Quantum criticality in correlated matter: a few surprises from a mesoscopic look

Serge Florens

ITKM - Karlsruhe

Recent collaboration with:
Lars Fritz
Rajesh Narayanan
Achim Rosch
Matthias Vojta

Older but related work with:
Antoine Georges
Gabriel Kotliar
Patrice Limelette
Sergei Pankov
Subir Sachdev
Motivation: the QPT problem

Classical phase transitions:
- A great challenge of last century
- Solved by K. Wilson in the ’70s
- New ideas (universality) and methods (renormalization)

Quantum phase transitions:
- An exciting problem in the new century
- Puzzling experiments, complex models
- Needs new ideas and methods!
Motivation: mesoscopic look

Idea:
go from a collection of quantum objects to an environmentally coupled single quantum object

Hope:
Simplify life, but preserve crucial aspects of QPT

Bonus:
relevant in mesoscopic physics!
Outline

• Reminder on classical phase transitions

• Introduction to quantum phase transitions

• QPT with dissipation: non-classical behavior

• Impurity QPT in superconductors: power of quantum renormalization

• Criticality in quantum dots: new paradigm

• Conclusion
Classical phase transitions
\(2^{\text{nd}}\) order classical PT

Ising magnet: \(E = \sum_{ij} J_{ij} S_i S_j\)

Associated free energy landscape:
Basic concepts

Order parameter: $\phi(\vec{x})$

Landau energy functional: $r \propto T - T_c$

$$E = \int d^Dx \left[ (\vec{\nabla}_x \phi)^2 + B\phi + r\phi^2 + u\phi^4 \right]$$

Mean field: neglect spatial fluctuations $\phi(\vec{x}) = M$

$M(T, B = 0) \propto (T_c - T)^\beta$ with $\beta = 1/2$

Universality: critical exponents do not depend on microscopic details
Beyond the mean-field picture

The difficulty:

- Spatial fluctuations are singular at $D < 4$ in the statistical mechanics of $Z = \sum \{\phi(\vec{x})\} \ e^{-E[\phi(\vec{x})]/k_BT}$

Wilson’s answer: renormalization

- integrate short wavelength fluctuations $a < \lambda < sa$
- iterate until $sa \sim \xi$: perturbation theory works

$\Rightarrow$ for $D < 4$ non trivial exponent $\beta_{RG}^{\text{exact}} = \frac{1}{2} - \frac{4-D}{6} + \ldots$

<table>
<thead>
<tr>
<th></th>
<th>$D = 2$</th>
<th>$D = 3$</th>
<th>$D \geq 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^{\text{exact}}$</td>
<td>$\frac{1}{16}$</td>
<td>$0.326 \pm 0.001$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\beta_{RG}$</td>
<td>$\frac{1}{6} + \ldots$</td>
<td>$\frac{1}{3} + \ldots$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Universal versus non-universal

Universal exponents seen in various systems:
(Anti)-ferromagnets, superfluids, liquid-gas transition and recently metal-insulator transition

$V_2O_3$ [P. Limelette Science ’04] $\kappa$-BEDT [Kagawa PRB ’04]

The whole thing: 
full transport in DMFT

[P. Limelette, SF, A. Georges, PRL ’04]
Quantum phase transitions
First look on QPT

What is a quantum phase transition?
A $T = 0$ transition driven by quantum fluctuations between two competing ground states

How can it be observed?
Change non-thermal parameter to drive $T_c$ to zero

Example: LiHoF$_4$ [Bitko et al. PRL '96]

3D Ising in transverse field

\[ H = \sum_{ij} J_{ij} S^z_i S^z_j + \sum_i B S^x_i \]

|Ferro\rangle = \bigotimes_i |\uparrow\rangle_i
|Para\rangle = \bigotimes_i (|\uparrow\rangle + |\downarrow\rangle)_i
Theory: QPT in insulators

Quantum partition function: \( Z = \text{Tr} \left[ e^{-(H_0 + H_1)/k_B T} \right] \)

\([H_0, H_1] \neq 0 \Rightarrow \text{dynamics and statics couple!} \)

