Percolation Physics from Local Spectroscopies and Transport in Quantum Hall Systems

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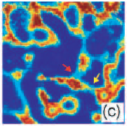
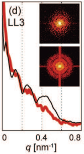
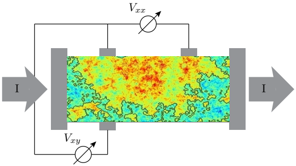
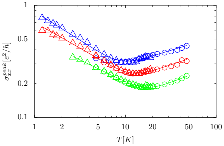








Acknowlegments

Theory part:

- Thierry Champel @ LPMMC - Grenoble/France
- Martina Flöser @ Néel Institute - Grenoble/France
- Mikhail Raikh @ University of Utah - Salt Lake City/USA
- Rudolf Roemer @ Warwick University - Coventry/UK

Experimental part:

- Katsushi Hashimoto @ Tohoku University - Sendai/Japan
- Markus Morgenstern @ Aachen University - Aachen/Germany
- Benjamin Piot & Duncan Maude @ LNCMI - Grenoble/France

Summary

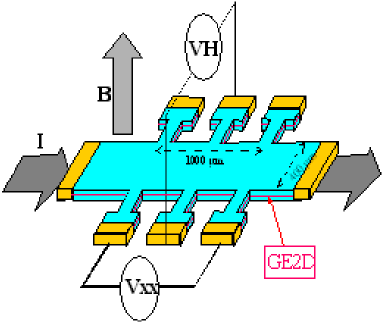
- Local spectroscopies of quantum Hall samples
  - Local density of states (LDoS) measurements by STM
  - Quantum formulation of the guiding center picture
  - Revealing the nodal structure of Landau states
  - Predictions for two-point correlations of the LDoS

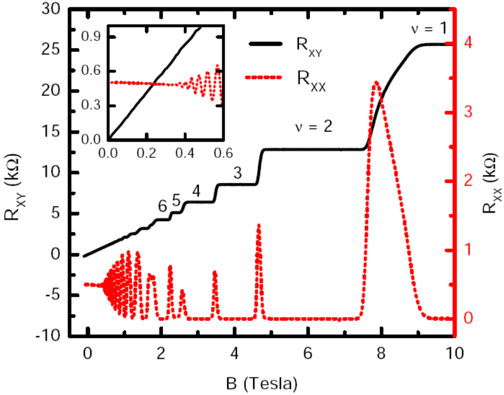
- Percolating transport in the high temperature regime of quantum Hall transitions
  - Effective medium approach to transport critical exponents
  - Scaling function of $\sigma_{xx}(T, B)$ at high temperature
  - Comparison to experiments

Motivation: from macroscopics to microscopics of QHE

Macroscopics of IQHE: transport

- High precision quantization of the Hall conductance
- Universal and non-universal features
- Disorder plays a central role in the phenomenon

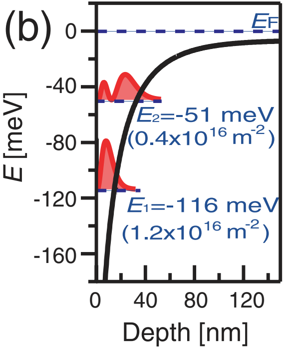


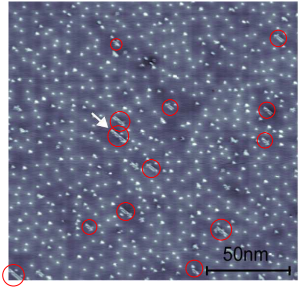


Microscopic view: local measurements

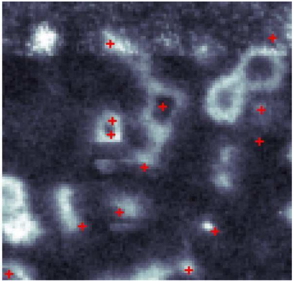
New STM experiment: Hashimoto *et al.*, PRL (2008)

- InSb surface states form a 2DEG (deposited Cs)
- High resolution, low temperature STM
- QHE in LL0 at B=12T





Surface topography

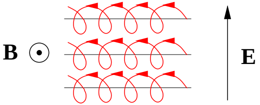


STS spectra

Classical motion in high perpendicular magnetic field

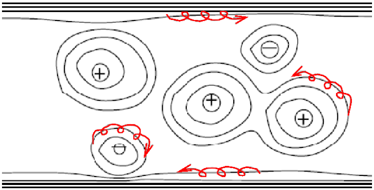
Two "degrees of freedom" with different timescales:

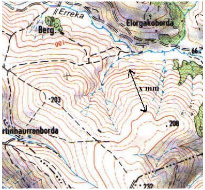
- fast cyclotron motion: $\frac{d\theta}{dt} = \omega_c = \frac{eB}{m^\star c}$
- slow drift velocity: $\mathbf{v}_d = \frac{c}{B}\mathbf{E} \times \hat{\mathbf{z}}$
- Decoupling at $B \to +\infty$



