Quantum optics in an universe with large fine structure constant

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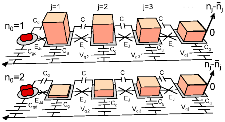
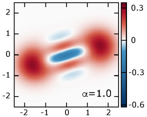
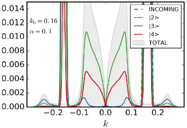






Non-perturbative QED in a nutshell

Fiat Lux

A bit of philosophy about the fine structure constant :

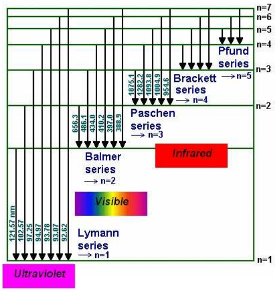
$$\alpha_{\mathrm{QED}} = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137} \quad \text{magic number!}$$

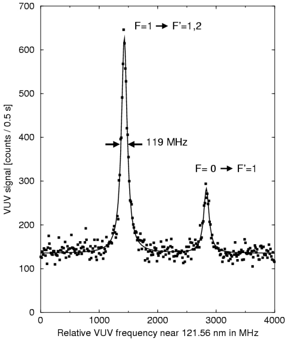
One fine quote by Feynman :

"God's hand wrote α, and we don't know how He pushed His pencil"

What if α_{QED} were much larger ?

- Spectroscopists would hate it :





[Eikema, Walz & Hänsch, PRL 2001]

- Natural linewidth Γ (relative to transition frequency Δ) for 3D atomic decay :

$$\frac{\Gamma}{\Delta} \simeq [\alpha_{\mathrm{QED}}]^3 \simeq 10^{-7}$$

For 1S→ 2P transition, $\Gamma = 10^2$ MHz and $\Delta = 10^9$ MHz

Anyway, playing with $\alpha_{\rm QED}$ is not such a good idea

Decrease $\alpha_{\rm QED}$ (with constant $\alpha_{\rm strong}$) by few % :

- Fast fusion p+p$\to$ ^{2}He takes place
- Stars exhaust fuel and quickly collapse to black holes

[Barrow,Tipler&Wheeler, *"The anthropic cosmological principle" (Oxford)*]

Increase $\alpha_{\rm QED}$ by few % :

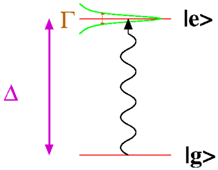
- Blocks nucleosynthesis of heavy elements
- Biology and life are no more possible

Safer approach : emulate this in a superconducting metamaterial !

Ultra-strong coupling of quantum optics

Linewidth of an atomic transition in vacuum :

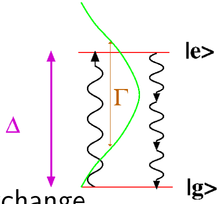
$$\frac{\Gamma}{\Delta} = \left(\frac{P}{e\lambda}\right)^2 \alpha_{\mathrm{QED}}$$
$$\simeq \left(\frac{a_{\mathrm{Bohr}}}{\lambda}\right)^2 \alpha_{\mathrm{QED}} \lll 1$$



- P = transition electric dipole
- λ = wavelength of resonant photon mode

Ultra-strong coupling of QED :

$$\frac{\Gamma}{\Delta} \simeq 1$$

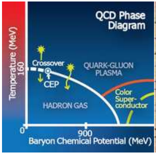


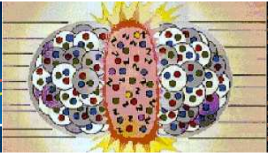
- Higher probability for multi-photon exchange
- Strong non-linearities at small power
- Many-body regime of QED

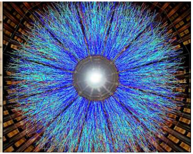
Some general thoughts about the quantum many-body problem

The quantum many-body problem is everywhere in physics

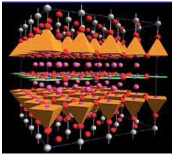
- From Nuclear Physics & Quantum Chromo Dynamics