New idea [Suzuki 1976]: \( Z = \sum \{ \phi(\vec{x}, \tau) \} e^{-E/\Delta} \)

with \( E = \int d\tau \int d^Dx \left[ (\partial_\tau \phi)^2 + (\vec{\nabla}_x \phi)^2 + r\phi^2 + u\phi^4 \right] \)

General statement: (for insulators)
Quantum PT in dim. \( D = \) Classical PT in dim. \( D+1 \)

Not so trivial: peculiar time correlations

Many successful applications:

- spin dynamics in undoped cuprates
- QPT in coupled ladder magnets
QPT in itinerant magnets

Magnetic critical point in a metal: $\text{CeCu}_{6-x}\text{Au}_x$

Competition between RKKY interaction and magnetic coupling to electrons

Anomalous response:
Theoretical puzzle

**Experiment:** CeCu$_{6-x}$Au$_x$

- $C/T \propto \log 1/T$
- $\chi^{-1} = \chi_0^{-1} + aT^{0.75}$
- $\rho = \rho_0 + AT$

**Theory:** electronic diffusion on spin fluctuations

Landau damping of $\phi$ mode:

$$E = \int d\omega \int d^Dq \left( \omega^2 + q^2 + i\omega \right) \phi^2 \Rightarrow D_{\text{eff}} = D + 2$$

3D SDW [Hertz; Moriya; Millis]

- $C/T = \gamma - \sqrt{T}$
- $\chi^{-1} = \chi_0^{-1} + aT$
- $\rho = \rho_0 + AT^{3/2}$

2D SDW [Rosch *et al.*; Pankov, SF, Georges, Kotliar, Sachdev]

- $C/T = \log 1/T$
- $\chi^{-1} = \chi_0^{-1} + a\frac{T}{\ln^2 T}$
- $\rho = \rho_0 + AT$

**Problem:** theory disagrees with experiment!
Itinerant criticality: an open problem

Many puzzling compounds:

- Heavy fermions antiferromagnets: $\text{CeCu}_{6-x}\text{Au}_x$, $\text{YbRh}_2(\text{Si}_{1-x}\text{Ge}_x)_2$, …
- Transition metal ferromagnets: $\text{MnSi}$, $\text{ZrZn}_2$
- High temperature superconductors

Some fascinating and fundamental questions:

- Non Fermi Liquids
- Exotic Superconductivity

Today: no sound theoretical framework!
**Mesoscopic QPT**

**Idea:**
go from a collection of quantum objects
to a environmentally coupled single one

**Hope:**
Simplify life, but preserve crucial aspects of QPT

**Remark:**
Mesoscopic many-body problem = quantum impurity
Strategy

Practical reason:
- Simpler to solve
- Develop new concepts and techniques
- Test with numerics (now possible)

Physical reason:
- Mesoscopic Effects + QPT = New Physics!
- Realization in the lab (quantum dots, STM)

Other important application:
- Basis of quantum mean field theory (DMFT)
- Self-consistent quantum impurity ↔ bulk
QPT with dissipation
Simple two-fluids model

Subohmic spin-boson: \[ H = \Delta S^x + \lambda S^z \Phi \]
\[ \rho_\Phi(\omega) \propto \omega^s \,(s \leq 1) \]

Ohmic case \( s = 1 \) standard [Leggett et al. 1987]
Subohmic proposal \( s = 1/2 \) [Tong and Vojta 2005]

Quantum to classical mapping:
Equivalence with long range Ising:
\[ E = \int d\tau \int d\tau' \frac{1}{|\tau - \tau'|^{1+s}} \phi(\tau)\phi(\tau') + \int d\tau [r\phi^2 + u\phi^4] \]

Quantum phase transition: RG analysis [Fisher-Ma ’72]

- For \( 1/2 < s \): \( u^* \neq 0 \Rightarrow \) Non trivial exponents
- For \( s < 1/2 \): \( u^* = 0 \Rightarrow \) Mean-field exponents
Non-classical PT

**Recent NRG simulations:** [Bulla, Tong, Vojta 2005]
$s > 1/2$: non-trivial exponents ok
$s < 1/2$: non mean-field exponents!