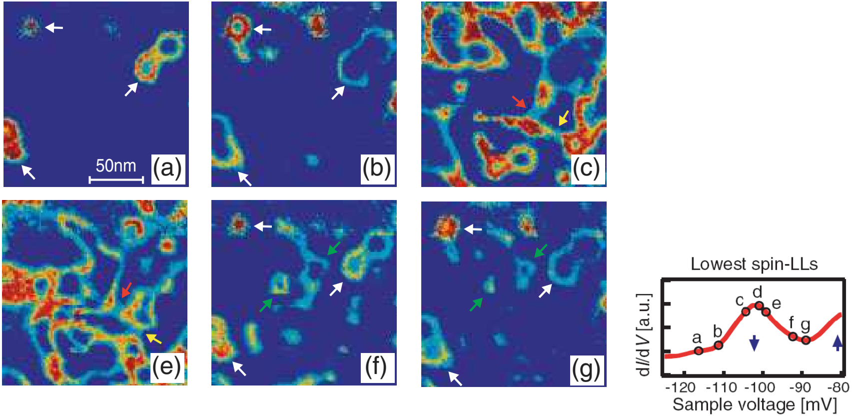
Motion:

- Disordered bulk: localization on closed equipotential lines
- Sharp edges: delocalized skipping orbits



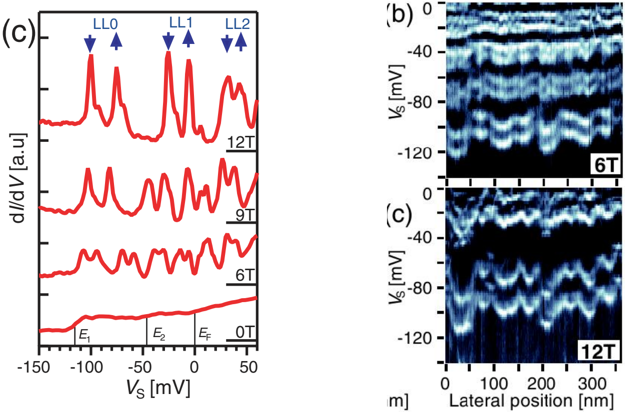


LDoS spatial maps at various energies Hashimoto *et al.*, PRL (2008)



- Thin spectral lines: wavefunction width $l_B = \sqrt{\hbar c/eB} \simeq 7$nm
- Disordered landscape: typical lengthscale $\xi \simeq 40$nm
- Percolation of wavefunction at the Landau band center

A closer look on energy and space dependence



- LDOS shows well-defined LLs with narrowing energy width at increasing B
- Successive LLs have different spatial energy dispersion

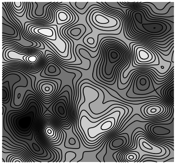
Standard theoretical approaches (I)

Wavefunctions

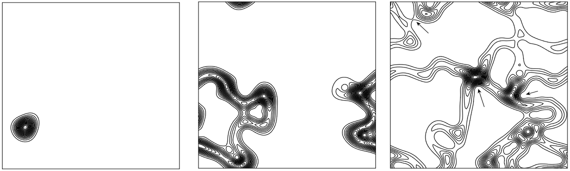
Schrödinger equation

$$H = \frac{1}{2m^\star}\left[\frac{\hbar}{i}\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right]^2 + V(\mathbf{r}) \quad \text{with} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$V(\mathbf{r})$



$|\Psi_\alpha(\mathbf{r})|^2$



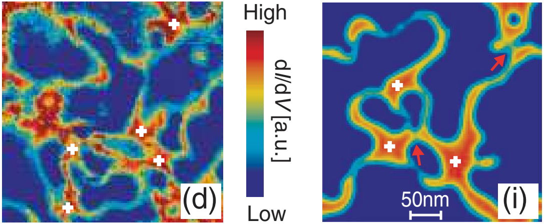
$E_\alpha \ll E_{cr.}$ $\quad E_\alpha \lesssim E_{cr.}$ $\quad E_\alpha \simeq E_{cr.}$

STS LDoS:

$$\rho^{\rm STS}(\epsilon, \mathbf{r}, T) \propto \sum_\alpha |\Psi_\alpha(\mathbf{r})|^2 \frac{\partial}{\partial \epsilon} n_F(E_\alpha - \epsilon)$$

Comparison theory/experiment

Numerical simulations: Hashimoto *et al.*, PRL (2008)



reproduce semi-quantitatively the data

but...

- Expensive numerical method
- Physical scales at play: non obvious!
- Unpractical inverse problem $\rho^{\mathrm{STS}}(\epsilon, \mathbf{r}, T) \rightarrow V(\mathbf{r})$

Aim: simpler analytical theory

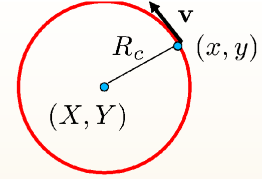
Standard theoretical approaches (II)

Semiclassical limit

Semi-classical guiding center picture

Basic idea:

- Quantum mechanical cyclotron motion: Landau levels
- Drift motion is described classically



New coordinates:

- $\hat{x} = \hat{X} + \delta\hat{x} = \hat{X} + \hat{v}_y/\omega_c$
- $\hat{y} = \hat{Y} + \delta\hat{y} = \hat{Y} - \hat{v}_x/\omega_c$
- Hamiltonian: $H = m^\star \hat{\mathbf{v}}^2/2 + V(\hat{X} + \delta\hat{x}, \hat{Y} + \delta\hat{y})$

Quantization:

- $[\hat{X}, \hat{Y}] = i{l_B}^2$ and $[\hat{v}_x, \hat{v}_y] = -i\hbar\omega_c/m^\star$
- Magnetic length: $l_B = \sqrt{\hbar c/eB}$
- Cyclotron frequency: $\omega_c = eB/m^\star c$