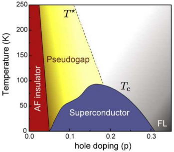


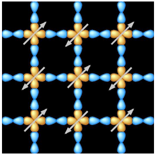




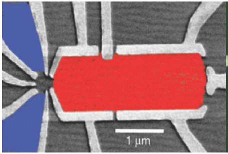
- To condensed (strongly correlated) matter

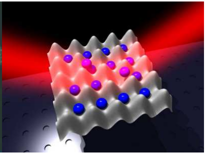






- And artificial systems : Nano-Electronics & Cold Atoms

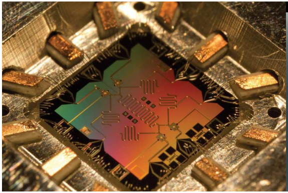


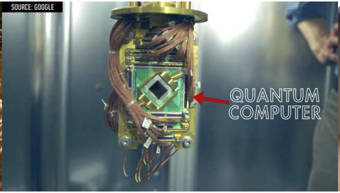


The many-body problem today : a philosophical question

Leave hope and build quantum machines ?

- The many-body wave function is too complex
- Simulate it with a dedicated quantum computer





Or should one stay optimistic ?

Max Born in *"The mechanics of the atom"* (1960).
"It would indeed be remarkable if Nature fortified herself against further advances in knowledge behind the analytical difficulties of the many-body problem."

The difficulty of the quantum many-body problem

A complicated puzzle with :

- Macroscopic number of particles
- Strong interaction between constituents
- Physics on many energy scales
- Individual vs. collective behavior
- Non-equilibrium processes

P. W. Anderson in *"More is different"*, Science **177**, 393 (1972).
*"The behavior of large and complex agregates of elementary particles is **not** to be understood in terms of a simple extrapolation of the properties of a few particles."*

The optimistic view : Life is simpler in the thermodynamic limit !

Many-body cats : Pointer states are selected by the system itself, limiting the degree of quantum superpositions

Superconducting circuits for ultra-strong coupling QED

[Puertas-Martinez *et al.*, in preparation]
[Snyman & Florens, PRB 2015]
[Peropadre, Lindkvist, Hoi, Wilson, Garcia-Ripoll, Delsing & Johansson, NJP 2013]
[Goldstein, Devoret, Houzet & Glazman, PRL 2013]
[LeHur, PRB 2012]

Fiat Lux Reloaded with large α

Step 1 : Increase density of states

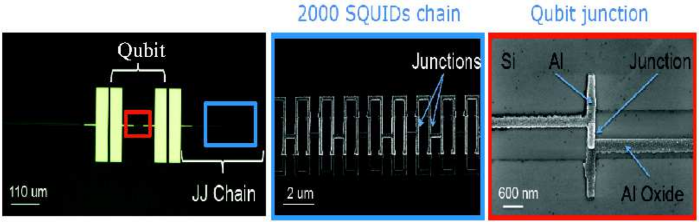
- 1D waveguides $\Rightarrow \Gamma/\Delta \simeq \alpha_{\rm QED}$ instead of $[\alpha_{\rm QED}]^3$

Step 2 : Slow down light to enhance interaction with matter

- Use large inductance medium $=$ Josephson arrays

Step 3 : Optimize atomic dipole P

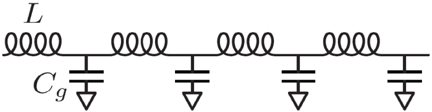
- Use tunable artificial atoms $=$ superconducting qubits



Why large α_{QED} = high inductance medium ?

