Local perspectives on disordered 2D electron gases at high magnetic fields

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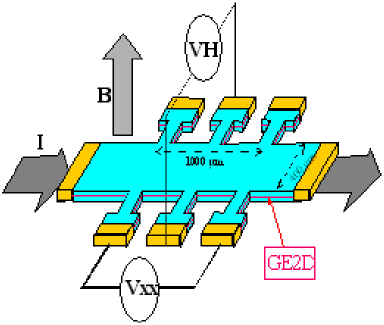
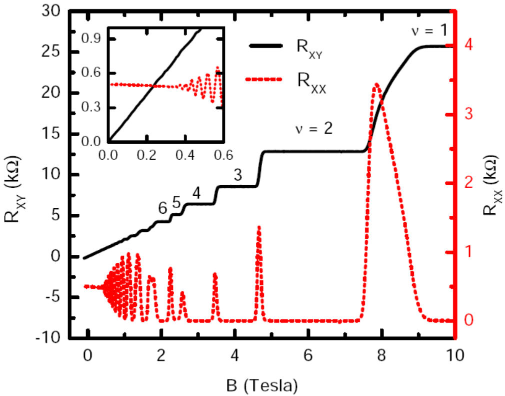
Motivation

What's so special about IQHE?

- High precision quantification of the Hall conductance
- Disorder plays a central and positive role

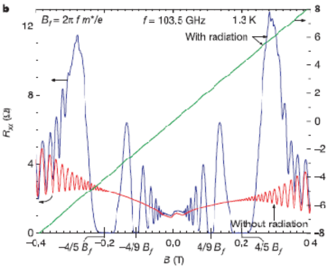




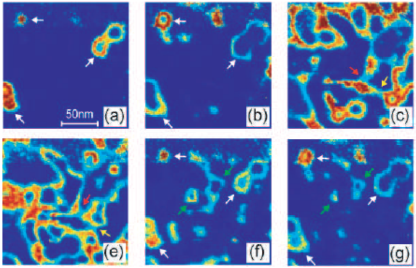
Why study now 2DEGs in a magnetic field?

Experiments:

- New systems: graphene
- New effects: microwave induced zero-resistance states
- New probes: local sensing techniques



Mani *et al.*, Nature (2002)



Hashimoto *et al.*, PRL (2008)

Is IQHE that well understood theoretically?

Unclear and difficult aspects:

- Quantum Hall breakdown at low field, high current...
- Plateau transitions
- Precision of quantized Hall conductance?

But more pragmatically: how do we calculate stuff??

- Weak coupling expansion in random smooth potential: invalid at high field! Raikh and Shahbazyan, PRB (1993)
- Edge state (scattering) picture: powerful for transport, but not practical with disorder Halperin PRB (1982), Büttiker PRB (1988)
- Guiding center (semi-classical) picture: Trugman PRB (1983) often used in practice, limited to high temperature

Summary

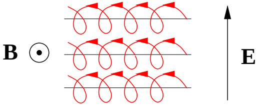
- Introduction: Landau levels and disorder
- The high magnetic field expansion:
  - Coherent states Green's functions formalism
  - Systematic semiclassical expansion
  - Quantum version of guiding center picture
  - Open vs closed quantum mechanics at high field
- Experimental implications:
  - Scanning Tunneling Spectroscopies
  - Local transport equations
- Outlook

Landau levels and disorder

Classical motion in high perpendicular magnetic field

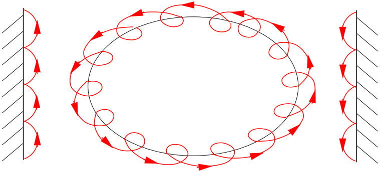
Two "degrees of freedom" with different timescales:

- fast cyclotron motion: $\frac{d\theta}{dt} = \omega_c = \frac{eB}{m^\star c}$
- slow drift velocity: $\mathbf{v}_d = \frac{c}{B}\mathbf{E} \times \hat{\mathbf{z}}$
- Decoupling at $B \to +\infty$



Transport:

- Disordered bulk: localization on closed equipotential lines
- Edges: delocalized skipping orbits



Quantum: translation invariant Landau eigenstates

Free Hamiltonian: no disorder, no interactions

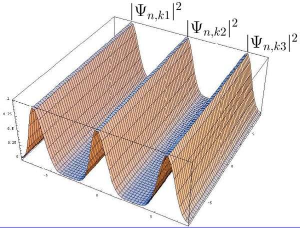
$$H_0 = \frac{1}{2m^\star}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 \quad \text{with} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Landau states:

$$E_{n,k} = \hbar\omega_c(n + \tfrac{1}{2})$$

$$\Psi_{n,k}(x,y) = e^{iky}\exp\left[-\frac{(x-kl_B^2)^2}{2l_B^2}\right] H_n\left(\frac{x-kl_B^2}{l_B}\right)$$

- Translationally invariant along y
- "Localized" along $x = kl_B^2$ on a scale $l_B = \sqrt{\hbar c/eB}$



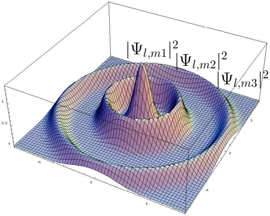
Another solution: circular eigenstates

Circular states:

$$E_{m,l} = \hbar\omega_c\left(l + \frac{m+|m|+1}{2}\right) = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

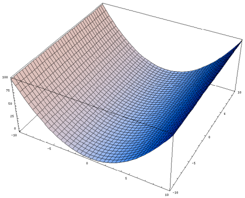
$$\Psi_{l,m}(r,\theta) = e^{im\theta} r^m \exp\left[\frac{-r^2}{4l_B^2}\right] L_l^m\left(\frac{r^2}{2l_B^2}\right)$$

- Rotationally invariant around the origin
- "Localized" on a scale l_B along r



The absence of an external potential leads to a huge degeneracy!