Implication for the LDoS

Semi-classical high field limit: $\omega_c \to +\infty$ and $l_B \to 0$

- Hamiltonian: $H \simeq m^\star \hat{\mathbf{v}}^2/2 + V(X,Y)$
- Classical guiding center: $[X,Y]=0$
- Energy: $E_{n,\mathbf{R}} = \hbar\omega_c(n+\frac{1}{2}) + V(\mathbf{R})$
- LDoS: $\rho^{\mathrm{STS}}(\epsilon,\mathbf{r},T) \propto \sum_n \frac{\partial}{\partial\epsilon} n_F(E_{n,\mathbf{r}}-\epsilon)$

Limitations:

- Same effective potential $E_{n,\mathbf{R}}$ for all n (incorrect)
- Classical states that are infinitely sharp ($l_B = 0$)
  $\to$ LDoS peaks width is set by temperature T (incorrect)

Quantum high field limit: $\omega_c \to +\infty$ only

- $[\hat{X},\hat{Y}] = i l_B^2$: Phase space is real space!
  This suggests the use of coherent states

The high magnetic field expansion:

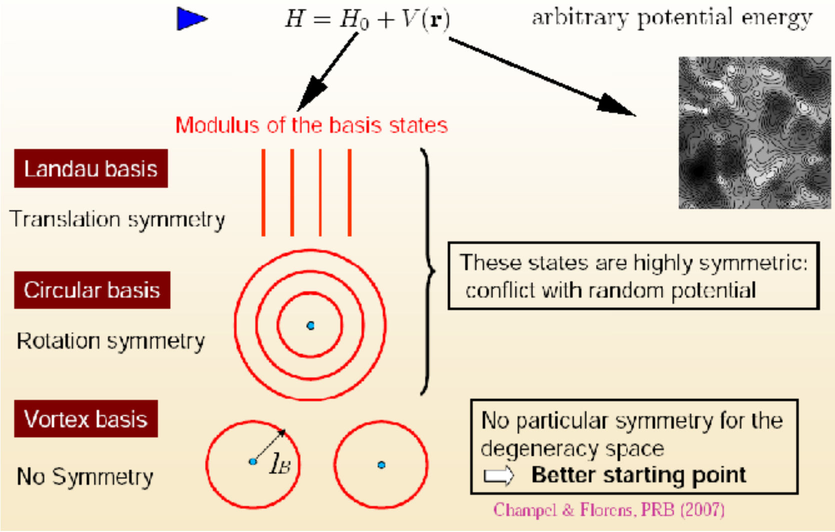
Coherent state Green's function formalism

[Champel & SF PRB (2007)]

[Champel, SF & Canet PRB (2008)]

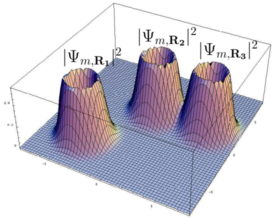
[Champel & SF PRB (2009)]

What states with a random potential?



Vortex coherent eigenstates

We need: states that can adapt to an arbitrary shape of $V(\mathbf{r})$, with no preferred symmetry
[Jain & Kivelson PRB (1988)]
[Champel & SF PRB (2007)]



Vortex states: $\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r}|\mathbf{R}, m\rangle$

$$E_{m,\mathbf{R}} = \hbar\omega_c(m + \frac{1}{2})$$

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = |\mathbf{r}-\mathbf{R}|^m \, e^{im\arg(\mathbf{r}-\mathbf{R})} \exp\left[-\frac{(\mathbf{r}-\mathbf{R})^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{r}\times\mathbf{R})}{4l_B^2}\right]$$

Remark: this is an overcomplete, coherent eigenstates basis!!

$$\langle \mathbf{R_1}, m_1|\mathbf{R_2}, m_2\rangle = \delta_{m_1,m_2} \exp\left[-\frac{(\mathbf{R}_1-\mathbf{R}_2)^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{R}_1\times\mathbf{R}_2)}{4l_B^2}\right]$$

What are the small parameters?

At large magnetic field:

- Magnetic length: $l_B = \sqrt{\hbar c/eB}$ =7nm at 12T
- Correlation length of disordered potential: $\xi^{\mathrm{GaAs}} > 40$nm

⇒ The random potential is smooth on the scale l_B!

- Cyclotron energy: at $B = 10$T, $\hbar\omega_c^{InSb} = 700$K and $\hbar\omega_c^{GaAs} = 200$K
- Typical disorder variation: $\sqrt{\langle V^2 \rangle} \simeq 200$K for surface of InSb (much less in GaAs)

⇒ Landau levels decouple!

Remark: previous authors have used a strict l_B/ξ expansion [Apenko & Lozovik J. Phys. (1984), Haldane & Yang PRL (1997)]

Challenge: develop a theory controlled at small but non-zero l_B/ξ

Vortex Green's functions

Real space Green function: $G(\mathbf{r},\mathbf{r}') = \langle \mathbf{r}|(\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1}|\mathbf{r}'\rangle$

- LDoS: $\rho^{\mathrm{STS}}(\varepsilon,\mathbf{r},T) \propto -\int \frac{d\omega}{\pi}\frac{\partial}{\partial\omega} n_F(\omega-\varepsilon)\mathrm{Im}\,G(\mathbf{r},\mathbf{r})$

Local vortex Green function:

$$g_{m;m'}(\mathbf{R}) = e^{-\left(l_B^2/4\right)\Delta_{\mathbf{R}}}\langle \mathbf{R},m|(\omega - \hat{H}_0 - \hat{V} + i0^+)^{-1}|\mathbf{R},m'\rangle$$

