Local density of states in disordered 2DEG at high magnetic fields

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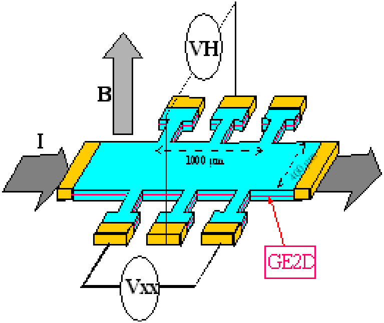
Summary

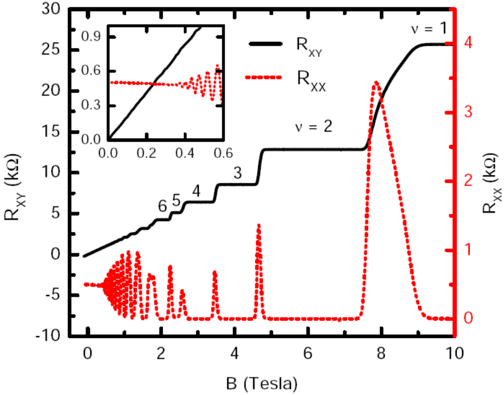
- Motivation: high resolution STS measurements
- Quantum formulation of the guiding center picture
- Experimental implications for LDoS
- Perspectives

Motivation

Macroscopics of IQHE: transport

- High precision quantization of the Hall conductance
- Universal and non-universal features
- Disorder plays a central role in the phenomenon

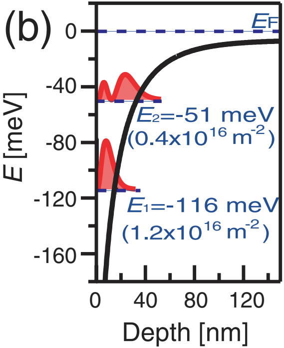


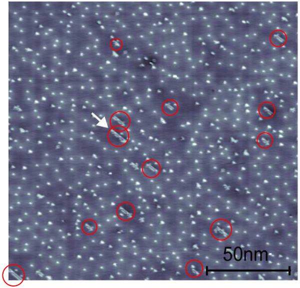


Microscopic view: local measurements

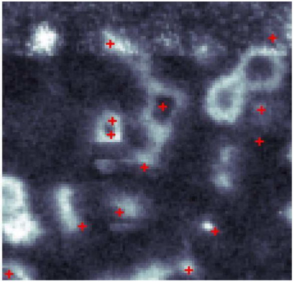
New STS experiment: Hashimoto *et al.*, PRL (2008)

- InSb surface states form a 2DEG (deposited Cs)
- High resolution, low temperature STM
- QHE in LL0 at B=12T



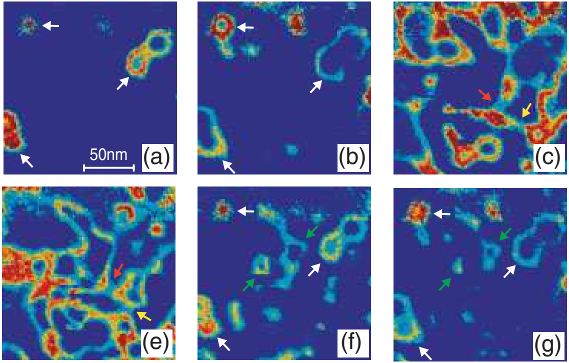


Surface topography



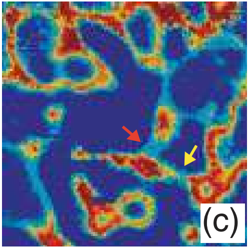
STS spectra

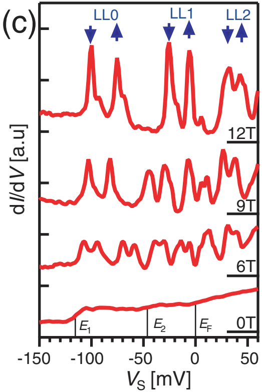
LDoS from STS spectra



- Thin spectral lines: wavefunctions of width $l_B \sim 7$nm
- Disordered landscape: typical lengthscale $\xi \sim 40$nm
- Percolation of lines at the threshold

Some other remarks





- wide structures: tunneling at saddle points?
- narrowing of Landau levels at high B