Alternative expression : $\alpha_{\text{QED}} = \frac{Z_0}{2R_K}$

- $Z_0 = \sqrt{\mu_0/\epsilon_0} \simeq 376\Omega$: vacuum impedance
- $R_K = h/e^2 \simeq 25812\Omega$: resistance quantum



Telegraph equation for LC waveguide : $Z_{\text{chain}} = \sqrt{L/C_g} = \sqrt{\ell/c_g}$

$$\frac{\partial V}{\partial x} = -\ell\frac{\partial I}{\partial t}\,, \quad \frac{\partial I}{\partial x} = -c_g\frac{\partial V}{\partial t}$$

$$\Rightarrow \quad I(x,t) = I_+ e^{i\omega[t-\sqrt{\ell c_g}x-t]} + I_- e^{i\omega[t+\sqrt{\ell c_g}x]}$$

$$\Rightarrow \quad V(x,t) = \sqrt{\ell/c_g}[I_+(x,t) - I_-(x,t)]$$

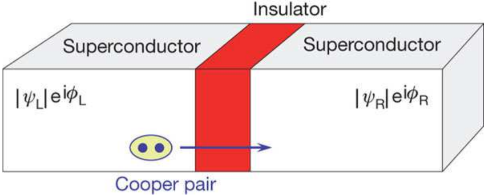
Other interpretation : light with slow velocity $v = 1/\sqrt{\ell c_g}$

Crash course on Josephson effect

Dynamics of the superconducting wavefunction :

$$i\hbar\frac{\partial\Psi_L}{\partial t} = 2eV_L\Psi_L + K\Psi_R$$

$$\hbar\frac{\partial\Psi_R}{\partial t} = 2eV_R\Psi_R + K\Psi_L$$



Ansatz : $\Psi_j = \sqrt{n_j}e^{i\Phi_j}$

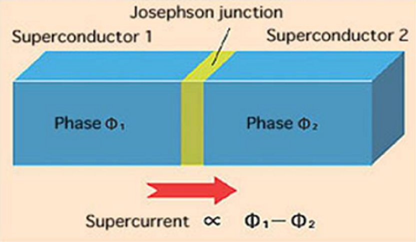
Current : $$I = \frac{\partial n_L}{\partial t} = \frac{2K\sqrt{n_L n_R}}{\hbar}\sin(\Phi_L - \Phi_R)$$

Voltage : $$V = V_L - V_R = \frac{\hbar}{2e}\frac{\partial[\Phi_L - \Phi_R]}{\partial t}$$

Energy : $$E = \int \mathrm{d}t\ VI = \int \mathrm{d}t\ E_J\sin(\Phi)\frac{\partial\Phi}{\partial t} = -E_J\cos(\Phi)$$

Josephson junction = high inductance lossless element

Josephson relations : Φ is phase difference across a junction



$$\boxed{I = I_c \sin\Phi \simeq I_c\Phi}\text{(linearized)}$$

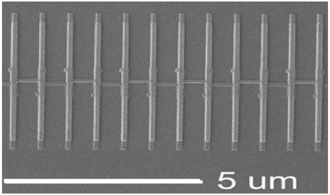
$$\boxed{V = \frac{\hbar}{2e}\frac{\partial\Phi}{\partial t}}$$

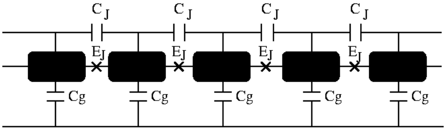
$$\Rightarrow V = \frac{\hbar}{2eI_c}\frac{\partial I}{\partial t} = L_J\frac{\partial I}{\partial t} \quad \Rightarrow \quad Z_J(\omega) = iL_J\omega$$

- Josephson inductance density : $\ell_J \simeq 1\ \mathrm{nH}/\mu\mathrm{m} = 10^4\ell_{\mathrm{geometric}}$
- Geometric self-inductance density :
  $\ell_{\mathrm{geometric}} = \mu_0/(4\pi) = 10^{-7}\ \mathrm{H/m} = 10^{-4}\ \mathrm{nH}/\mu\mathrm{m}$

Josephson junction arrays

Waveguide : a chain of tunnel-coupled superconducting islands





Generic Hamiltonian : valid for $T \ll T_c$

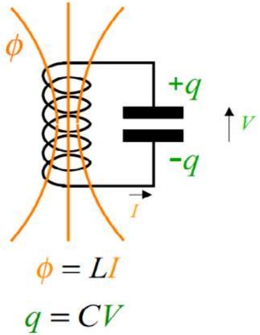
$$H = \frac{1}{2} \sum_{i,j} (2e)^2 n_i \left(C^{-1}\right)_{ij} n_j - E_J \cos(\Phi_i - \Phi_{i+1})$$

