Quantum criticality in correlated matter: a few surprises from a mesoscopic look

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Recent collaboration with:

- Lars Fritz
- Rajesh Narayanan
- Achim Rosch
- Matthias Vojta

Older but related work with:

- Antoine Georges
- Gabriel Kotliar
- Patrice Limelette
- Sergei Pankov
- Subir Sachdev

Motivation: the QPT problem

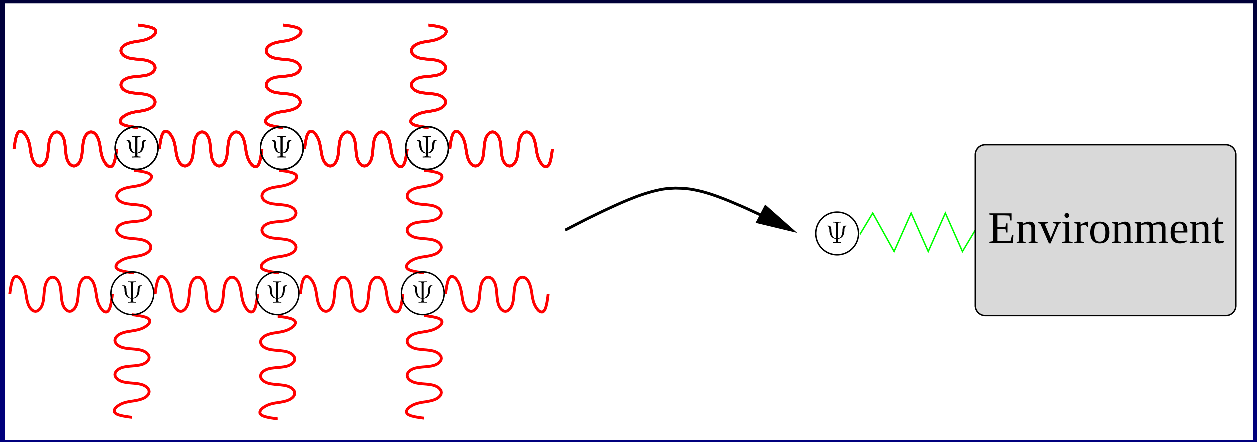
Classical phase transitions:

- A great challenge of last century
- Solved by K. Wilson in the ’70s
- New ideas (universality) and methods (renormalization)

Quantum phase transitions:

- An exciting problem in the new century
- Puzzling experiments, complex models
- Needs new ideas and methods!

Motivation: mesoscopic look



Idea:
go from a collection of quantum objects to an environmentally coupled single quantum object

Hope:
Simplify life, but preserve crucial aspects of QPT

Bonus:
relevant in mesoscopic physics!

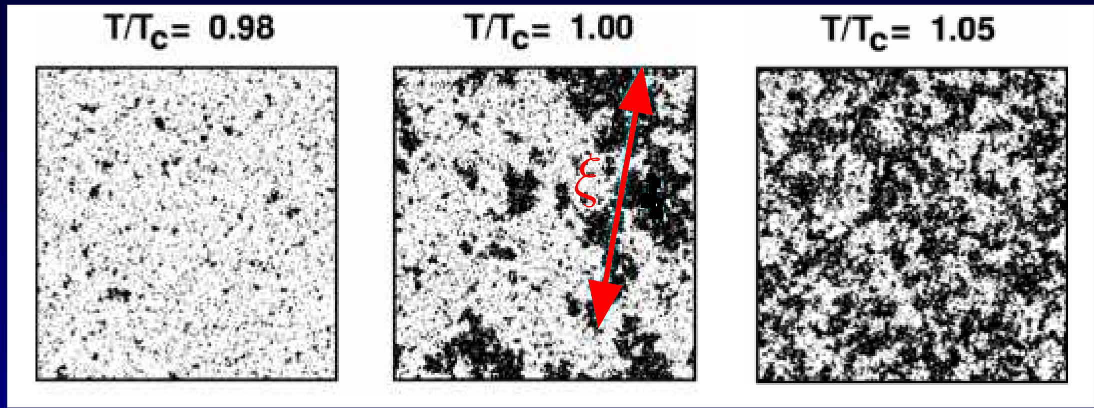
Outline

- Reminder on classical phase transitions
- Introduction to quantum phase transitions
- QPT with dissipation: non-classical behavior
- Impurity QPT in superconductors: power of quantum renormalization
- Criticality in quantum dots: new paradigm
- Conclusion