**New result:** [SF & R. Narayanan (unpublished)]
**Trick:** Majorana $\vec{S} = -(i/2)\vec{\eta} \times \vec{\eta}$
Reconsider Landau: $u$ singular at $s < 1/2$!!

**Interpretation:** non-analytic free energy

$$F(M) = hM + rM^2 + \nu M^{2/(1-s)}$$

$$\Rightarrow \beta = \frac{1-s}{2s} \neq \frac{1}{2} \text{ fits numerics!}$$

**Future challenges:** higher dimension generalization
Impurity QPT in superconductors
Pseudogap Kondo model

**Kondo problem:** spin $\vec{S}$ coupled to a Fermi sea

$$H = H_{\text{el.}} + J \vec{S} \cdot \vec{S}_{\text{el.}} (\vec{x} = \vec{0})$$

** Metals:**
- finite DoS $\rho_c(\epsilon) = \text{const.}$
- $\Rightarrow$ Screening of the spin for all $J > 0$

** HTc superconductors:**
- linear DoS $\rho_c(\epsilon) = |\epsilon|$
- $\Rightarrow$ Screening??
Quantum Phase Transition

Quantum phase transition: [Withoff & Fradkin 1990]

Power-law DoS $\rho_c(\epsilon) \propto |\epsilon|^r$

Trick: $r = \text{small parameter}$

Quantum critical point: at $j = j_c$

- Fractional spin $S' < 1/2$: $\chi_{imp}(T) = \frac{S'(S'+1)}{3T}$
- Critical exponents: $\chi_{loc}(\omega) = \omega^{-1+\eta_x}$

Universal crossover: for $j < j_c$ [Fritz, SF, Vojta 2006]
Other progresses

Global approach: expansions around $r = 0, 1/2, 1$

Finite temperature dynamics:
Still a challenge for the NRG at $\omega < T$

Possible openings:
- Clearer experimental studies (STM, nanobridges...)
- Application of our method to more complex models
- Key to understand failure of bosonic approach?
Criticality in quantum dots
Two channel Kondo effect

**Model:** spin $\vec{S}$ coupled to two Fermi seas [Nozières 1980]

$$H = H_{el.1} + H_{el.2} + J_1 \vec{S} \cdot \vec{S}_{el.1} + J_2 \vec{S} \cdot \vec{S}_{el.2}$$

**Critical state:** if $J_1 = J_2 \Rightarrow S_{imp}(T=0) = \log(\sqrt{2})$

**Conclusion:** generic instability!
Can one realize 2CK in dots?

**Experiments on 1CK:** [Goldhaber-Gordon, Kastner 1998]

Mixing of electrodes ⇒ 1CK!

**New idea for 2CK:** [Oreg & Goldhaber-Gordon 2003]

Freeze charge exchange between leads 1 and 2 by capacitive energy cost $E_c$ (Coulomb blockade)

⇒ 2CK possible if large $E_c$
Surprising interplay of correlations

**Solution:** [SF & A. Rosch PRL 2004]

Phase $\theta$ dual to charge: $c_{\sigma m}^\dagger = a_{\sigma m}^\dagger e^{i\theta}$ [SF & A. Georges ’02]

**Results:** inverse crossover $1\text{CK} \rightarrow 2\text{CK}$ if small $E_c$

---

### Implications:

- New paradigm for stability of NFL states
- Optimize experiments: need $E_c \sim T_K^{1C}$
Conclusion

QPT in mesoscopic domain: a promising field!

- QPT are $T = 0$ quantum-driven transitions
- QPT have fascinating consequences
- New ideas and methods for their understanding
- Mesoscopic QPT are easier to understand, but still very surprising!
- Cross-fertilization of ideas between traditional condensed matter and mesoscopic physics
Future directions of research

Quantum phase transitions:

• Challenge: bulk QPT in itinerant magnets
• Non equilibrium quantum phase transitions
• $T = 0$ Mott metal-insulator transition

Other problems:

• Transport in novel mesoscopic magnets
• Connection between many-body & ab-initio