1D confinement



1D Parabolic potential:

$$H = H_0 + V(x) = H_0 + \frac{1}{2} m^\star \omega_0^2 x^2$$

Modified Landau states:

$$E_{n,k} = \hbar\Omega\left(n + \frac{1}{2}\right) + V(kL^2)$$

$$\Psi_{nk}(\mathbf{r}) = e^{-iky} \exp\left[-\frac{\left(x - \frac{\omega_c}{\Omega} kL^2\right)^2}{2L^2}\right] H_n\left(\frac{x - \frac{\omega_c}{\Omega} kL^2}{L}\right)$$

where $\Omega = \sqrt{\omega_c^2 + \omega_0^2} \simeq \omega_c$ and $L = \sqrt{\hbar/m^\star\Omega} \simeq l_B$

- Degeneracy is fully lifted by $V(x)$
- Wavefunction live around equipotential lines: $X = kl_B^2$
- Drift velocity: $v_y(X) = \frac{1}{\hbar}\frac{dEn,k}{dk}$

2D confinement

2D Parabolic potential: $H = H_0 + V(\mathbf{r}) = H_0 + \frac{1}{2} m^\star \omega_0^2 (x^2 + y^2)$

Fock-Darwin states:

$$\begin{aligned} E_{nl} &= \hbar\Omega \left(n + \frac{|l| + 1}{2} \right) + \frac{l}{2} \hbar\omega_c \\ &\simeq \hbar\omega_c \left(n + \frac{1}{2} \right) + \hbar \frac{\omega_0^2}{\omega_c} l \\ \Psi_{n,l}(\mathbf{r}) &= A \left(\frac{r}{\sqrt{2}L} \right)^{|l|} L_n^{|l|} e^{-\frac{r^2}{4L^2}} \left(\frac{r^2}{2L^2} \right) \frac{e^{il\theta}}{\sqrt{2\pi}} \end{aligned}$$

where $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2} \simeq \omega_c$ and $L = \sqrt{\hbar / m^\star \Omega} \simeq l_B$

- Energies are quantized
- … but one recovers continuous drift picture at $\omega_c \gg \omega_0$

Semi-classical guiding center picture

Basic idea: Trugman PRB (1983)

- treat cyclotron motion quantum mechanically: allows Landau levels formation
- drift is described classically

How it's usually done (with Landau states):

- $X = kl_B^2$: center of gaussian wavepacket
- $Y = -il_B^2 \frac{d}{dX}$: conjugate variable as $[X, Y] = il_B^2$
- Energy $E_{n,X} = \hbar\omega_c(n + \frac{1}{2}) + V(X)$ if $[X, Y] \simeq 0$

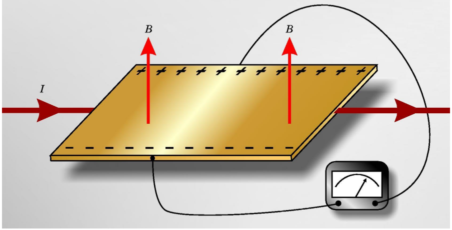
Limitations:

- No quantization of energies for a closed system
- No irreversibility for an open system (QPC)
- Problems to formulate consistent transport theory

Classical Hall effect

Local current density: $\mathbf{j}(\mathbf{r}) = -en_e(\mathbf{r})\mathbf{v}_d = -\frac{e}{B} n_e \mathbf{E} \times \hat{\mathbf{z}} = -\sigma_{xy} \mathbf{E}$

$\longrightarrow$ local Hall conductivity: $\sigma_{xy}(\mathbf{r}) = \frac{e}{B} n_e(\mathbf{r})$



For a homogeneous sample: $I = G_{xy} V_{xy}$

where $G_{xy} = \frac{e}{B} n_e$ is the Hall conductance

$\longrightarrow$ G_{xy} gives information on the carriers charge and density

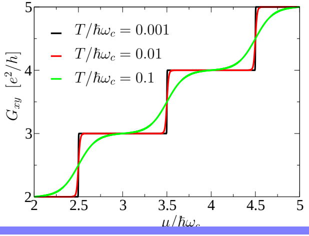
Quantum effects on Hall transport

Back to conductance:
$G_{xy} = \frac{e}{B} n_e = \frac{e^2}{h}\frac{h}{eB} n_e = \frac{e^2}{h}\nu$
$\nu = \frac{h}{eB} n_e = 2\pi l_B^2 n_e$: dimensionless density (filling factor)

Landau level quantization: $E_m = \hbar\omega_c(m + \frac{1}{2})$
with the cyclotron energy: $\hbar\omega_c = 20$meV at 10T for GaAs

Too naive picture of IQHE: successive filling of Landau level with integer $\nu = \sum_m n_F(E_m - \mu)$ leads to successive G_{xy} plateaus:

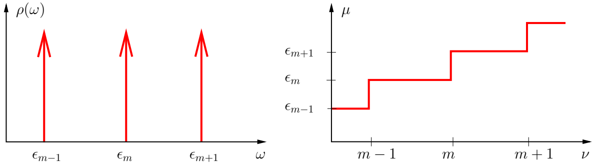
$$G_{xy} = \sum_m n_F(E_m - \mu)$$



Homogeneous system

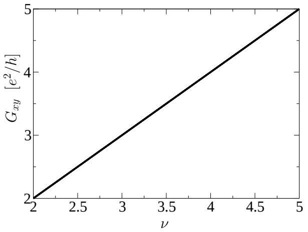
As a function of density:

- sharp Landau levels
- μ sticks to E_m and jumps between LL



Conductance:

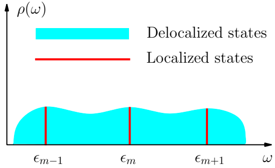
No plateau!