Typical energy scales below 1K : $f < 20$ GHz range (microwaves)

$n - \Phi$ are conjugate variables : fluctations controlled by ratio of

$E_C \sim (2e)^2/C$ and $E_J = \hbar^2/[(2e)^2 L_J]$

Crash course on circuit-QED for LC-resonator



Classical energy : harmonic oscillator

$$H = \frac{Q^2}{2C} + \frac{L}{2} I^2 = \frac{Q^2}{2C} + \frac{L}{2} (\dot{Q})^2$$

Conjugate classical variables : charge/flux

$$\frac{\partial H}{\partial \dot{Q}} = L\dot{Q} = LI = \phi$$

Quantum regime : $[\hat{Q}, \hat{\phi}] = i\hbar$ What does it mean ?

- Tiny electromagnetic signals generated by charge fluctuations of order $\delta Q \simeq 2e$ and flux fluctuations of order $\delta \phi \simeq h/2e$
- Vacuum reached when $k_B T \ll \hbar/\sqrt{LC}$ and low loss

Bath engineering

$$H = \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left(C^{-1}\right)_{ij} n_j - E_J \cos(\Phi_i - \Phi_{i+1})$$

Harmonic regime :

- For $E_J \gg (2e)^2/(2C_J + C_g)$, weak phase fluctuations :

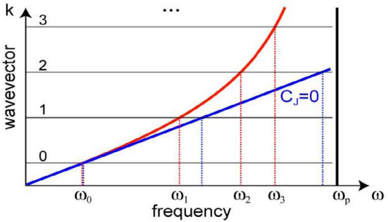
$$H_{\text{chain}} \simeq \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left(C^{-1}\right)_{ij} n_j + \sum_i \frac{E_J}{2}(\Phi_i - \Phi_{i+1})^2 = \sum_k \omega_k a_k^\dagger a_k$$

Spectrum :

$$\omega_k = 2\sin(\tfrac{k}{2})\sqrt{\frac{(2e)^2 E_J}{C_g + 4C_J \sin^2(k/2)}}$$

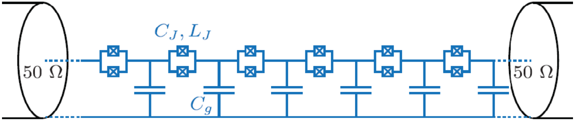
$$\omega_k \underset{k\to 0}{\simeq} k\sqrt{(2e)^2 E_J/C_g}$$

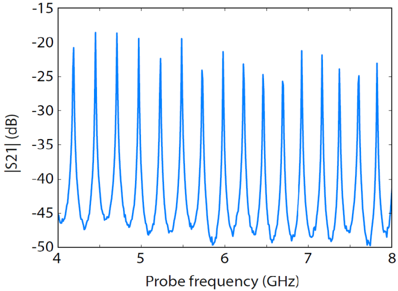
$$\omega_k \underset{k\to 0}{\propto} k/\sqrt{L_J}$$

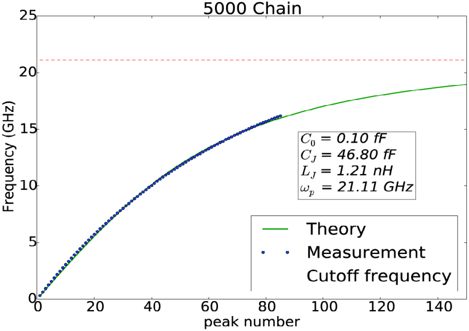


Seeing the modes !