Aim: simple analytical theory for LDoS at high magnetic fields

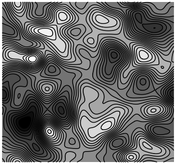
Standard theoretical approaches (I)

Wavefunctions

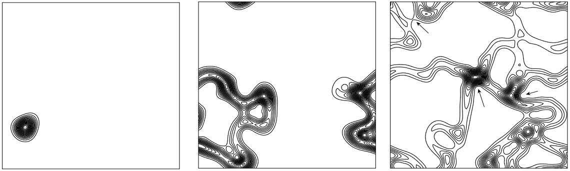
Schroedinger equation

$$H = \frac{1}{2m^\star}\left[\frac{\hbar}{i}\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right]^2 + V(\mathbf{r}) \quad \text{with} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$V(\mathbf{r})$



$|\Psi_\alpha(\mathbf{r})|^2$



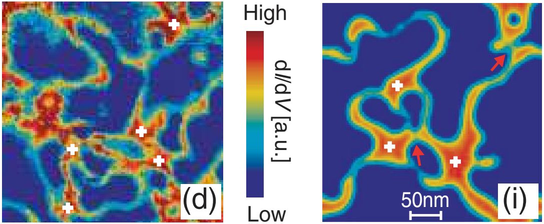
$E_\alpha \ll E_{cr.}$ $E_\alpha \lesssim E_{cr.}$ $E_\alpha \simeq E_{cr.}$

STS LDoS:

$$\rho^{\mathrm{STS}}(\epsilon, \mathbf{r}, T) \propto \sum_\alpha |\Psi_\alpha(\mathbf{r})|^2 \frac{\partial}{\partial\epsilon} n_F(E_\alpha - \epsilon)$$

Comparison theory/experiment

Qualitative LDoS: Hashimoto *et al.*, PRL (2008)



but...

- Expensive numerical method
- Physical scales at play: non obvious!
- Unpractical inverse problem $\rho^{\mathrm{STS}}(\mathbf{r}) \to V(\mathbf{r})$

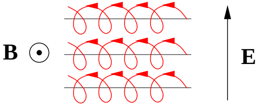
Standard theoretical approaches (II)

Semiclassical limit

Classical motion in high perpendicular magnetic field

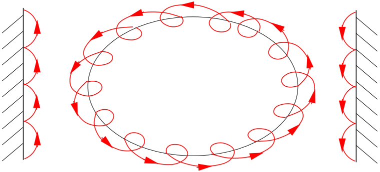
Two "degrees of freedom" with different timescales:

- fast cyclotron motion: $\frac{d\theta}{dt} = \omega_c = \frac{eB}{m^\star c}$
- slow drift velocity: $\mathbf{v}_d = \frac{c}{B}\mathbf{E} \times \hat{\mathbf{z}}$
- Decoupling at $B \to +\infty$



Motion:

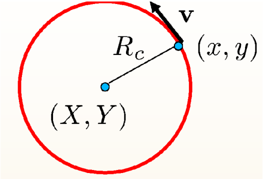
- Disordered bulk: localization on closed equipotential lines
- Edges: delocalized skipping orbits



Semi-classical guiding center picture

Basic idea:

- Quantum mechanical cyclotron motion: Landau levels
- Drift motion is described classically



New coordinates:

- $\hat{x} = \hat{X} + \delta\hat{x} = \hat{X} + \hat{v}_y/\omega_c$
- $\hat{y} = \hat{Y} + \delta\hat{y} = \hat{Y} - \hat{v}_x/\omega_c$
- Hamiltonian: $H = m^\star \hat{\mathbf{v}}^2/2 + V(\hat{X} + \delta\hat{x}, \hat{Y} + \delta\hat{y})$

Quantization:

- $[\hat{X}, \hat{Y}] = i l_B^2$ and $[\hat{v}_x, \hat{v}_y] = -i\hbar\omega_c/m^\star$
- Magnetic length: $l_B = \sqrt{hc/eB}$
- Cyclotron frequency: $\omega_c = eB/m^\star c$

Implication for the LDoS

High field limit: $B \to +\infty$

- Hamiltonian: $H \simeq m^\star \hat{\mathbf{v}}^2/2 + V(X,Y)$
- Energy: $E_{n,\mathbf{r}} = \hbar\omega_c(n + \frac{1}{2}) + V(\mathbf{r})$
- LDoS: $\rho^{\mathrm{STS}}(\epsilon, \mathbf{r}, T) \propto \sum_n \frac{\partial}{\partial \epsilon} n_F(E_{n,\mathbf{r}} - \epsilon)$

Limitations:

- Classical states with no transverse spatial spread ($l_B = 0$)
  $\to$ LDoS peaks of constant width (set by temperature T)
- No quantization of energies for a closed system
- No dissipation associated to tunneling

What states?