Connexion to real space observables: (exact relation)

$$G(\mathbf{r},\mathbf{r}') = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m_1,m_2} g_{m_1;m_2}(\mathbf{R}) e^{-\left(l_B^2/4\right)\Delta_{\mathbf{R}}} \left[\Psi^\star_{m_2,\mathbf{R}}(\mathbf{r}')\Psi_{m_1,\mathbf{R}}(\mathbf{r})\right]$$

Advantages:

- Wave-function transverse spread on scale l_B naturally encoded
- $g(\mathbf{R})$ can be systematically developed in powers of l_B/ξ

Guiding center and cyclotron motion decoupling

Vortex view of LDoS: $g_{m,m'} = g_m \delta_{m,m'}$ diagonal at large ω_c

$$\rho(\mathbf{r}, E) = -\frac{1}{\pi} Im \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{n=0}^{+\infty} F_n(\mathbf{R} - \mathbf{r})\, g_n(\mathbf{R}, E)$$

$$\text{with structure factor : } \; F_n(\mathbf{R}) = \frac{(-1)^n}{\pi l_B^2} L_n\left(\frac{2\mathbf{R}^2}{l_B^2}\right) e^{-\mathbf{R}^2/l_B^2},$$

Interpretation: quantum dynamics of the electron results from convolution of guiding center drifting and cyclotron orbit

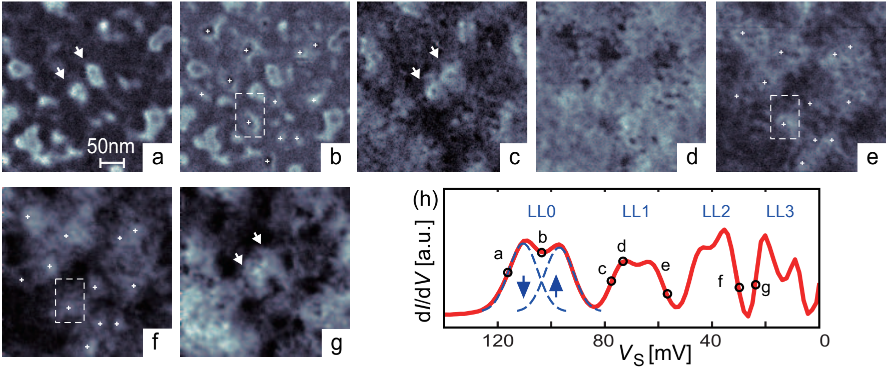
- LDoS factorizes in momentum-space: $\widetilde{\rho}(\mathbf{q}, E) = -\frac{1}{\pi} Im \sum_{n=0}^{+\infty} \widetilde{F}_n(\mathbf{q}) \tilde{g}_n(\mathbf{q}, E)$
- F_n lives on small scale l_B, $\tilde{g}_n$ on large scale ξ
- F_n encodes the nodal structure of Landau levels

Question: can we unveil the nodes in the experiment?

Nodal structure of Landau levels

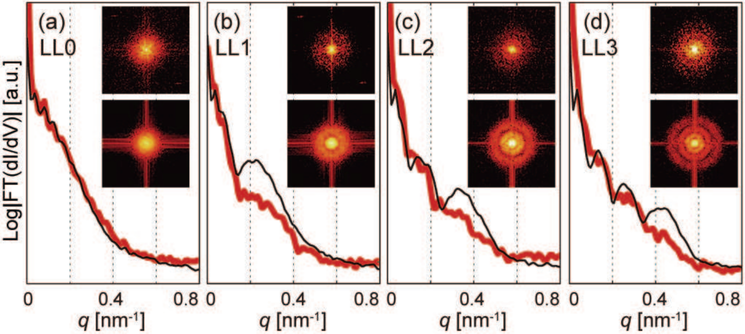
Hashimoto, Champel, SF, Sohrmann, Wiebe, Hirayama, Roemer, Wiesendanger, Morgenstern, arXiv (2012)

Real space LDoS data at $B = 6\,T$



- 4 successive LLs are observed (spin resolved)
- The drift trajectories are blurred in the high LLs
  ... but no obvious signature of the nodal structure

Momentum-space LDoS data at $B = 6T$



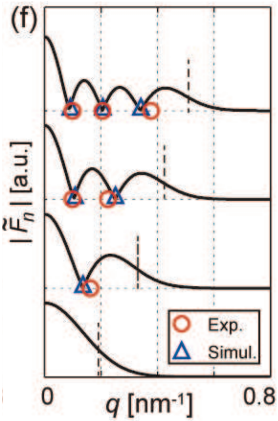
- Structures appear at scale $1/l_B \simeq 0.1\text{nm}^{-1}$
- Spectra are rotationnally invariant
- LLn shows n kinks in the momentum-dependence
- Good comparison experiment/simulations

Extraction of the nodes

Comparison with guiding center theory:

$$\widetilde{\rho}(\mathbf{q}, E) = -\frac{1}{\pi} Im \sum_{n=0}^{+\infty} \widetilde{F}_n(\mathbf{q}) \tilde{g}_n(\mathbf{q}, E)$$

Kinks in $\widetilde{\rho}(\mathbf{q}, E)$ follow the nodes of $\widetilde{F}_n(\mathbf{q})$

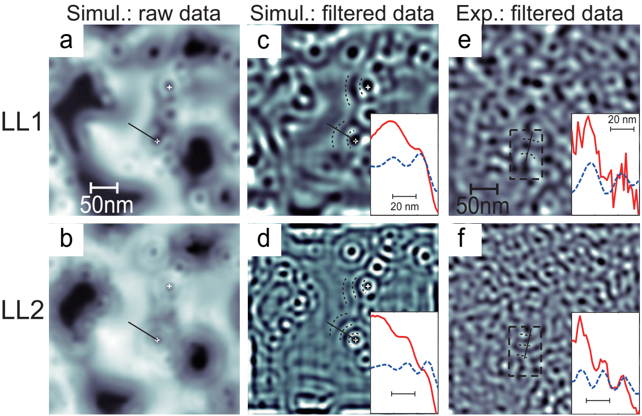


Conclusion:

- the nodal structure of LLs is robust to disorder
- Key property of quantum Hall states!