Finite chain coupled to 50 Ω lines : "giant Fabry-Perot cavity"





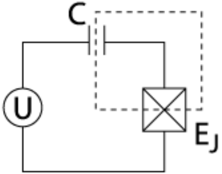


Fitting : $\omega_k = vk$ with $v = c/50$ (slow light)

Artificial atom engineering

Cooper pair box : gate-tuned Josephson grain

- For $E_C \gg E_J$: charge locked $\Rightarrow$ phase fluctuates
- Maximize non-linear effects, but not optimal w.r.t. noise



At charge degeneracy point :

$$H = \frac{(2e)^2}{2C}\left[\hat{N} - N_g\right]^2 - E_J \cos\hat{\Phi} \Rightarrow H = E_J \frac{\hat{\sigma}_x}{2}$$

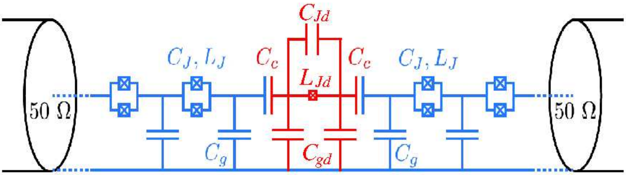
Two-level system :

$$\left|g/e\right\rangle = \frac{\left|N+1\right\rangle \pm \left|N\right\rangle}{\sqrt{2}} = \frac{\left|\uparrow\right\rangle \pm \left|\downarrow\right\rangle}{\sqrt{2}}$$

Atom+bath on a chip

Our proposal :

- Couple capacitively a charge qubit to waveguide $\Longrightarrow$ Two-level system + harmonic bath



Effective Hamiltonian : [Leggett *et al.* RMP (1987)]

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

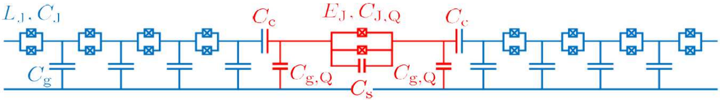
Two-level system : relative charge$=\sigma_z$ with splitting $\Delta = E_{Jd}$

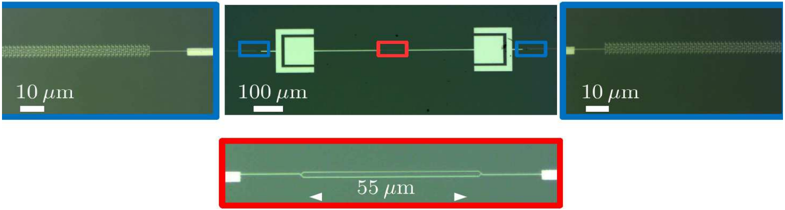
Reaching the ultra-strong coupling regime

Spectral density : $J(\omega) \equiv \sum_k g_k^2 \delta(\omega - \omega_k)$

- Ohmic spectrum : $J(\omega) = 2\pi\alpha\omega$ for $\omega \ll \omega_P$
- Relative linewidth from Golden Rule : $\frac{\Gamma}{\Delta} = \frac{\pi}{2}\alpha$
- α controlled by geometric capacitances and chain impedance :

$$\alpha = \left(\frac{C_c}{C_{\mathrm{tot}}}\right)^2 \frac{2Z_{\mathrm{chain}}}{R_K}$$

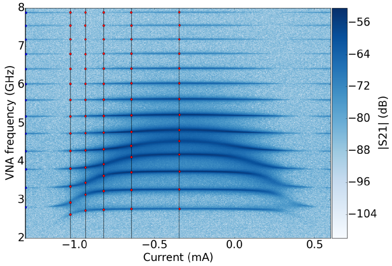


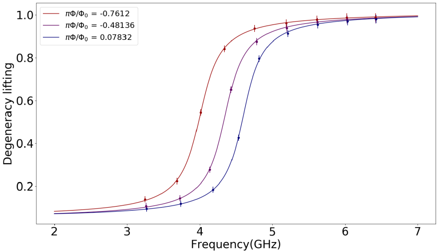


Experimental status

<u>First evidence for multi-mode hybridization :</u>

Sample with moderate impedance (500Ω) $\Longrightarrow \alpha \simeq 0.1$





We measure indeed $\Gamma \simeq 400$MHz and $\Delta \simeq 4$GHz

<u>Current and future work :</u> [Puertas-Martinez *et al.* (unpublished)]