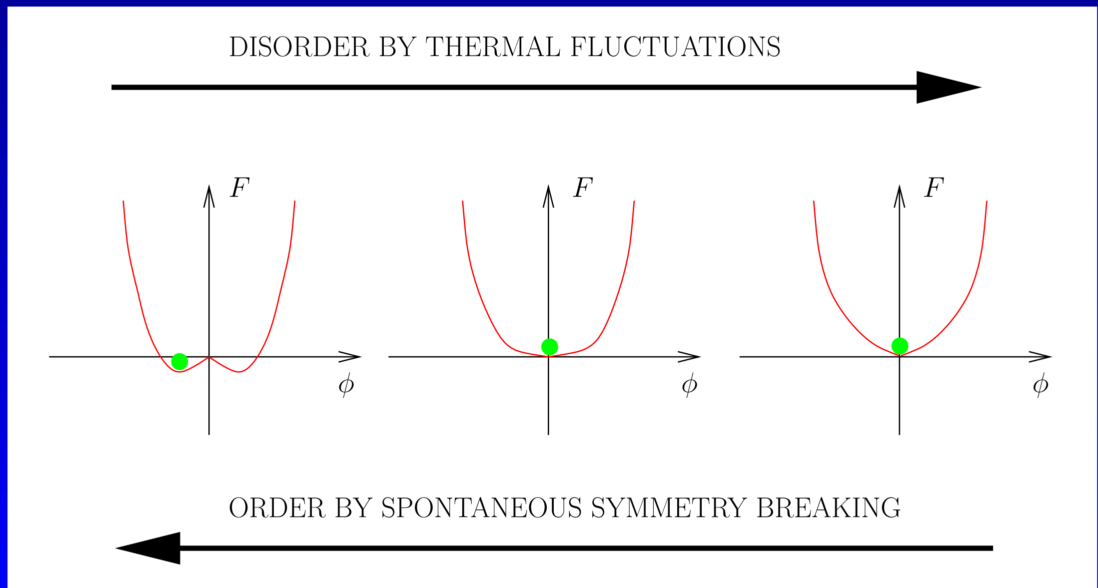
Classical phase transitions

2^{nd} order classical PT

Ising magnet: $E = \sum_{ij} J_{ij} S_i S_j$



Associated free energy landscape:



Basic concepts

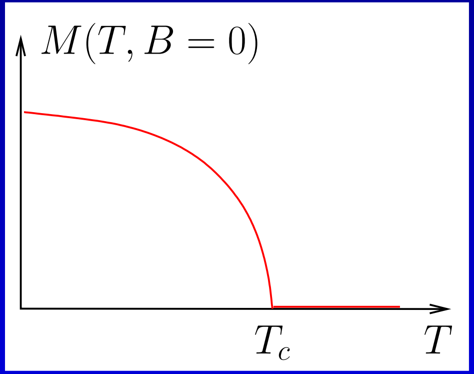
Order parameter: $\phi(\vec{x})$

Landau energy functional: $r \propto T - T_c$

$$E = \int d^D x \left[(\vec{\nabla}_x \phi)^2 + B\phi + r\phi^2 + u\phi^4 \right]$$

Mean field: neglect spatial fluctuations $\phi(\vec{x}) = M$

$M(T, B=0) \propto (T_c - T)^\beta$ with $\beta = 1/2$



Universality: critical exponents do not depend on microscopic details

Beyond the mean-field picture

The difficulty:

- Spatial fluctuations are singular at $D < 4$ in the statistical mechanics of $Z = \sum_{\{\phi(\vec{x})\}} e^{-E[\phi(\vec{x})]/k_B T}$

Wilson's answer: renormalization

- integrate short wavelength fluctuations $a < \lambda < sa$
- iterate until $sa \sim \xi$: perturbation theory works

$\Rightarrow$ for $D < 4$ non trivial exponent $\beta^{\mathrm{RG}} = \frac{1}{2} - \frac{4-D}{6} + \ldots$

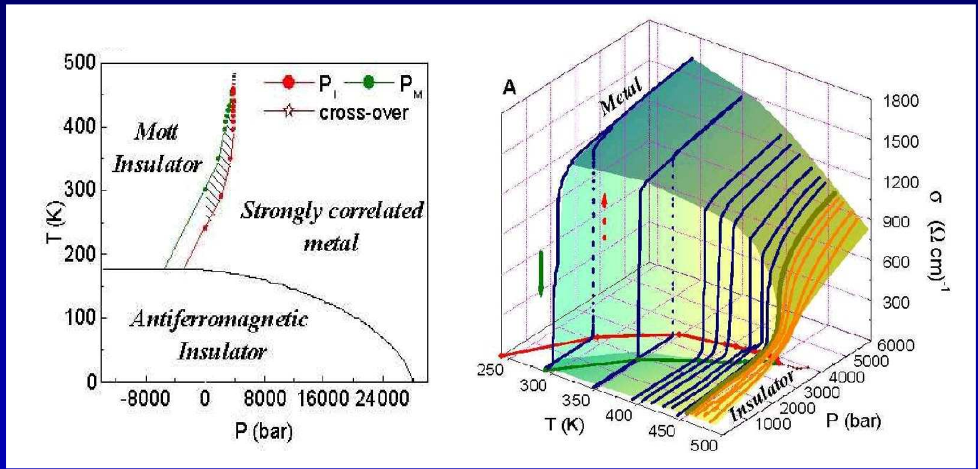
	$D = 2$	$D = 3$	$D \geq 4$
β^{exact}	$\frac{1}{16}$	0.326 ± 0.001	$\frac{1}{2}$
β^{RG}	$\frac{1}{6} + \ldots$	$\frac{1}{3} + \ldots$	$\frac{1}{2}$