Can we see the nodes in real space?

<u>Trick:</u> bandpass for momenta $|\mathbf{q}| \simeq 1/l_B$ in $\widetilde{\rho}(\mathbf{q}, E)$ and Fourier transform back to real space



This improves resolution: shadow lines appear!

Correlations of the LDoS

Champel, SF, Raikh, PRB (2011)

Theory at finite l_B

Dyson equation: simpler at $\omega_c \to +\infty$ (no Landau level mixing)

$$(\omega - E_m + i0^+) g_m(\mathbf{R}) = 1 + v_m(\mathbf{R}) \star g_m(\mathbf{R})$$

where $\star = \exp\left[i \frac{l_B^2}{2} \left(\overleftarrow{\partial}_X \overrightarrow{\partial}_Y - \overleftarrow{\partial}_Y \overrightarrow{\partial}_X \right)\right]$: star product

Rigorous phase space (=real space) quantization!

Effective potential: $v_m(\mathbf{R}) = \int d^2\mathbf{u}\; F_m(\mathbf{R}-\mathbf{u}) V(\mathbf{u})$

Check with classical limit:

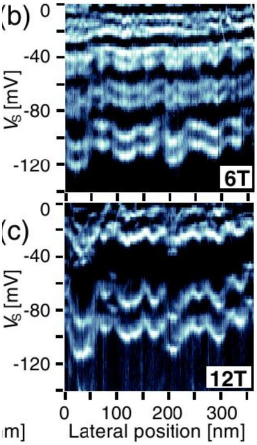
- Take $l_B \to 0$ and $m \to +\infty$ with $L_m = l_B\sqrt{2m+1}$ fixed
- $\tilde{v}_m(\mathbf{R}) = \int d^2\mathbf{u}\; \delta(|\mathbf{R}-\mathbf{u}| - L_m) V(\mathbf{u})$: classical orbit (OK)

Solving Dyson equation: $\tilde{g}_m(\mathbf{R}) = [\omega + i0^+ - E_m - \tilde{v}_m(\mathbf{R})]^{-1}$

up to curvature terms of order $(l_B/\xi)^2 \sqrt{\langle V^2 \rangle}$

Effective potential in the experiment

Spatial variation of the LDoS in InSb: Hashimoto *et al.* PRL (2009)



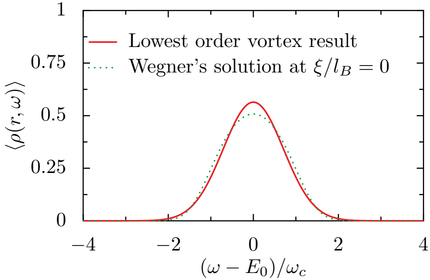
- Well-separated Landau levels
- Spin resolved
- Potential amplitude shrinks with increasing LL index and decreasing B (since l_B grows)

- Prospects:
  - Get bare disoder $V(\mathbf{R})$ from $v_0(\mathbf{R})$
  - Test relations between $v_m(\mathbf{R})$'s: $v_1(\mathbf{R}) = \hbar\omega_c + [1 + (l_B^2/2)\Delta_{\mathbf{R}}]v_0(\mathbf{R})$

How controlled is the theory?

Sample (disorder) averaged DoS: $\langle \rho(\mathbf{r}, \omega) \rangle$

- The lowest order vortex Green's function is exact at $\xi \gg l_B$
- Test: opposite limit $\xi \ll l_B$ analytically solved by Wegner



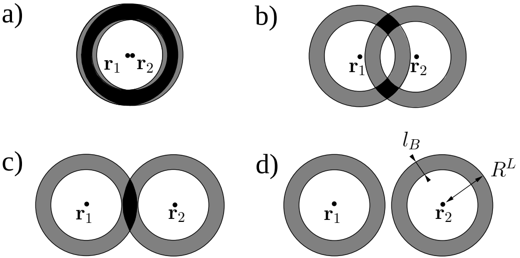
- Lowest order result already quite good and improves (asymptotic) by getting higher order corrections.
- Small parameter $= l_B^2 / (\xi^2 + 4 l_B^2)$

Geometrical interpretation of spatial LDOS correlations

Definition: perform the following sample averaging

$$\chi(|\mathbf{r}_1 - \mathbf{r}_2|, \omega_1, \omega_2) \equiv \langle \rho(\mathbf{r}_1, \omega_1)\rho(\mathbf{r}_2, \omega_2)\rangle - \langle \rho(\mathbf{r}_1, \omega_1)\rangle \langle \rho(\mathbf{r}_2, \omega_2)\rangle$$

Overlap of quantum rings: consider LLn with $n > 0$



Area for c) $>$ Area for b) $\Rightarrow \chi$ peaks again when $|\mathbf{r}_1 - \mathbf{r}_2| \simeq 2R_L$

Robust way to reveal the nodes in real space!