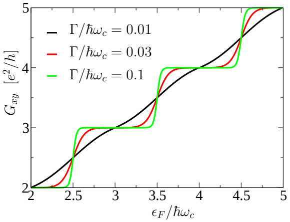


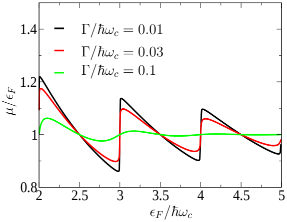
Inhomogeneous system

<u>With disorder:</u> μ is pinned by bulk localized states

$$\bar{\nu} = \sum_n \int \frac{d^2\mathbf{r}}{\mathcal{A}rea} n_F(E_m - \mu - V(\mathbf{r}))$$



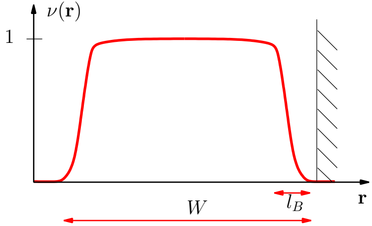




Disorder is essential to plateaus formation!

One more caveat...

Hall bar: quantization of σ_{xy} does not guarantee G_{xy} quantized



- width W $\sim$1mm
- non-homogeneous region near edge of width l_B $\sim$8nm

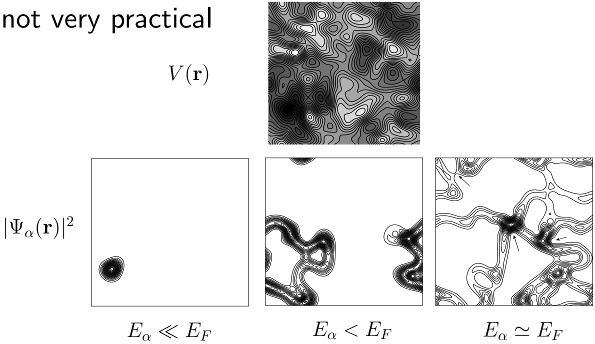
Deviation to quantization: of the order $l_B/W \sim 10^{-5}$
$\longrightarrow$ in contradiction with experiment: $\delta G_{xy}/G_{xy} < 10^{-9}$

Scattering transport theory (Landauer-Büttiker) is better!

With disordered potential

Numerical solutions:

- Confirms the intuition
- Can be coupled to Landauer formalism
- But not very practical



Is there a simple analytical approach at high magnetic field?

The high magnetic field expansion:

Coherent state Green's function formalism

[Champel & Florens PRB (2007)]

What is the small parameter?

At large magnetic field:

- Magnetic length: $l_B = \sqrt{\hbar c / eB}$ =8nm at 10T
- Correlation length of the disordered potential: $\xi \gtrsim$ 100nm in clean AsGa heterostructures

The random potential is smooth on the scale l_B!

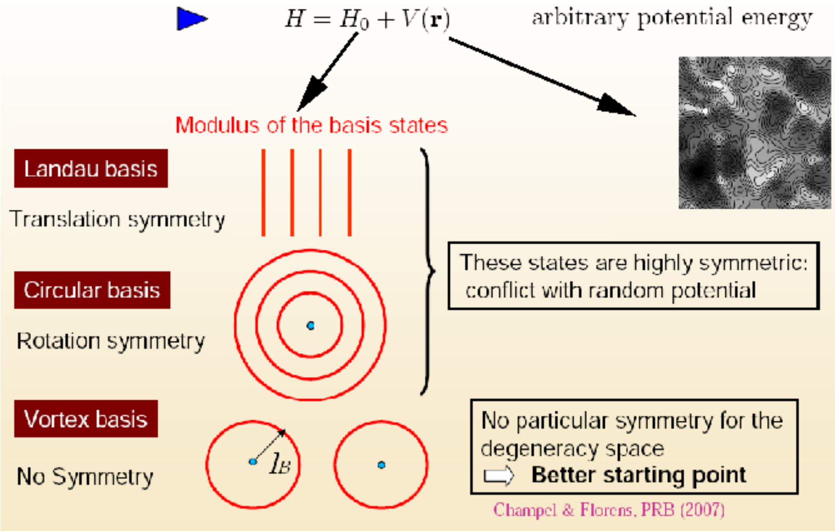
Remark: the idea of an l_B/ξ expansion is not new

- Effective Hamiltonian at order l_B^2 in the limit $l_B \to 0$ [Apenko & Lozovik J. Phys. (1984), Haldane & Yang PRL (1997)]
- Lowest Landau level projection [Girvin & Jach PRB (1983), Jain & Kivelson PRB (1988)]

The challenge:

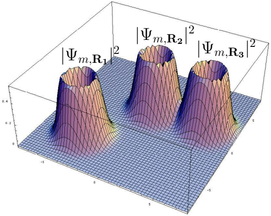
- Go beyond the strict $l_B/\xi \to 0$ limit
- Include Landau level mixing: Crucial for transport since $\langle n | \mathbf{j} | n' \rangle \propto \delta_{n,n' \pm 1}$