Translation invariant Landau eigenstates

Free Hamiltonian: no disorder, no interactions

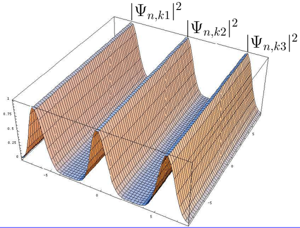
$$H_0 = \frac{1}{2m^\star}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 \quad \text{with} \quad \mathbf{B} = \nabla\times\mathbf{A}$$

Landau states:

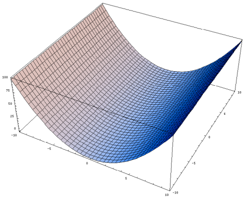
$$E_{n,k} = \hbar\omega_c(n+\tfrac{1}{2})$$

$$\Psi_{n,k}(x,y) = e^{iky}\exp\left[-\frac{(x-kl_B^2)^2}{2l_B^2}\right] H_n\left(\frac{x-kl_B^2}{l_B}\right)$$

- Translationally invariant along y
- "Localized" along $x = kl_B^2$ on a scale $l_B = \sqrt{\hbar c/eB}$



1D confinement



1D Parabolic potential:

$$H = H_0 + V(x) = H_0 + \frac{1}{2} m^\star \omega_0^2 x^2$$

Modified Landau states:

$$E_{n,k} = \hbar\Omega\left(n + \frac{1}{2}\right) + V(kL^2)$$

$$\Psi_{nk}(\mathbf{r}) = e^{-iky} \exp\left[-\frac{\left(x - \frac{\omega_c}{\Omega} kL^2\right)^2}{2L^2}\right] H_n\left(\frac{x - \frac{\omega_c}{\Omega} kL^2}{L}\right)$$

where $\Omega = \sqrt{\omega_c^2 + \omega_0^2} \simeq \omega_c$ and $L = \sqrt{\hbar / m^\star \Omega} \simeq l_B$

- Degeneracy is fully lifted by $V(x)$
- Wavefunction live around equipotential lines: $X = k l_B^2$
- Drift velocity: $v_y(X) = \frac{1}{\hbar} \frac{dEn,k}{dk}$

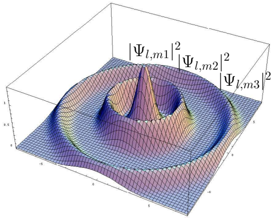
Circularly invariant eigenstates

Circular states: (no disorder, no confinement)

$E_{m,l} = \hbar\omega_c(l + \frac{m+|m|+1}{2}) = \hbar\omega_c(n + \frac{1}{2})$

$\Psi_{l,m}(r,\theta) = e^{im\theta} r^m \exp\left[\frac{-r^2}{4l_B^2}\right] L_l^m\left(\frac{r^2}{2l_B^2}\right)$

- Rotationally invariant around the origin
- "Localized" on a scale l_B along radius

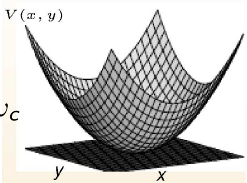


The absence of an external potential leads to a huge degeneracy!

2D confinement

2D Parabolic potential: $H = H_0 + V(\mathbf{r}) = H_0 + \frac{1}{2} m^\star \omega_0^2 (x^2 + y^2)$

Fock-Darwin states:



$$E_{nl} = \hbar\Omega \left(n + \frac{|l|+1}{2} \right) + \frac{l}{2}\hbar\omega_c$$