Computation of the LDoS correlations

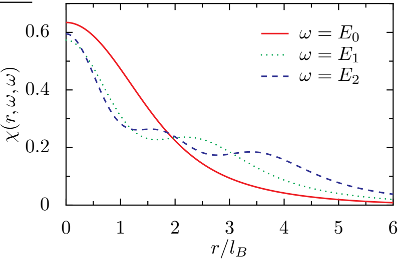
Procedure:

$$\langle\rho(\mathbf{r}_1,\omega_1)\rho(\mathbf{r}_2,\omega_2)\rangle = \int\frac{d^2\mathbf{R}_1}{2\pi l_B^2}\int\frac{d^2\mathbf{R}_2}{2\pi l_B^2}\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty} F_{n_1}(\mathbf{R}_1-\mathbf{r}_1)F_{n_2}(\mathbf{R}_2-\mathbf{r}_2)$$

$$\times \int\frac{dt_1}{2\pi}\int\frac{dt_2}{2\pi} e^{i(\omega_1-E_{n_1})t_1+i(\omega_2-E_{n_2})t_2}\left\langle e^{-i[V_{n_1}(\mathbf{R}_1)t_1+V_{n_2}(\mathbf{R}_2)t_2]}\right\rangle$$

This can be evaluated analytically!

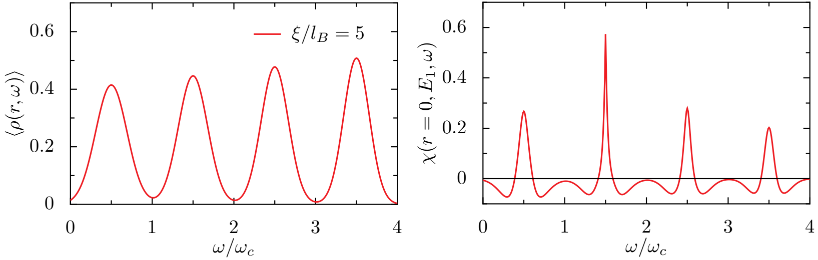
Spatial dependence: confirms previous expectations



$\Rightarrow$ robust view (sample averaged) of the “nodes” in real space

Energy dependence

DoS vs LDoS correlations at equal position:



- DoS $\langle\rho(\omega)\rangle$ has broad peaks with width $\sqrt{\langle V^2\rangle}$
- $\chi(\omega)$ has narrow resonances with width $(l_B/\xi)\sqrt{\langle V^2\rangle}$
- Positive correlations if $\Delta\omega \simeq \hbar\omega_c n$, negative otherwise

Prospects:

- Compare analytic theory with experiments and numerics

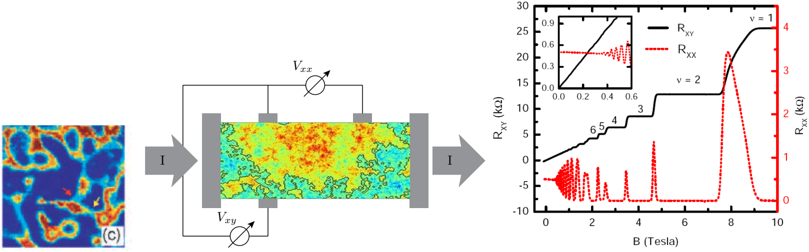
Percolating transport in the quantum Hall regime

Flöser, SF, Champel, PRL (2011)

Status of transport in IQHE

Origin of the percolation problem:

- Disorder induces density inhomogeneities
- Guiding center trajectories follow equipotential contours



- Percolation physics plays a key role at the plateau transitions

Available approaches for transport

Fully quantum mechanical: low temperature regime

- Coherent tunneling between valleys
- Solve numerically Schrödinger equation (expensive)
- Use Kubo formula or Landauer formalism
- Does quantum percolation explain experiments? (unsettled)

Semiclassical guiding center approach: high temperature regime

- Incoherent tunneling between valleys
- Local Ohm's law: $\mathbf{j}(\mathbf{r}) = -\hat{\sigma}(\mathbf{r}).\nabla_{\mathbf{r}}\Phi(\mathbf{r})$
- Drift-diffusion local conductivity $\hat{\sigma}(\mathbf{r})$ (next slide)
- Solve continuity equation $\nabla_{\mathbf{r}}.\mathbf{j}(\mathbf{r}) = 0$

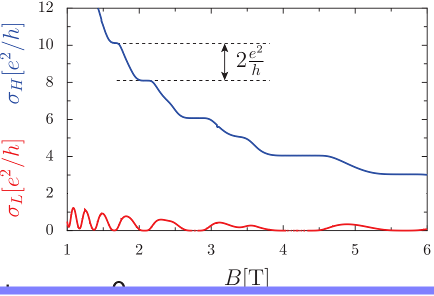
Drift-diffusion model for IQHE

Dissipative part (phonons): $\sigma_{xx}(\mathbf{r}) = \sigma_0$

Semiclassical drift part: $\sigma_{xy}(\mathbf{r}) = \frac{e^2}{h} \sum_m n_F(E_m + V(\mathbf{r}))$

- Smooth disorder $\Rightarrow E_m(\mathbf{R}) = \hbar\omega_c(m + 1/2) + V(\mathbf{R})$
- Current: $\mathbf{j}(\mathbf{R}) = -en(\mathbf{R})\mathbf{v}_{\mathrm{drift}} = \frac{e^2}{h} 2\pi l_B^2 n(\mathbf{R}) \mathbf{E} \times \hat{\mathbf{z}}$
  with $n(\mathbf{R})$=density of filled states

Theoretical challenge: large Hall conductivity fluctuations



$\sigma_0 \ll \sqrt{\langle \delta\sigma_{xy}^2 \rangle} \simeq 2e^2/h$

Classical percolation problem for IQHE

Pure drift: limit $\sigma_0 = 0$

- Closed trajectories do not contribute to transport
- Percolating trajectories must go through saddle points
  $\Rightarrow$ drift velocity $\mathbf{v}_d = -\frac{ec}{B}\nabla V \times \hat{\mathbf{z}}$ vanishes!