Clues for a high magnetic field expansion



Vortex (coherent) eigenstates

We need: states that can adapt to an arbitrary shape of $V(\mathbf{r})$, with no preferred symmetry
[Girvin & Jach PRB (1984)]
[Champel & Florens PRB (2007)]



Vortex states: $\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r} | \mathbf{R}, m \rangle$

$$E_{m,\mathbf{R}} = \hbar\omega_c(m + \frac{1}{2})$$

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = |\mathbf{r} - \mathbf{R}|^m \, e^{im \arg(\mathbf{r}-\mathbf{R})} \exp\left[-\frac{(\mathbf{r}-\mathbf{R})^2 - 2i\hat{\mathbf{z}} \cdot (\mathbf{r} \times \mathbf{R})}{4 l_B^2} \right]$$

Remark: this is an overcomplete, coherent eigenstates basis!!

$$\langle \mathbf{R_1}, m_1 | \mathbf{R_2}, m_2 \rangle = \delta_{m_1,m_2} \exp\left[-\frac{(\mathbf{R}_1 - \mathbf{R}_2)^2 - 2i\hat{\mathbf{z}} \cdot (\mathbf{R}_1 \times \mathbf{R}_2)}{4 l_B^2} \right]$$

Vortex Green's functions

How to proceed:

- Define $G_{\mathbf{R}_1,m_1;\mathbf{R}_2,m_2} = \langle \mathbf{R_1}, m_1 | (\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1} | \mathbf{R_2}, m_2 \rangle$
- Use unicity and closure relation:

$$\int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m=0}^{+\infty} |m, \mathbf{R}\rangle\langle m, \mathbf{R}| = \hat{1}$$

- Sandwich Dyson equation:

$$\begin{aligned}(\omega - E_{m_1} + i0^+) G_{\mathbf{R}_1,m_1;\mathbf{R}_2,m_2} &= \langle \mathbf{R_1}, m_1 | \mathbf{R_2}, m_2 \rangle \\ &+ \sum_{m_3=0}^{+\infty} \int \frac{d^2\mathbf{R}_3}{2\pi l_B^2} \ \langle \mathbf{R_1}, m_1 | \hat{V} | \mathbf{R_3}, m_3 \rangle G_{\mathbf{R}_3,m_3;\mathbf{R}_2,m_2}\end{aligned}$$

- Peculiar structure in the vortex coordinates

$$\langle \mathbf{R_1}, m_1 | \hat{V} | \mathbf{R_3}, m_3 \rangle = \langle \mathbf{R_1}, m_1 | \mathbf{R_3}, m_3 \rangle \ v_{m_1;m_3} \left(\frac{\mathbf{R_1}+\mathbf{R_3}}{2} + i \frac{\mathbf{R_3}-\mathbf{R_1}}{2} \times \hat{\mathbf{z}} \right)$$

The high magnetic field expansion

Systematic semiclassical expansion

[Champel, Florens & Canet PRB (2008)]

Solution of Dyson equation by l_B expansion

- Important relation:

$$G_{\mathbf{R}_1,m_1;\mathbf{R}_2,m_2} = \langle \mathbf{R_1}, m_1 | \mathbf{R_2}, m_2 \rangle \, g_{m_1;m_2} \left(\frac{\mathbf{R_1}+\mathbf{R_2}}{2} + i \frac{\mathbf{R_2}-\mathbf{R_1}}{2} \times \hat{\mathbf{z}} \right)$$

$g_{m_1;m_2}$ depends on a single (complex) vortex coordinate!

- Expand in powers of l_B: $g_{m_1;m_2}(\mathbf{R}) = \sum_{n=0}^{+\infty} \left(\frac{l_B}{\sqrt{2}}\right)^n g^{(n)}_{m1;m2}(\mathbf{R})$
- Perform integral over $\mathbf{R_3}$ and collect l_B^n terms: closed recursion

$$g^{(n)}_{m1;m2}(\mathbf{R}) = g^{(0)}_{m1;m2}(\mathbf{R}) \sum_{l=0}^{n-1} \sum_{j,k,p} \sum_{m_3} \frac{\delta_{n,2k+j+l}}{k!} \frac{\delta_{m_1+p,m_3+j-p}}{p!(j-p)!} \frac{(m_1+p)!}{\sqrt{m_1! m_3!}}$$
$$\times [(\partial_X - i\partial_Y)^{k+j-p} (\partial_X + i\partial_Y)^p V(\mathbf{R})] (\partial_X + i\partial_Y)^k g^{(l)}_{m3;m2}(\mathbf{R})$$

Lowest order result: semi-classical guiding center result

$$g^{(0)}_{m1;m2}(\mathbf{R}) = \frac{\delta_{m_1,m_2}}{\omega - E_{m_1} - V(\mathbf{R}) - i0^+}$$

How to get physical quantities at equilibrium?