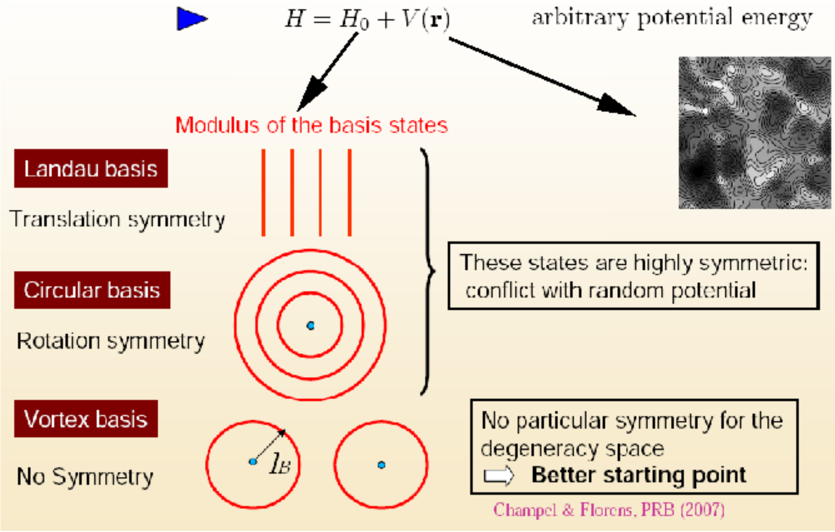
$$\simeq \hbar\omega_c \left(n + \frac{1}{2} \right) + \hbar\frac{\omega_0^2}{\omega_c} l$$

$$\Psi_{n,l}(\mathbf{r}) = A \left(\frac{r}{\sqrt{2}L} \right)^{|l|} e^{-\frac{r^2}{4L^2}} L_n^{|l|} \left(\frac{r^2}{2L^2} \right) \frac{e^{il\theta}}{\sqrt{2\pi}}$$

where $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2} \simeq \omega_c$ and $L = \sqrt{\hbar / m^\star \Omega} \simeq l_B$

- Energies are quantized
- ... but one recovers continuous drift picture at $\omega_c \gg \omega_0$

With a random potential

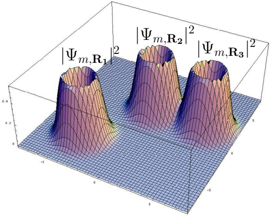


Vortex (coherent) eigenstates

We need: states that can adapt to an arbitrary shape of $V(\mathbf{r})$, with no preferred symmetry

[Girvin & Jach PRB (1984)]

[Champel & Florens PRB (2007)]



Vortex states: $\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r} | \mathbf{R}, m \rangle$

$$E_{m,\mathbf{R}} = \hbar\omega_c(m + \frac{1}{2})$$

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = |\mathbf{r}-\mathbf{R}|^m \, e^{im\arg(\mathbf{r}-\mathbf{R})} \exp\left[-\frac{(\mathbf{r}-\mathbf{R})^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{r}\times\mathbf{R})}{4l_B^2}\right]$$

Remark: this is an overcomplete, coherent eigenstates basis!!

$$\langle \mathbf{R_1}, m_1 | \mathbf{R_2}, m_2 \rangle = \delta_{m_1,m_2} \exp\left[-\frac{(\mathbf{R}_1-\mathbf{R}_2)^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{R}_1\times\mathbf{R}_2)}{4l_B^2}\right]$$

The high magnetic field expansion:

Coherent state Green's function formalism

[Champel & Florens PRB (2007)]

[Champel, Florens & Canet PRB (2008)]

[Champel & Florens PRB (2009)]

What is the small parameter?

At large magnetic field:

- Magnetic length: $l_B = \sqrt{\hbar c/eB}$ =7nm at 12T
- Correlation length of the disordered potential: $\xi > 100$nm in clean AsGa heterostructures

The random potential is smooth on the scale l_B!

Remark: the idea of an l_B/ξ expansion is not new

- Effective energy up order l_B^2 in the limit $l_B \to 0$ [Apenko & Lozovik J. Phys. (1984), Haldane & Yang PRL (1997)]
- LDoS is still too sharply peaked

The challenge: Go beyond the strict $l_B \to 0$ limit

- Develop a theory controlled at small non-zero l_B/ξ

Vortex representation

Real space Green function: $G(\mathbf{r},\mathbf{r}') = \langle \mathbf{r}|(\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1}|\mathbf{r}'\rangle$

- LDoS: $\rho^{\mathrm{STS}}(\varepsilon,\mathbf{r},T) \propto - \int \frac{d\omega}{\pi}\frac{\partial}{\partial\omega} n_F(\omega-\varepsilon)\mathrm{Im}\, G(\mathbf{r},\mathbf{r})$

Local vortex Green function:

$\tilde{g}_{m;m'}(\mathbf{R}) = e^{-\left(l_B^2/2\right)\Delta_{\mathbf{R}}}\langle \mathbf{R}, m|(\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1}|\mathbf{R}, m'\rangle$