Extra processes (encoded in σ_0) are required for transport

Expectations for longitudinal conductivity: power law at small σ_0

$\sigma_{xx} \propto \sigma_0^{1-\kappa} \left\langle \delta\sigma_{xy}^2 \right\rangle^{\kappa/2}$ with κ transport percolation exponent

Conjecture: $\kappa = 10/13 \simeq 0.7692$ [Isichenko RMP (1992), Simon&Halperin PRL (1994)]

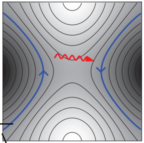
Goal:

- Compute κ microscopically
- Obtain scaling form of $\sigma_{xx}(T, B)$ in order to extract critical exponent from experimental data

High temperature conductivity model

Longitudinal component: [Zhao & Feng PRL (1993), Floeser *et al.*]

$\sigma_0(T) = A_{\mathrm{ph.}} T$
(phonon contribution)



Hall component: at $T \gg \sqrt{\langle V^2 \rangle}$

$\sigma_{xy}(\mathbf{r}) \simeq \frac{e^2}{h} \sum_m [n_F(E_m) + V(\mathbf{r}) n'_F(E_m)]$ has Gaussian fluctuations

At plateau transition, one finds: $\delta\sigma_{xy}(\mathbf{r}) \simeq \frac{e^2}{h} \left[\frac{1}{4T} + \frac{1}{\hbar\omega_c} \right] V(\mathbf{r})$

Scaling function: $\sigma_{xx} \propto [\sigma_0(T)]^{1-\kappa} \left[\frac{e^2}{h} \sqrt{\langle V^2 \rangle} \right]^\kappa \left[\frac{1}{4T} + \frac{1}{\hbar\omega_c} \right]^\kappa$

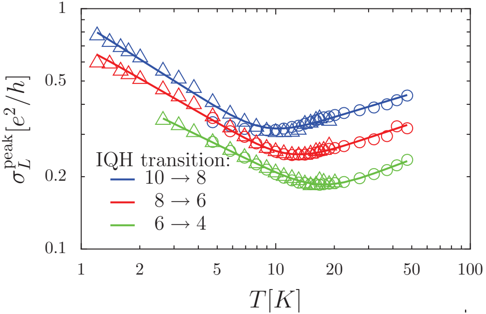
- $\sigma_{xx} \propto T^{1-2\kappa} \simeq T^{-0.5}$ at $T < \hbar\omega_c/4$
- $\sigma_{xx} \propto T^{1-\kappa} \simeq T^{0.2}$ at $T > \hbar\omega_c/4$

σ_{xx} should go through a minimum at $T \simeq \hbar\omega_c/4$

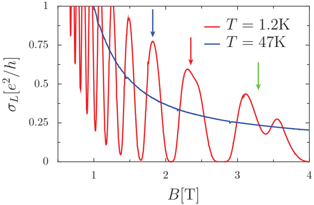
Comparison to experiments

[Data from B. Piot (unpublished)]

Peak conductivity vs temperature



Examine $\sigma_{xx}^{\rm peak}(T)$ for three IQHE transitions:



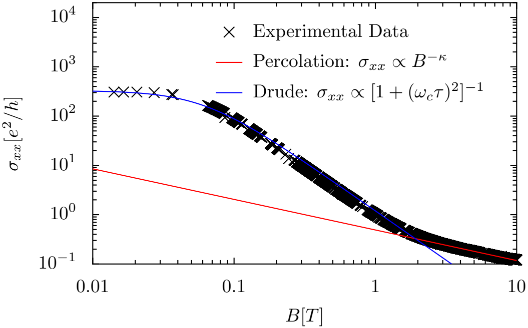
- Two power-laws: $T^{1-2\kappa} = T^{-0.5}$ (low T)
  $T^{1-\kappa} = T^{0.2}$ (high T)

  $\Rightarrow$ crossover from classical to quantized cyclotron motion
- The minimum appears as predicted ($\hbar\omega_c$ is not fitted)
- Quantitative agreement with the scaling function for $\sigma_{xx}(T)$
  We extract: $\kappa = 0.73 \pm 0.03$

Anomalous magneto-transport

Prediction of percolation theory: Polyakov *et al.* PRB (2001); Flöser, SF, Champel PRL (2011). $\sigma_{xx} \propto B^{-\kappa}$ at $T > \hbar\omega_c/4 \Rightarrow \rho_{xx} \propto B^{2-\kappa}$
This is very different from Drude result: $\sigma_{xx} \propto B^{-2}$ and $\rho_{xx} \propto B^0$

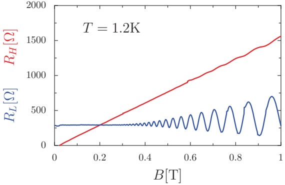
Experimental data: taken at $T = 47$K

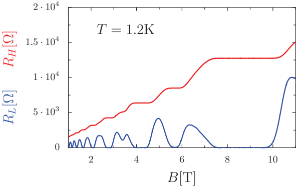


Clear crossover from Drude to classical percolating transport!