Electronic Green function: $G(\mathbf{r},\mathbf{r}') = \langle \mathbf{r}|(\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1}|\mathbf{r}'\rangle$

Local observables:

- Local charge density: $\rho(\mathbf{r}) = -\int \frac{d\omega}{\pi} n_F(\omega) \mathrm{Im}\, G(\mathbf{r},\mathbf{r})$
- Local current density:
  $\mathbf{j}(\mathbf{r}) = -\int \frac{d\omega}{\pi} n_F(\omega) \left[\frac{e\hbar}{2m^\star}(\nabla_{\mathbf{r}'} - \nabla_{\mathbf{r}}) + i\frac{e^2}{m^\star c}\mathbf{A} \right] \mathrm{Im}\, G(\mathbf{r},\mathbf{r}')\Big|_{\mathbf{r}'=\mathbf{r}}$

Change of representation: same trick as before

$$G(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{R_1}}{2\pi l_B^2} \int \frac{d^2\mathbf{R_2}}{2\pi l_B^2} \sum_{m_1,m_2} \Psi^\star_{m_2,\mathbf{R_2}}(\mathbf{r}')\Psi_{m_1,\mathbf{R_1}}(\mathbf{r})\, G_{\mathbf{R_1},m_1;\mathbf{R_2},m_2}$$

$$G(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m_1,m_2} \Psi^\star_{m_2,\mathbf{R}}(\mathbf{r}')\Psi_{m_1,\mathbf{R}}(\mathbf{r}) \sum_{k=0}^{+\infty} \frac{1}{k!} \left(-\frac{l_B^2}{2}\Delta_{\mathbf{R}} \right)^k g_{m_1;m_2}(\mathbf{R})$$

Simple connexion to local vortex Green function $g(\mathbf{R})$

Checking accuracy: local charge density

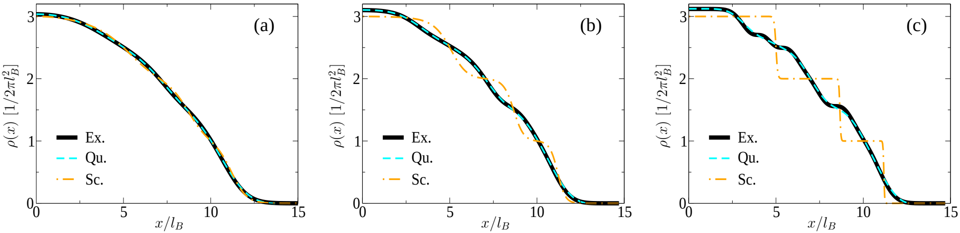
Quantum expansion: define $\xi_m(\mathbf{R}) = E_m + V(\mathbf{R}) - \mu$

$$\rho_{\mathrm{Qu.}}(\mathbf{r}) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_m n_F[\xi_m(\mathbf{R})] |\Psi_{m,\mathbf{R}}(\mathbf{r})|^2 + O(l_B^2)$$

Semiclassical result: point-like wavefunction for $l_B = 0$

$$\rho_{\mathrm{Sc.}}(\mathbf{r}) = \frac{1}{2\pi l_B^2} \sum_m n_F[\xi_m(\mathbf{r})]$$

Checking a on solvable 1D model: for $k_B T/\hbar\omega_c = 0.2, 0.1, 0.01$



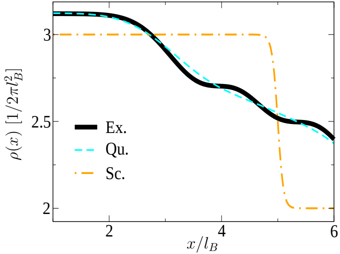
The high magnetic field expansion:

Quantum version of guiding center picture

[Champel & Florens arxiv:condmat (2009)]

What's missing in the strict l_B expansion?

Zooming in: deviations from terms in $|l_B \nabla_{\mathbf{R}} V|^{2n}$ associated to contributions of order $l_B^{2n} \Delta_{\mathbf{r}}^n \rho(\mathbf{r}) \sim \rho(\mathbf{r})$



Main discrepancy: vortex states are almost correct at high field

New viewpoint: instead of expanding order by order in l_B

- Resum all processes like $|l_B^p \partial_{\mathbf{R}}^p V(\mathbf{R})|^n$ to infinite order in n, but order by order in p

How to do it?

Non crucial simplification: $\hbar\omega_c = \infty$ kills Landau level mixing
$\Rightarrow$ Vortex Green functions become diagonal in m

First truncation: keep all terms of order $|l_B \nabla_{\mathbf{R}} V(\mathbf{R})|^n$ in g_m

$$1 = (\omega - E_m - V(\mathbf{R}) + i0^+) g_m(\mathbf{R}) + \frac{l_B^2}{2} \nabla_{\mathbf{R}} V . \nabla_{\mathbf{R}} g_m$$

Solution: introduce a modified Green's function $h_m(\mathbf{R})$

$$g_m(\mathbf{R}) = \sum_{p=0}^{+\infty} \frac{1}{p!} \left(\frac{l_B^2}{4} \Delta_{\mathbf{R}} \right)^p h_m(\mathbf{R})$$

Electronic Green function: new "vortex-Hermite" states

$$G(\mathbf{r}, \mathbf{r}) = \sum_{m=0}^{+\infty} \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \left| \Phi_m(\mathbf{R} - \mathbf{r}) \right|^2 h_m(\mathbf{R})$$

$$\left| \Phi_m(\mathbf{R}) \right|^2 = \frac{1}{\pi\, m! l_B^2} \frac{\partial^m}{\partial s^m} \left. \frac{e^{-A_s \mathbf{R}^2 / l_B^2}}{1+s} \right|_{s=0} \text{ with } A_s = (1-s)/(1+s)$$

Quantum formulation of the guiding center picture

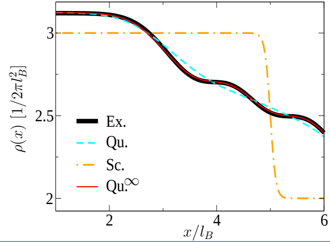
Modified vortex-Hermite Green's function:

Guiding center becomes exact: $h_m(\mathbf{R}) = [\omega + i0^+ - E_m - V(\mathbf{R})]^{-1}$

Rigorous formulation of an early idea by [Trugman PRB (1983)]

Back to 1D model: use new expression in the local density

$$\rho^{\infty}_{\mathrm{Qu.}}(\mathbf{r}) = \sum_{m=0}^{+\infty} \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \left|\Phi_m(\mathbf{R}-\mathbf{r})\right|^2 n_F(E_m + V(\mathbf{R}) - \mu) + O(l_B^2)$$



Need to go beyond the guiding center

Some non trivial questions:

- How to get quantized energies for a closed system?
- How to get irreversibility in an open system (QPC)?