Connexion:

$$G(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{R}}{2\pi l_B^2}\sum_{m_1,m_2}\tilde{g}_{m_1;m_2}(\mathbf{R}) e^{-\left(l_B^2/4\right)\Delta_{\mathbf{R}}}\left[\Psi^\star_{m_2,\mathbf{R}}(\mathbf{r}')\Psi_{m_1,\mathbf{R}}(\mathbf{r})\right]$$

Advantages:

- Wave-function spread naturally encoded
- $\tilde{g}(\mathbf{R})$ contains all the information about the spectrum
- $\tilde{g}(\mathbf{R})$ can be systematically developed in powers of l_B

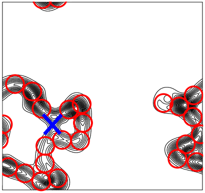
Quantum formulation of the guiding center picture

Locally flat (yet arbitrary) potential:

Guiding center becomes exact: $\tilde{g}_m(\mathbf{R}) = [\omega + i0^+ - E_m - V(\mathbf{R})]^{-1}$

Rigorous formulation of the early idea by [Trugman PRB (1983)]

$$G(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_m \frac{e^{-(l_B^2/4)\Delta_{\mathbf{R}}} \left[\Psi^{\star}_{m,\mathbf{R}}(\mathbf{r}')\Psi_{m,\mathbf{R}}(\mathbf{r})\right]}{\omega + i0^+ - E_m - V(\mathbf{R})}$$



- Powerful expression describing the physics in the Landau tails [Raikh-Shahbazyan 1995, Fogler-Chklovskii 1998]
- $\overline{\log|G(\mathbf{r},\mathbf{r}')|} = -|\mathbf{r}-\mathbf{r}'|/L_{\mathrm{loc.}}$

Need to go beyond the guiding center

Some missing quantum effects:

- How to get quantized energies for a closed system?
- How to get dissipation at tunneling trajectories?

Everything is encoded already in quadratic (curvature) terms!

Truncate Dyson: keep all terms of order $|l_B^2 \partial_{\mathbf{R}}^2 V(\mathbf{R})|^n$

$$1 = \left[\omega + i0^+ - E_m - V(\mathbf{R}) - \frac{2m+1}{4} l_B^2 \Delta_{\mathbf{R}} V\right] \tilde{g}_m(\mathbf{R}) + \frac{l_B^4}{8}\left[\partial_Y^2 V \partial_X^2 + \partial_X^2 V \partial_Y^2 - 2\partial_X \partial_Y V \partial_X \partial_Y\right] \tilde{g}_m(\mathbf{R})$$

This EDP can be exactly solved! Champel & Florens PRB (2009)

Experimental implications:

Scanning tunneling spectroscopy

STS current: interpretation (I)

Generic expression in lowest LL ($m=0$):

$$\rho^{\rm STS}(\varepsilon,\mathbf{r},T) = \frac{1}{2\pi l_B^2}{\rm Re}\int_0^{+\infty} dt\, e^{i[\varepsilon-E_0-\zeta/2-V(\mathbf{r})]t} \times \frac{Tt}{\sinh[\pi T t]} \frac{e^{i\frac{\eta}{\gamma}[t-\tau(t)]-\frac{\tau^2(t)}{4}\frac{l_B^2|\nabla_{\mathbf{r}}V|^2+4i\eta\tau(t)}{1+i\zeta\tau(t)-\gamma\tau^2(t)}}}{\cos(\sqrt{\gamma}t)\sqrt{1+i\zeta\tau(t)-\gamma\tau^2(t)}}$$

LDoS lines: Fourier transform peaks the LDoS around lines of constant (effective) energy: $V(\mathbf{r})+\zeta(\mathbf{r})/2=\varepsilon$
Here: $\zeta(\mathbf{r})=(l_B^2/2)\Delta_{\mathbf{r}}V$ [Haldane & Yang PRL (1997)]

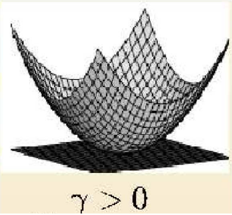
LDoS width: set by two obvious cutoffs

- Thermal cutoff at scale $\omega_{\rm therm.}=\pi T$
- Drift cutoff at scale $\omega_{\rm drift}=l_B|\nabla_{\mathbf{r}}V|$

STS current: interpretation (II)