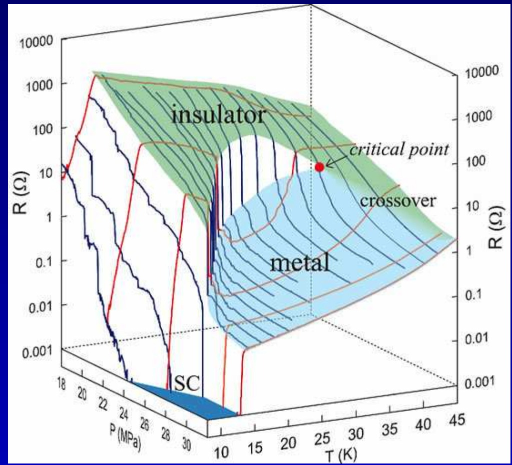
Universal versus non-universal

Universal exponents seen in various systems:

(Anti)-ferromagnets, superfluids, liquid-gas transition and recently metal-insulator transition

V_2O_3 [P. Limelette Science '04] κ-BEDT [Kagawa PRB '04]

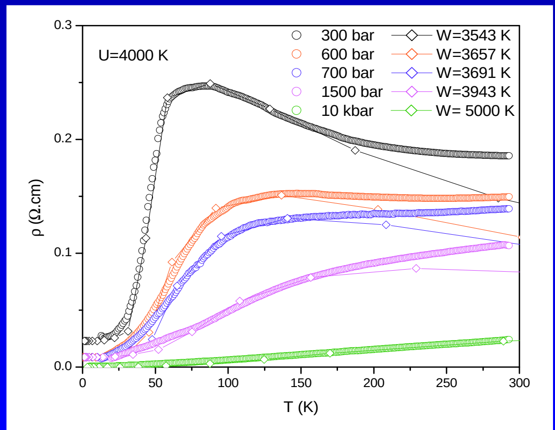




The whole thing:

full transport in DMFT

[P. Limelette, SF, A. Georges, PRL '04]



Quantum phase transitions

First look on QPT

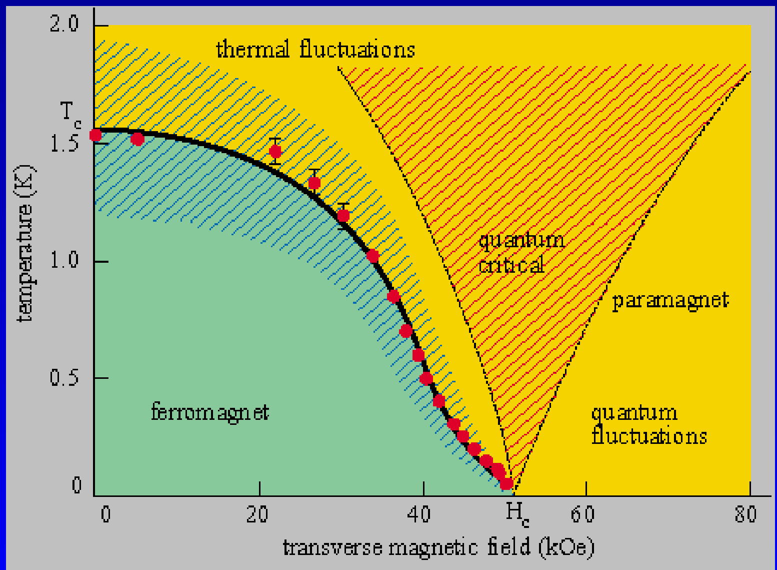
What is a quantum phase transition?

A $T = 0$ transition driven by quantum fluctuations between two competing ground states

How can it be observed?

Change non-thermal parameter to drive T_c to zero

Example: $LiHoF_4$ [Bitko *et al.* PRL '96]



$3D$ Ising in transverse field

$$H = \sum_{ij} J_{ij} S_i^z S_j^z + \sum_i B S_i^x$$

$$|\mathrm{Ferro}\rangle = \bigotimes_i |\uparrow\rangle_i$$
$$|\mathrm{Para}\rangle = \bigotimes_i (|\uparrow\rangle + |\downarrow\rangle)_i$$

Theory: QPT in insulators

Quantum partition function: $Z = \mathrm{Tr}\left[e^{-(H_0+H_1)/k_B T}\right]$

$[H_0, H_1] \neq 0 \Rightarrow$ dynamics and statics couple!

New idea [Suzuki 1976]: $Z = \sum_{\{\phi(\vec{x},\tau)\}} e^{-E/\Delta}$

with $E = \int d\tau \int d^D x \left[(\partial_\tau \phi)^2 + (\vec{\nabla}_x \phi)^2 + r\phi^2 + u\phi^4\right]$

General statement: (for insulators)

Quantum PT in dim. D = Classical PT in dim. D+1

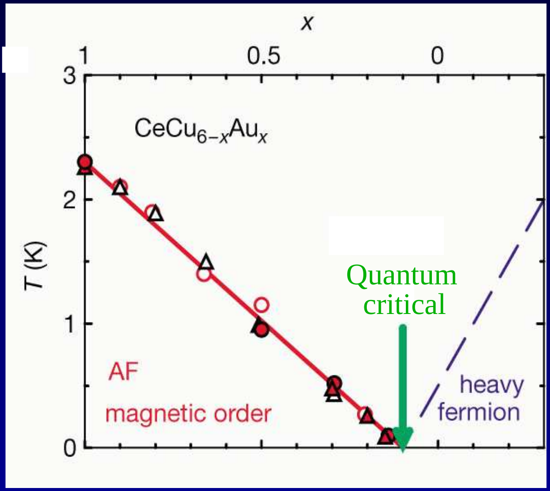
Not so trivial: peculiar time correlations