Everything is encoded already in quadratic (curvature) terms!

Second truncation: keep all terms of order $|l_B^2 \partial_{\mathbf{R}}^2 V(\mathbf{R})|^n$ in h_m

$$\begin{aligned} 1 &= \left[\omega + i0^+ - E_m - V(\mathbf{R}) - \frac{2m+1}{4} l_B^2 \Delta_{\mathbf{R}} V\right] h_m(\mathbf{R}) \\ &+ \frac{l_B^4}{8} \left[\partial_Y^2 V \partial_X^2 + \partial_X^2 V \partial_Y^2 - 2\partial_X \partial_Y V \partial_X \partial_Y\right] h_m(\mathbf{R}) \end{aligned}$$

How do we solve this new EDP?

Dynamics of equipotential lines

Mapping: set $h_m(\mathbf{R}) = f_m\left[E(\mathbf{R})\right]$ with $E(\mathbf{R}) = V(\mathbf{R}) - V(\mathbf{R}_0)$

$$1 = \left[(\tilde{\omega}_m + i0^+ - E) + (\gamma E + \eta) \frac{d^2}{dE^2} + \gamma \frac{d}{dE} \right] f_m(E)$$

$$\begin{aligned}
\tilde{\omega}_m &= \omega - E_m - V(\mathbf{R}_0) - (m + 1/2)\zeta \\
\gamma &= \frac{l_B^4}{4}\left[\partial_{XX} V \partial_{YY} V - \partial_{XY} V \partial_{XY} V\right]\big|_{\mathbf{R}=\mathbf{R}_0} \\
\eta &= \frac{l_B^4}{8}\left[\partial_{XX} V (\partial_Y V)^2 + \partial_{YY} V (\partial_X V)^2 - 2\partial_{XY} V \partial_X V \partial_Y V\right]\big|_{\mathbf{R}=\mathbf{R}_0} \\
\zeta &= \frac{l_B^2}{2} \Delta_{\mathbf{R}} \, V\big|_{\mathbf{R}=\mathbf{R}_0}
\end{aligned}$$

γ, related to the curvature of the potential, provide the damping!

Solving the dynamical equation

Fourier transform gives the answer:

$$f_m(E) = -i \int_0^{+\infty} dt \, \frac{e^{-i(E+\eta/\gamma)\tau(t)}}{\cos(\sqrt{\gamma} t)} \, e^{i(\tilde{\omega}_m + i0^+ + \eta/\gamma)t}$$

with $\tau(t) = (1/\sqrt{\gamma}) \tan\left(\sqrt{\gamma} t\right)$

Interpretation:

- $\gamma > 0$ (quantum dot): $\tau(t)$ is periodic $\Rightarrow$ quantized energies!
- $\gamma < 0$ (QPC): $1/\cosh(\sqrt{-\gamma} t)$ cutoff $\Rightarrow$ irreversibility!

What was achieved: local quantum theory at high fields
[Champel & Florens arxiv:condmat (2009)]

Open problem (tough): non local aspects for arbitrary potential

Experimental implications:

Scanning tunneling spectroscopy

Champel & Florens arxiv:condmat (2009)

STS current

Generic expression:

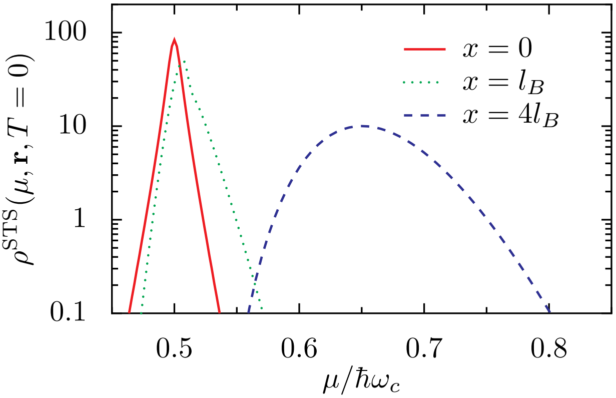
$$\rho^{\mathrm{STS}}(\mu, \mathbf{r}, T) = \frac{1}{2\pi l_B^2} \mathrm{Re} \sum_{m=0}^{+\infty} \frac{1}{m!} \frac{\partial^m}{\partial s^m} \int_0^{+\infty} dt \, \frac{Tt}{\sinh[\pi T t]} \times \left. \frac{e^{i[\mu - E_m - (m+1/2)\zeta - V(\mathbf{r})]t + i\frac{\eta}{\gamma}[t - \tau(t)] - \frac{\tau^2(t)}{4} \frac{A_s l_B^2 |\nabla_{\mathbf{r}} V|^2 + 4i\eta\tau(t)}{A_s^2 + iA_s\zeta\tau(t) - \gamma\tau^2(t)}}}{(1+s)\cos(\sqrt{\gamma}t)\sqrt{A_s^2 + iA_s\zeta\tau(t) - \gamma\tau^2(t)}} \right|_{s=0}$$

Remarks:

