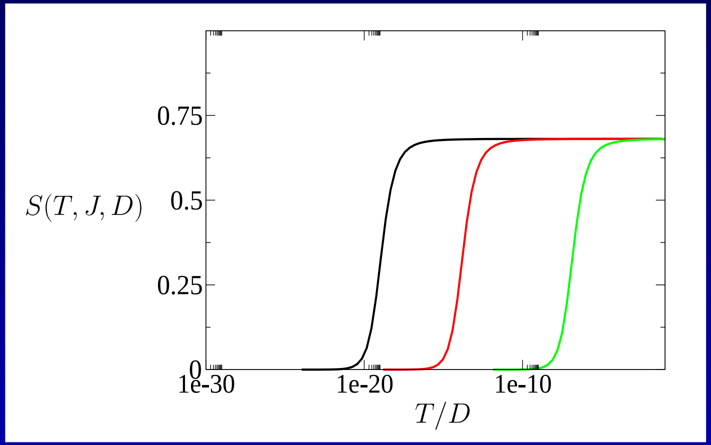
Saturation: $\chi(T = 0) \propto 1/T_K$
$\Rightarrow T_K = Z\Gamma$ is also the FL coherence scale

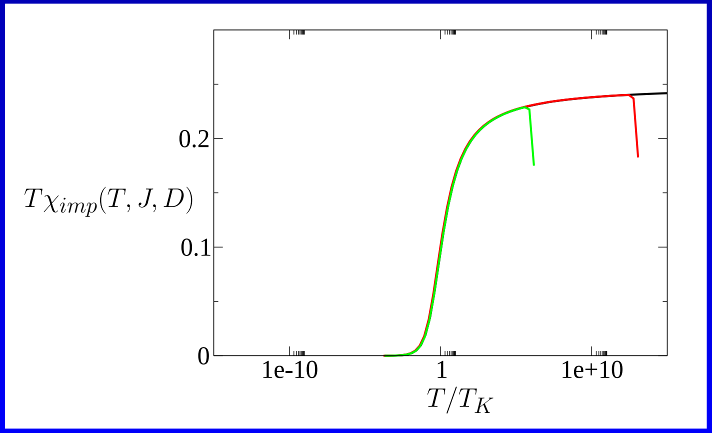
Universality

For $T \ll D$: $T\chi_{imp}(T/D, J/D) = \Phi(T/T_K)$

NRG runs: for three values of U/Γ







data collapse at $T \ll D$

Kondo signature in transport

T-matrix: embodies scattering properties

$$\mathcal{T}(\tau) = V^2 G_d(\tau)$$
$$= J^2 \left\langle \left(\vec{S} \cdot \vec{\tau}_{\sigma\sigma'} c^\dagger_\sigma \right)_\tau \left(\vec{S} \cdot \vec{\tau}_{\rho\rho'} c_\rho \right)_0 \right\rangle$$

Universal resistivity of Kondo alloys:

$$R(T) \propto \mathcal{I}m\,\mathcal{T}(T, \omega = 0) \propto \rho_d(T, \omega = 0) \propto j_R(T)^2$$