Many successful applications:

- spin dynamics in undoped cuprates
- QPT in coupled ladder magnets

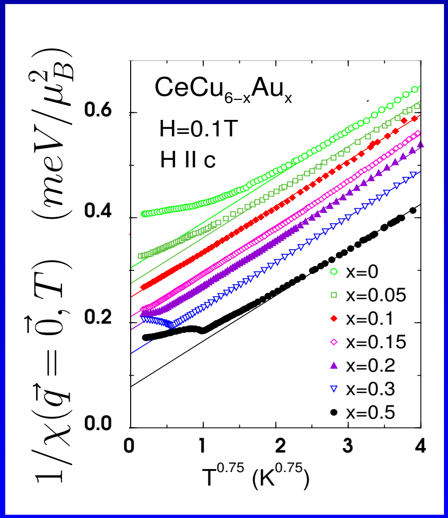
QPT in itinerant magnets

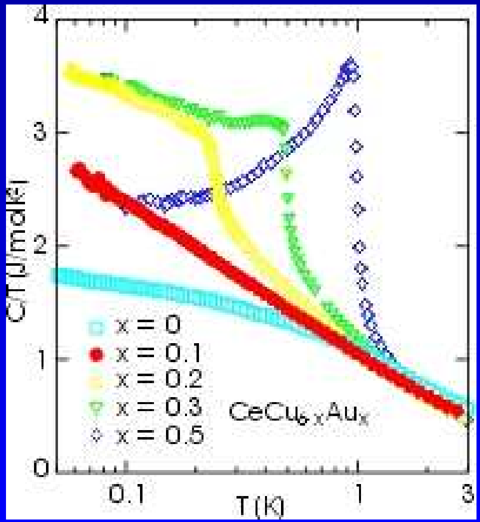
Magnetic critical point in a metal: $CeCu_{6-x}Au_x$



Competition between RKKY interaction and magnetic coupling to electrons

Anomalous response:





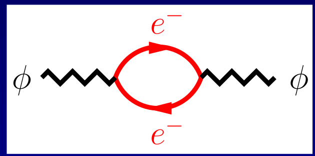
Theoretical puzzle

Experiment: $\mathrm{CeCu}_{6-x}\mathrm{Au}_x$

- $C/T \propto \log 1/T$
- $\chi^{-1} = \chi_0^{-1} + aT^{0.75}$
- $\rho = \rho_0 + AT$

Theory: electronic diffusion on spin fluctuations

Landau damping of ϕ mode:



$$E = \int d\omega \int d^D q \; (\omega^2 + q^2 + i\omega)\phi^2 \Rightarrow D_{\text{eff}} = D + 2$$

3D SDW [Hertz; Moriya; Millis]

- $C/T = \gamma - \sqrt{T}$
- $\chi^{-1} = \chi_0^{-1} + aT$
- $\rho = \rho_0 + AT^{3/2}$

2D SDW [Rosch *et al.*; Pankov, SF, Georges, Kotliar, Sachdev]

- $C/T = \log 1/T$
- $\chi^{-1} = \chi_0^{-1} + a\frac{T}{\ln^2 T}$
- $\rho = \rho_0 + AT$

Problem: theory disagrees with experiment!

Itinerant criticality: an open problem

Many puzzling compounds:

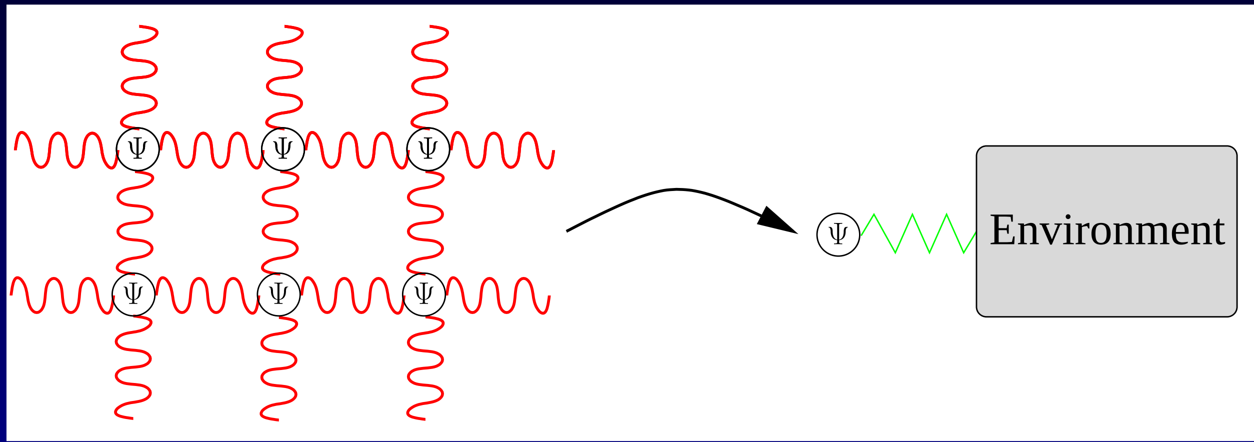
- Heavy fermions antiferromagnets: $CeCu_{6-x}Au_x$, $YbRh_2(Si_{1-x}Ge_x)_2$, ...
- Transition metal ferromagnets: MnSi, $ZrZn_2$
- High temperature superconductors

Some fascinating and fundamental questions:

- Non Fermi Liquids
- Exotic Superconductivity

Today: no sound theoretical framework!