- Valid for any Landau level
- Valid for any potential locally described up to its second derivatives
- Valid for high and low temperature

Spectral properties

Model saddle point: $V(\mathbf{R}) = \omega_0 XY$



Quite different lineshapes/linewidths depending on tip position (note spectral asymmetries)

Interpretation

Various regimes

- Thermal dominated (semiclassical): $\omega_T = \pi T$

$$\rho^{\mathrm{STS}}(\mu, \mathbf{r}, T) \approx \frac{1}{2\pi l_B^2} \frac{\mathrm{sech}^2\left(\frac{\mu-\omega_c/2-V(\mathbf{r})}{2T}\right)}{4T}$$

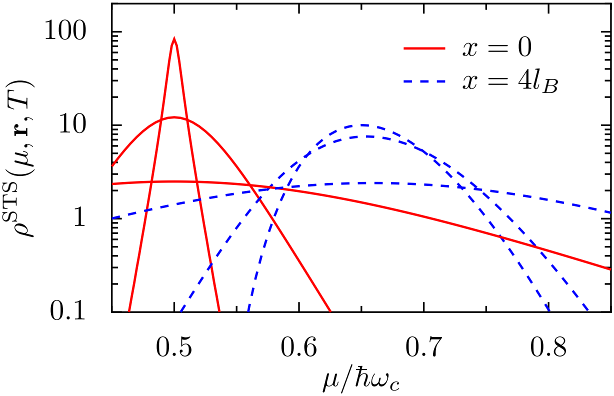
- Drift dominated: $\omega_{\mathrm{drift}} = l_B |\nabla_{\mathbf{r}} V(\mathbf{r})|$

$$\rho^{\mathrm{STS}}(\mu, \mathbf{r}, T) \approx \frac{1}{2\pi l_B^2} \frac{\exp\left[-\left(\frac{\mu-\omega_c/2-V(\mathbf{r})}{\omega_{\mathrm{drift}}}\right)^2\right]}{\sqrt{\pi}\omega_{\mathrm{drift}}}$$

- Curvature dominated: $\omega_{\mathrm{saddle}} = 2\sqrt{-\gamma}$

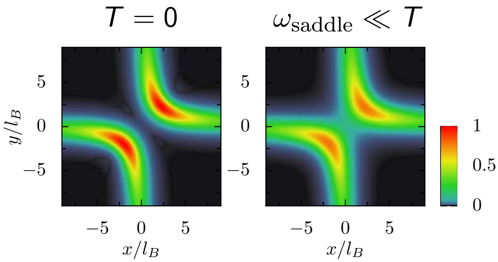
$$\rho^{\mathrm{STS}}(\mu, \mathbf{r}, T) \approx \frac{P_{-1/2+ia}(0)}{2\pi l_B^2} \frac{\mathrm{sech}\left(\frac{\mu-\omega_c/2-V(\mathbf{r})}{\omega_{\mathrm{saddle}}/\pi}\right)}{\sqrt{2}\omega_{\mathrm{saddle}}}$$

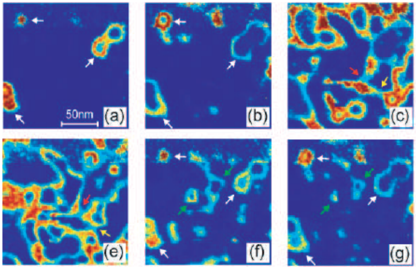
Temperature effects



Thermal smearing more effective near saddle-points

What should one see?





Champel & Florens condmat (2009) Hashimoto *et al.* PRL (2009)

True quantum tunneling states are hard to see experimentally!

Experimental implications:

Transport equations

[Champel, Florens & Canet PRB (2008)]

Various transport regimes

Low temperature:

- Tunneling dominated
- Landauer-Büttiker is a good picture, but unpractical
- Open problem for vortex theory (non-locality)

High temperature:

- Landau level mixing dominated
- Guiding center is a good picture

Next question:

What about semiclassical transport?

Local current density: semiclassical result

Current at leading order: for $l_B \to 0$

- Drift (conduction)

$$\mathbf{j}^{(\mathbf{0})}_{\mathrm{drift}}(\mathbf{r}) = \frac{e}{h} \sum_{m=0}^{+\infty} n_F[\xi_m(\mathbf{r})] \nabla_{\mathbf{r}} V(\mathbf{r}) \times \hat{\mathbf{z}}$$

- Density gradient (diffusion) [Geller & Vignale PRB (1994)]

$$\mathbf{j}^{(\mathbf{0})}_{\mathrm{grad}}(\mathbf{r}) = \frac{e}{h} \sum_{m=0}^{+\infty} \hbar\omega_c \left(m + \frac{1}{2}\right) \nabla_{\mathbf{r}} n_F[\xi_m(\mathbf{r})] \times \hat{\mathbf{z}}$$

Sub-leading current: new terms!
[Champel, Florens & Canet PRB (2008)]

$$\mathbf{j}^{(\mathbf{2})}_{\mathrm{drift}}(\mathbf{r}) = l_B^2 \frac{e}{h} \sum_{m=0}^{+\infty} n_F[\xi_m(\mathbf{r})] \left[\frac{(\nabla_{\mathbf{r}} V \cdot \nabla_{\mathbf{r}})}{\hbar\omega_c} \nabla_{\mathbf{r}} V + \frac{3}{2}\left(m + \frac{1}{2}\right) \Delta_{\mathbf{r}} \nabla_{\mathbf{r}} V \right] \times \hat{\mathbf{z}}$$