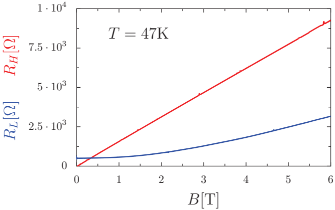
Low vs high field & Low vs high temperature

High temperature regime: onset of QHE at $B \gtrsim 1\text{T}$





Low temperature regime: onset of classical percolation at $B \gtrsim 1\text{T}$



Similar crossover from rough to smooth disorder?

Computation of transport exponent κ

Flöser, SF, Champel, PRL (2011)

How to compute κ?

Why is it difficult? The transport equation is ill-defined at $\sigma_0 = 0$
$\Rightarrow$ a small σ_0 expansion is not possible

Effective conductivity formalism: [Dreizin & Dykhne JETP (1972), Stroud PRB (1975)]. Decompose $\hat{\sigma}(\mathbf{r}) = \hat{\sigma}_0 + \delta\hat{\sigma}(\mathbf{r})$ and aim to solve:

$$\nabla \cdot [\hat{\sigma}_0 \nabla \Phi(\mathbf{r})] = -\nabla \cdot [\delta\hat{\sigma}(\mathbf{r}) \nabla \Phi(\mathbf{r})]$$

Introduce Green's function: $\nabla \cdot [\hat{\sigma}_0 \nabla G(\mathbf{r}, \mathbf{r}')] = -\delta(\mathbf{r} - \mathbf{r}')$
After some manipulation: $\hat{\sigma}_{\text{eff}} = \hat{\sigma}_0 + \langle \hat{\chi} \rangle$
with $\hat{\chi}(\mathbf{r}) = \delta\hat{\sigma}(\mathbf{r}) + \delta\hat{\sigma}(\mathbf{r}) \int d^d r' \, \hat{\mathcal{G}}_0(\mathbf{r}, \mathbf{r}') \hat{\chi}(\mathbf{r}')$
and $\left[\hat{\mathcal{G}}_0\right]_{ij} = \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} G(\mathbf{r}, \mathbf{r}')$

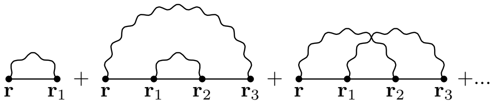
Idea: iterating the equation for $\langle \hat{\chi} \rangle$ allows to expand the conductivity perturbatively in powers of $1/\sigma_0$

General formalism

Disorder averaging the conductivity:

$$\langle \hat{\chi}(\mathbf{r}) \rangle = \langle \delta\hat{\sigma}(\mathbf{r}) \rangle + \int d^d \mathbf{r}_1 \langle \delta\hat{\sigma}(\mathbf{r}) \hat{\mathcal{G}}_0(\mathbf{r}, \mathbf{r}_1) \delta\hat{\sigma}(\mathbf{r}_1) \rangle +$$

$$\int d^d \mathbf{r}_1 \int d^d \mathbf{r}_2 \langle \delta\hat{\sigma}(\mathbf{r}) \hat{\mathcal{G}}_0(\mathbf{r}, \mathbf{r}_1) \delta\hat{\sigma}(\mathbf{r}_1) \hat{\mathcal{G}}_0(\mathbf{r}_1, \mathbf{r}_2) \delta\hat{\sigma}(\mathbf{r}_2) \rangle + \dots$$

$\langle \hat{\chi}(\mathbf{r}) \rangle =$ 

Result at six-loop order:

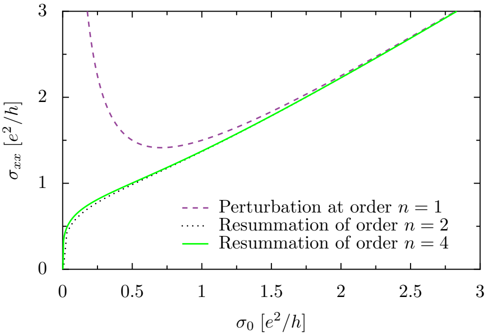
$$\sigma_{xx} = \sigma_0 + \langle \chi \rangle = \sigma_0 + \sum_{n=1}^{\infty} a_n \frac{\langle \delta\sigma^2 \rangle^n}{\sigma_0^{2n-1}}$$

Order	Method	Coefficient a_n
1	Analytical	$\frac{1}{2}$
2	Analytical	$\frac{1}{8} - \frac{1}{2}\log(2)$
3	Analytical	0.2034560502
4	Numerical	-0.265 ± 0.001
5	Numerical	0.405 ± 0.001
6	Numerical	-0.694 ± 0.001

Padé resummation of the series

Method: extrapolate series to strong coupling $\sigma_0 \to 0$

Order	Method	Exponent κ
2	Padé	0.72 ± 0.09
4	Padé	0.779 ± 0.006
4	n-fit	0.767 ± 0.002
∞	Conjecture	$10/13 \simeq 0.7692$



Good convergence: supporting the conjecture!

Conclusion

- STS showed in real space the percolating states for 2DEG at high magnetic fields
- STS revealed the robust nodal structure of higher Landau levels
- Local observables can be calculated accurately from systematic gradient expansion using coherent state Green's functions both for given disorder landscape and for disorder-averages
- The classical percolation transport regime of IQHE was addressed, with accurate calculation of critical exponents and comparison to recent experiment
- Phonons seem to provide the main dissipation mechanism for a wide range of temperatures

Extra slides