Gaussian curvature effects: $\gamma(\mathbf{r}) = \frac{l_B^4}{4}[\partial_{xx}V\partial_{yy}V - \partial_{xy}V\partial_{xy}V]$

$$\rho^{\mathrm{STS}}(\varepsilon,\mathbf{r},T) = \frac{1}{2\pi l_B^2}\mathrm{Re}\int_0^{+\infty} dt\, e^{i[\varepsilon - E_0 - \zeta/2 - V(\mathbf{r})]t} \times \frac{Tt}{\sinh[\pi T t]} \frac{e^{i\frac{\eta}{\gamma}[t-\tau(t)] - \frac{\tau^2(t)}{4}\frac{l_B^2|\nabla_{\mathbf{r}}V|^2 + 4i\eta\tau(t)}{1+i\zeta\tau(t)-\gamma\tau^2(t)}}}{\cos(\sqrt{\gamma}t)\sqrt{1+i\zeta\tau(t)-\gamma\tau^2(t)}}$$



- Positive curvature: confinement
- Periodic function of time: $\tau(t) = (1/\sqrt{\gamma})\tan\left(\sqrt{\gamma}t\right)$ $\rightarrow$ quantized energy levels!

STS current: interpretation (II)

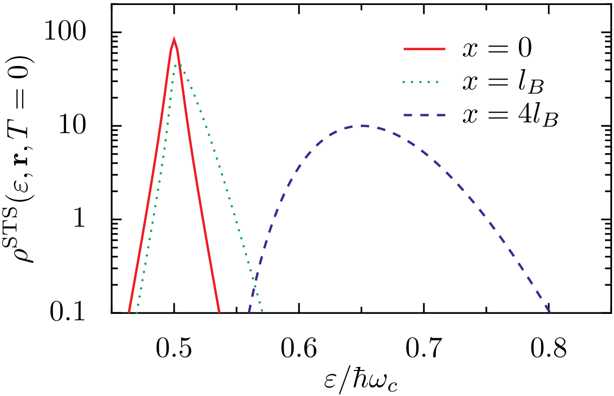
Gaussian curvature effects: $\gamma(\mathbf{r}) = \frac{l_B^4}{4}[\partial_{xx}V\partial_{yy}V - \partial_{xy}V\partial_{xy}V]$

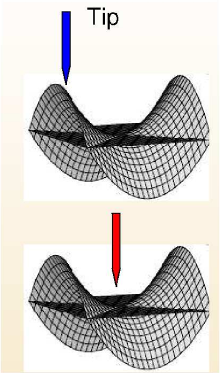
$$\rho^{\mathrm{STS}}(\varepsilon, \mathbf{r}, T) = \frac{1}{2\pi l_B^2}\mathrm{Re}\int_0^{+\infty} dt\, e^{i[\varepsilon - E_0 - \zeta/2 - V(\mathbf{r})]t} \times \frac{Tt}{\sinh[\pi T t]} \frac{e^{i\frac{\eta}{\gamma}[t-\tau(t)] - \frac{\tau^2(t)}{4}\frac{l_B^2|\nabla_{\mathbf{r}}V|^2 + 4i\eta\tau(t)}{1+i\zeta\tau(t)-\gamma\tau^2(t)}}}{\cosh(\sqrt{-\gamma}t)\sqrt{1+i\zeta\tau(t)-\gamma\tau^2(t)}}$$

$\gamma < 0$

- Negative curvature: tunneling
- New cutoff energy at scale
  $\rightarrow$ Lifetime! $\omega_{\mathrm{saddle}} = 2\sqrt{-\gamma(\mathbf{r})}$

Zero temperature LDoS for saddle point





Quite different lineshapes/linewidths depending on tip position

Interpretation

Various regimes

- Thermal dominated (semiclassical): $\omega_{\text{therm.}} = \pi T$

$$\rho^{\text{STS}}(\varepsilon, \mathbf{r}, T) \approx \frac{1}{2\pi l_B^2} \frac{\operatorname{sech}^2\left(\frac{\varepsilon - \omega_c/2 - V(\mathbf{r})}{2T}\right)}{4T}$$

- Drift dominated: $\omega_{\text{drift}} = l_B |\nabla_{\mathbf{r}} V(\mathbf{r})|$