Local equilibrium: Ohm's law

Total potential: $V(\mathbf{r}) = V_{\rm eff}(\mathbf{r}) + e\Phi(\mathbf{r})$

- $V_{\rm eff}$: confinement and impurity (screened) potential
- $\Phi(\mathbf{r})$: local out-of-equilibrium potential

Local conductivity tensor: purely transverse at $l_B \to 0$

- $\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r})\mathbf{E} = -\sigma_H(\mathbf{r})\nabla\Phi(\mathbf{r}) \times \hat{\mathbf{z}}$
- $\sigma_H(\mathbf{r}) = \sum_m n_F[E_m + V_{\rm eff}(\mathbf{r}) - \mu]$

Transport equation: $\nabla.\mathbf{j} = 0$ (continuity equation) gives

$$\big(\nabla\sigma_H(\mathbf{r}) \times \nabla\Phi(\mathbf{r})\big).\hat{\mathbf{z}} = 0$$

- Equipotentials coincide with lines of constant filling factor
- Indeterminacy at points where $\nabla\sigma_H(\mathbf{r}) = \mathbf{0}$

Conduction beyond the drift contribution

- Non-local contribution to current:

$$\delta \mathbf{j}(\mathbf{r}) = l_B^2 \frac{e^2}{h} \sum_{m=0}^{+\infty} n_F[\xi_m(\mathbf{r})] \frac{3}{2} \left(m + \frac{1}{2} \right) \Delta_{\mathbf{r}} \nabla_{\mathbf{r}} \Phi \times \hat{\mathbf{z}}$$

Originates from quantum tunneling (negligeable at high T)

- Longitudinal and transverse corrections to the conductivity:

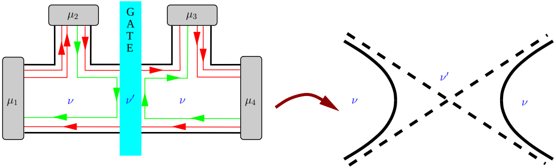
$$\delta \hat{\sigma}(\mathbf{r}) = \frac{l_B^2}{\hbar \omega_c} \sigma_H(\mathbf{r}) \begin{pmatrix} -\partial_{xy} V_{\text{eff}} & \partial_{yy} V_{\text{eff}} \\ \partial_{xx} V_{\text{eff}} & \partial_{xy} V_{\text{eff}} \end{pmatrix}$$

Local conductivities may not obey Onsager's relation!

Transport equation: keeping local terms only

$$(\nabla_{\mathbf{r}} \sigma_H \times \nabla_{\mathbf{r}} \Phi) \cdot \hat{\mathbf{z}} - \frac{l_B^2}{\hbar \omega_c} \sigma_H \mathrm{Tr} \left\{ \delta \hat{\sigma} . \begin{pmatrix} \partial_{xx} \Phi & \partial_{xy} \Phi \\ \partial_{xy} \Phi & \partial_{yy} \Phi \end{pmatrix} \right\} = 0$$

Checking bulk conduction against Büttiker picture



Toy model: $V_{\rm eff}(\mathbf{r}) = V_{\rm eff}(\mathbf{0}) + a\frac{x^2}{2} + b\frac{y^2}{2}$

- Non-trivial potential drop [for saddle point only $(ab < 0)$]:

  $$\Phi(\mathbf{r}) = \left[A + B\int_0^{x/\lambda} dt\,\exp(-t^2)\right]\left[C + D\int_0^{y/\lambda} dt\,\exp(-t^2)\right]$$

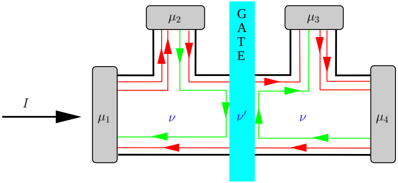
  where $\lambda^2 = -2\frac{l_B^2}{\hbar\omega_c}\frac{\sum_m n_F(\xi_m(\mathbf{0}))}{\sum_m n_F'(\xi_m(\mathbf{0}))}$

- Two-point conductance: $G_{2P} = \frac{e^2}{h}\sigma_H(\mathbf{0})$ Edge state result!

Remark: the conductance is independent of microscopic aspects

Scattering: a simple example

4 terminals with single "scatterer":



Transmissions:

$T_{2\leftarrow 1} = \nu,\ T_{3\leftarrow 2} = \nu',\ T_{1\leftarrow 2} = \nu - \nu'$, etc...

Resistances:

- $R_{14} = \frac{h}{e^2\nu'}$: two-point resistance
- $R_{34} = \frac{h}{e^2\nu}$: Hall resistance
- $R_{23} = \frac{h}{e^2}(\frac{1}{\nu'} - \frac{1}{\nu})$: four-point resistance

Transport: conduction vs. diffusion

Bottomline:

- Transport in IQHE regime may be investigated on the basis of simple bulk equations
- Microscopic details of the equilibrium density and current inhomogeneities are naturally taken into account: practical approach

Interesting directions to investigate:

- General connection to edge state formalism
- Study bulk transport equations for complex geometries (i.e. disordered)
- Role of non-local corrections: low temperature regime
- Coupling to self-consistent screening calculations

Conclusion

- Vortex wavefunctions are the naturally selected quantum states in the high field limit
- The mathematical foundation of vortex Green's functions was established:
  - generates trivially the semiclassical expansion
  - provides a fully quantum approach to guiding center ideas
  - unifies closed and open systems (quantization vs. irreversibility)
- Local equilibrium observables can be calculated accurately from simple and controlled density functionals
- Semi-classical (high temperature) transport equations were proposed and investigated for a simplified scattering problem