Mesoscopic QPT



Idea:
go from a collection of quantum objects
to a environmentally coupled single one

Hope:
Simplify life, but preserve crucial aspects of QPT

Remark:
Mesoscopic many-body problem = quantum impurity

Strategy

Practical reason:

- Simpler to solve
- Develop new concepts and techniques
- Test with numerics (now possible)

Physical reason:

- Mesoscopic Effects + QPT = New Physics!
- Realization in the lab (quantum dots, STM)

Other important application:

- Basis of quantum mean field theory (DMFT)
- Self-consistent quantum impurity $\Leftrightarrow$ bulk

QPT with dissipation

Simple two-fluids model

Subohmic spin-boson: $H = \Delta S^x + \lambda S^z \Phi$

$$\rho_\Phi(\omega) \propto \omega^s \ (s \leq 1)$$

Ohmic case $s = 1$ standard [Leggett *et al.* 1987]
Subohmic proposal $s = 1/2$ [Tong and Vojta 2005]

Quantum to classical mapping:
Equivalence with long range Ising:

$$E = \int d\tau \int d\tau' \frac{1}{|\tau - \tau'|^{1+s}} \phi(\tau)\phi(\tau') + \int d\tau \left[r\phi^2 + u\phi^4 \right]$$

Quantum phase transition: RG analysis [Fisher-Ma '72]

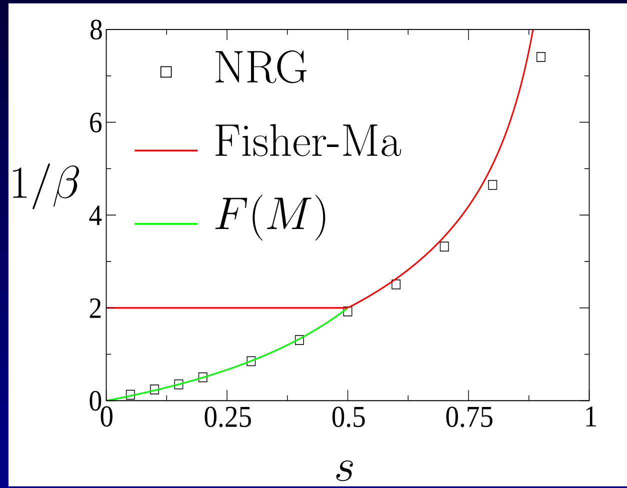
- For $1/2 < s$: $u^\star \neq 0 \Rightarrow$ Non trivial exponents
- For $s < 1/2$: $u^\star = 0 \Rightarrow$ Mean-field exponents

Non-classical PT

Recent NRG simulations: [Bulla, Tong, Vojta 2005]

$s > 1/2$: non-trivial exponents ok

$s < 1/2$: non mean-field exponents!



New result: [SF & R. Narayanan (unpublished)]

Trick: Majorana $\vec{S} = -(i/2)\vec{\eta} \times \vec{\eta}$

Reconsider Landau: u singular at $s < 1/2$!!

Interpretation: non-analytic free energy

$$F(M) = hM + rM^2 + vM^{2/(1-s)}$$

$\Rightarrow \beta = \frac{1-s}{2s} \neq \frac{1}{2}$ fits numerics!

Future challenges: higher dimension generalization

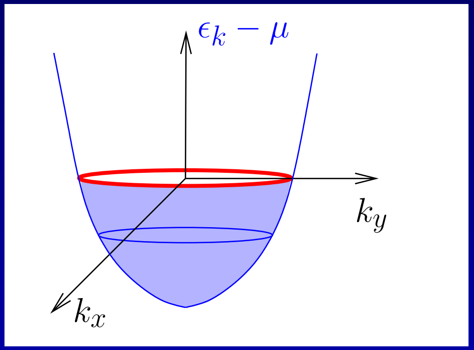
Impurity QPT in superconductors

Pseudogap Kondo model

Kondo problem: spin $\vec{S}$ coupled to a Fermi sea

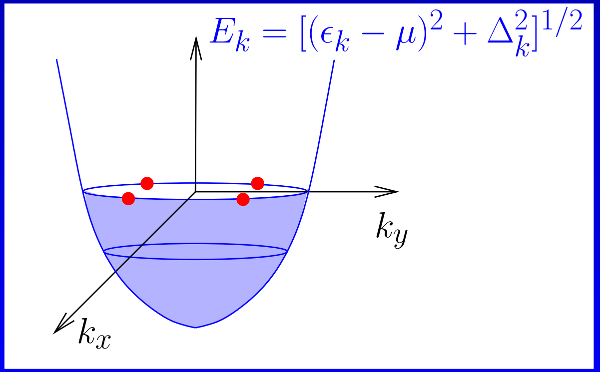
$$H = H_{\text{el.}} + J\vec{S}\cdot\vec{S}_{\text{el.}}(\vec{x}=\vec{0})$$

Metals:



- finite DoS $\rho_c(\epsilon) = const.$
- $\Rightarrow$ Screening of the spin for all $J > 0$

HTc superconductors:



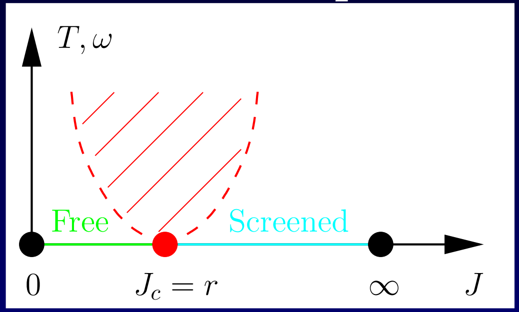
- linear DoS $\rho_c(\epsilon) = |\epsilon|$
- $\Rightarrow$ Screening??