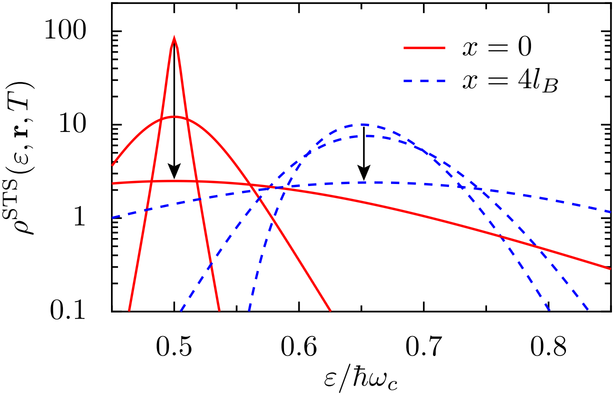
$$\rho^{\text{STS}}(\varepsilon, \mathbf{r}, T) \approx \frac{1}{2\pi l_B^2} \frac{\exp\left[-\left(\frac{\varepsilon - \omega_c/2 - V(\mathbf{r})}{\omega_{\text{drift}}}\right)^2\right]}{\sqrt{\pi}\omega_{\text{drift}}}$$

- Curvature dominated: $\omega_{\text{saddle}} = 2\sqrt{-\gamma}$

$$\rho^{\text{STS}}(\varepsilon, \mathbf{r}, T) \approx \frac{P_{-1/2+ia}(0)}{2\pi l_B^2} \frac{\operatorname{sech}\left(\frac{\varepsilon - \omega_c/2 - V(\mathbf{r})}{\omega_{\text{saddle}}/\pi}\right)}{\sqrt{2}\omega_{\text{saddle}}}$$

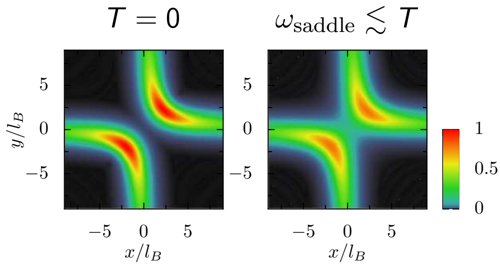
Temperature effects on LDoS

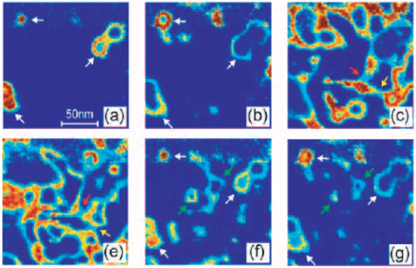
Increasing temperature: $T/\omega_c = 0, 0.02, 0.1$



Thermal smearing more effective near saddle-points

What should one see experimentally?

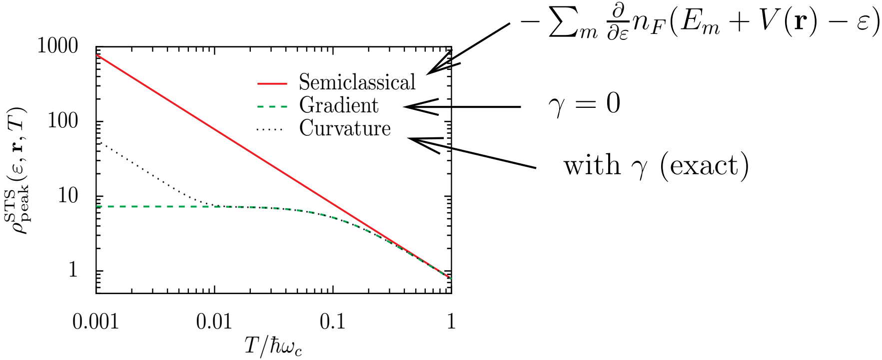




Champel & Florens condmat (2009) Hashimoto *et al.* PRL (2009)

- Thermal smearing is more important near saddle points
- True quantum tunneling states are hard to see experimentally!

How controlled?



- Existence of a hierachy of local energy scales
- Previous LDoS expression valid down to scale $V_{\mathrm{typ.}}(l_B/\xi)^3$

Conclusion

- The mathematical formulation of a quantum guiding center theory was established
- Closed and open systems (quantization vs. tunneling) can be unified in this picture
- Local equilibrium observables can be calculated accurately from simple and controlled density functionals
- A generic expression for the LDoS at high magnetic field was proposed

Perspectives

- Quantitative comparison to STS experiments
- Direct application to self-consistent (Hartree-Fock or LDA) calculations for 2DEGs
- Correlate local to global properties: transport near the percolation threshold