Quantum Phase Transition

Quantum phase transition: [Withoff & Fradkin 1990]

Power-law DoS $\rho_c(\epsilon) \propto |\epsilon|^r$

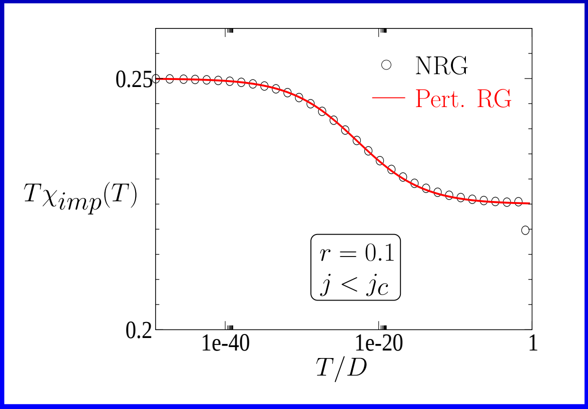
Trick: r = small parameter

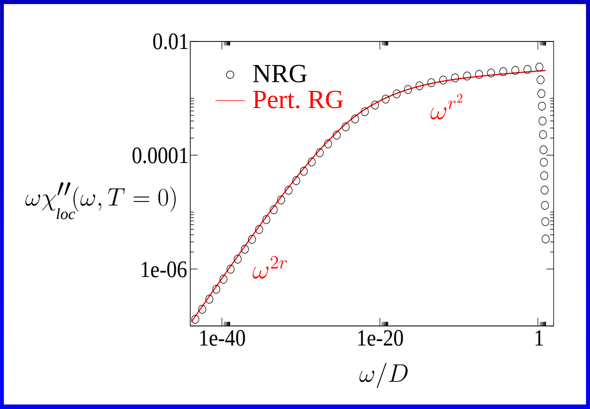


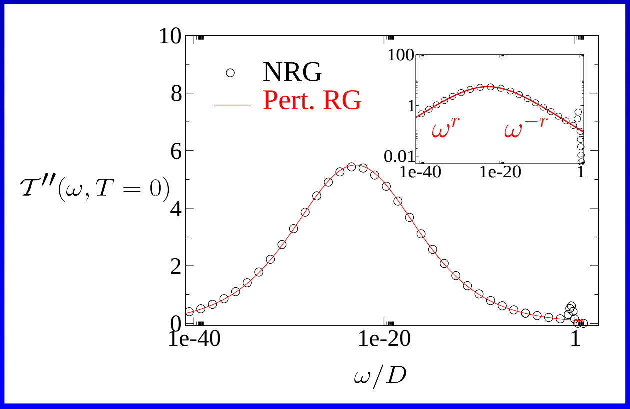
Quantum critical point: at $j = j_c$

- Fractional spin $S' < 1/2$: $\chi_{imp}(T) = \frac{S'(S'+1)}{3T}$
- Critical exponents: $\chi_{loc}(\omega) = \omega^{-1+\eta_\chi}$

Universal crossover: for $j < j_c$ [Fritz, SF, Vojta 2006]

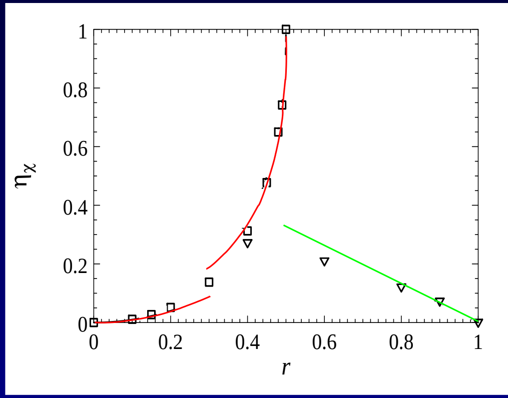


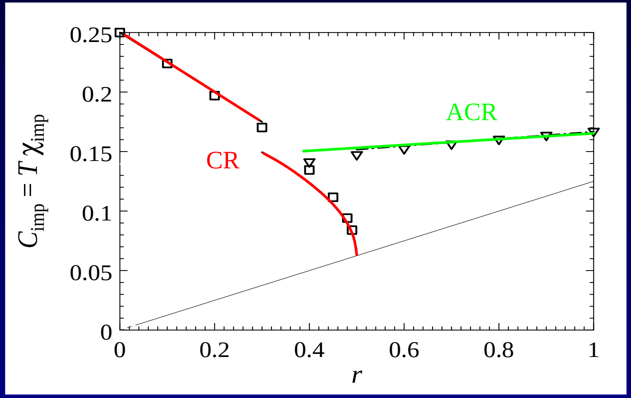


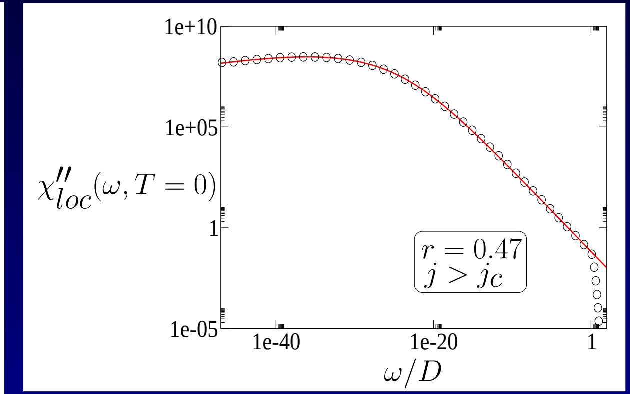


Other progresses

Global approach: expansions around $r = 0, 1/2, 1$

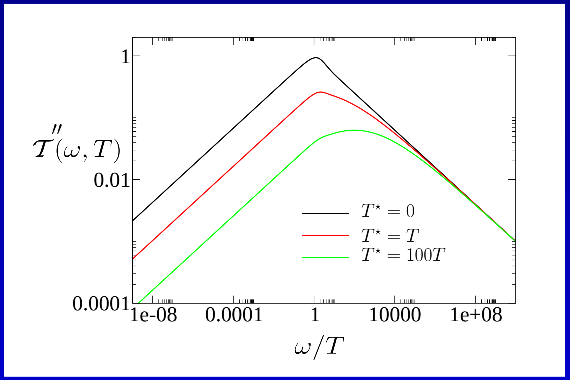






Finite temperature dynamics:

Still a challenge for the NRG at $\omega < T$



Possible openings:

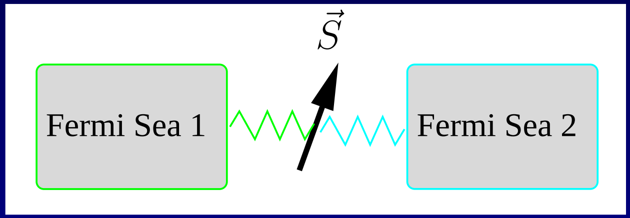
- Clearer experimental studies (STM, nanobridges...)
- Application of our method to more complex models
- Key to understand failure of bosonic approach?

Criticality in quantum dots

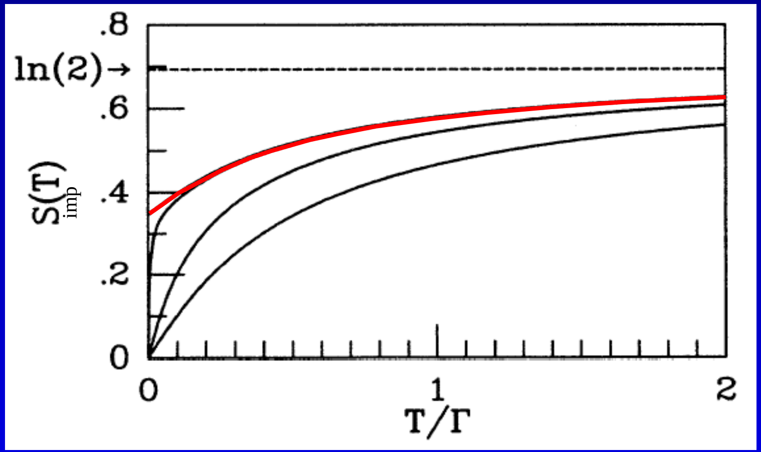
Two channel Kondo effect

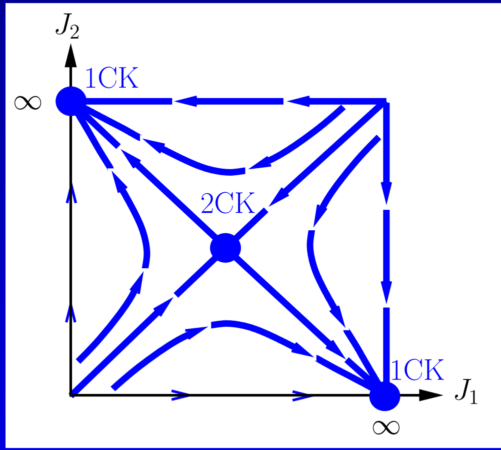
Model: spin $\vec{S}$ coupled to two Fermi seas [Nozières 1980]

$$H = H_{el.1} + H_{el.2} + J_1 \vec{S} \cdot \vec{S}_{el.1} + J_2 \vec{S} \cdot \vec{S}_{el.2}$$



Critical state: if $J_1 = J_2 \Rightarrow S_{imp}(T=0) = \log(\sqrt{2})$



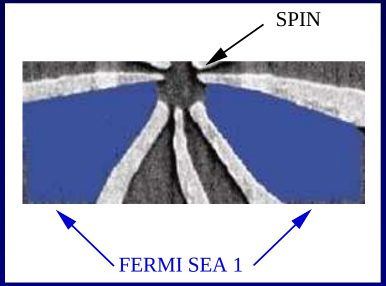


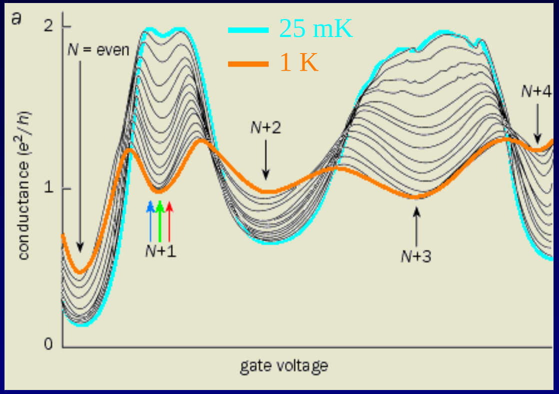
Conclusion: generic instability!

Can one realize 2CK in dots ?

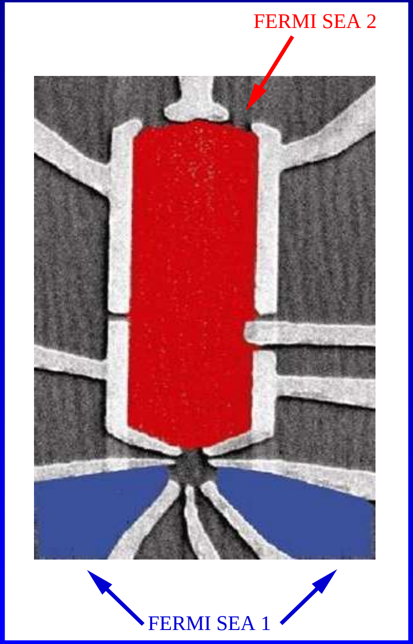
Experiments on 1CK: [Goldhaber-Gordon, Kastner 1998]

Mixing of
electrodes
$\Rightarrow$ 1CK!





New idea for 2CK: [Oreg & Goldhaber-Gordon 2003]



Freeze charge exchange
between leads 1 and 2
by capacitive energy cost E_c
(Coulomb blockade)

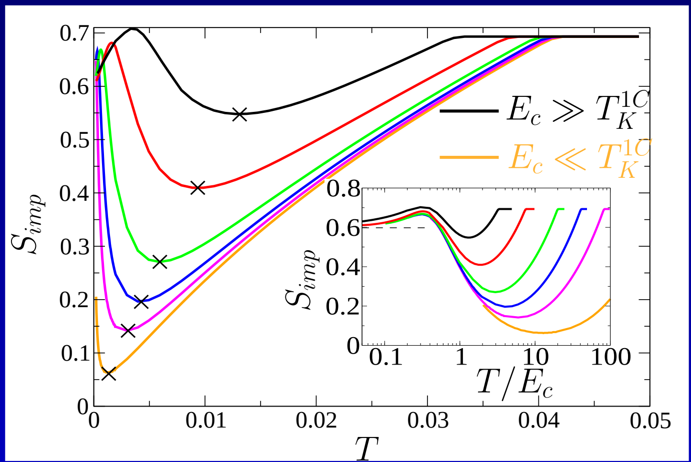
$\Rightarrow$ 2CK possible if large E_c

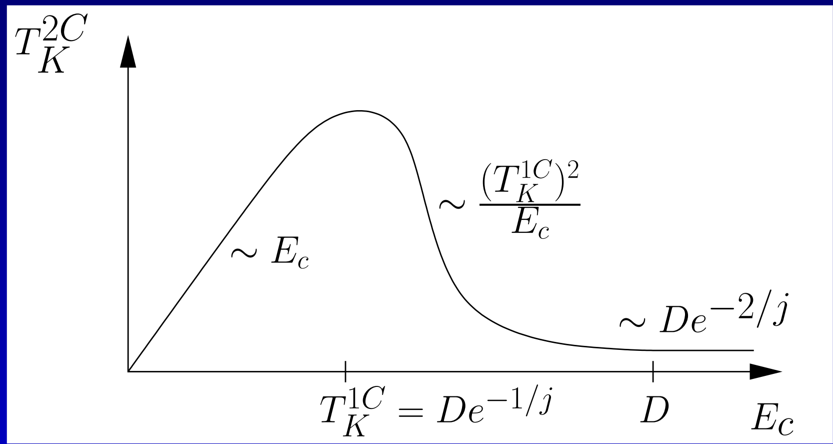
Surprising interplay of correlations

Solution: [SF & A. Rosch PRL 2004]

Phase θ dual to charge: $c^\dagger_{\sigma m} = a^\dagger_{\sigma m} e^{i\theta}$ [SF & A. Georges '02]

Results: inverse crossover 1CK $\longrightarrow$ 2CK if small E_c





Implications:

- New paradigm for stability of NFL states
- Optimize experiments: need $E_c \sim T_K^{1C}$

Conclusion

QPT in mesoscopic domain: a promising field!

- QPT are $T = 0$ quantum-driven transitions
- QPT have fascinating consequences
- New ideas and methods for their understanding
- Mesoscopic QPT are easier to understand, but still very surprising!
- Cross-fertilization of ideas between traditional condensed matter and mesoscopic physics

Future directions of research

Quantum phase transitions:

- Challenge: bulk QPT in itinerant magnets
- Non equilibrium quantum phase transitions
- $T = 0$ Mott metal-insulator transition

Other problems:

- Transport in novel mesoscopic magnets
- Connection between many-body & ab-initio