- Study of many-body non-linear effects

Vacuum at large α : dressed atom

[Snyman & Florens, PRB 2015]
[Bera, Florens, Baranger, Roch, Nazir & Chin, PRB 2014]

The traditional vacuum in quantum optics

Re-write Hamiltonian : in atom eigenbasis

$$H = \frac{\Delta}{2}\tau_z - \sum_k \frac{g_k}{2}[(\tau^+ + \tau^-)a_k^\dagger + (\tau^+ + \tau^-)a_k] + \sum_k \omega_k a_k^\dagger a_k$$

with $\tau_z = |e\rangle\langle e| - |g\rangle\langle g|$ and $\tau^+ = |e\rangle\langle g|$

RWA approximation : a photon is absorbed when exciting the atom

$$H \simeq \frac{\Delta}{2}\tau_z - \sum_k \frac{g_k}{2}(\tau^- a_k^\dagger + \tau^+ a_k) + \sum_k \omega_k a_k^\dagger a_k$$

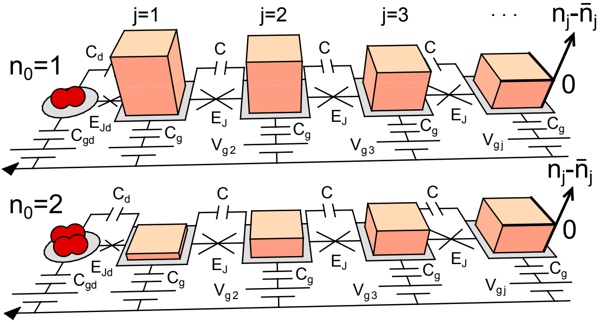
"Exact" ground state (under RWA) :

$$|\Psi\rangle = |g\rangle \otimes |0\rangle = \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}} \otimes |0\rangle = \frac{|\uparrow\rangle \otimes |0\rangle}{\sqrt{2}} - \frac{|\downarrow\rangle \otimes |0\rangle}{\sqrt{2}}$$

This seems intuitive, but...

True vacuum : dressed atom

Physics : each atomic state induces its own charge polarization



$$|\Psi\rangle \simeq \frac{|\uparrow\rangle \otimes |\Psi_\uparrow\rangle}{\sqrt{2}} - \frac{|\downarrow\rangle \otimes |\Psi_\downarrow\rangle}{\sqrt{2}}$$

Entangled state between atom and Josephson chain ! !

[Snyman and Florens (2015)]

The classical limit $\Delta = 0$

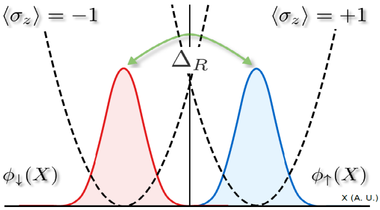
No tunneling : frozen charge $\Rightarrow$ doubly-degenerate ground state

$$H = \sum_k \omega_k a_k^\dagger a_k - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) = \sum_k \omega_k \left[a_k^\dagger - \frac{\sigma_z g_k}{2\omega_k}\right]\left[a_k - \frac{\sigma_z g_k}{2\omega_k}\right]$$

$|\Psi_\uparrow\rangle = |f^{\mathrm{bare}}\rangle$
$|\Psi_\downarrow\rangle = |-f^{\mathrm{bare}}\rangle$

with coherent state
$|\pm f\rangle \equiv e^{\pm \sum_k f_k (a_k^\dagger - a_k)}|0\rangle$

and displacement
$f_k^{\mathrm{bare}} = g_k/(2\omega_k)$



Physical interpretation : two different macroscopic charge polarisations of the array depending whether the qubit contains N or $N+1$ Cooper pairs

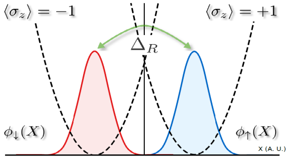
Approximate ground state for $\Delta \neq 0$ and finite α

The qubit is simply dressed by coherent states :

$$|\Psi\rangle \simeq \frac{|\uparrow\rangle \otimes |+f\rangle - |\downarrow\rangle \otimes |-f\rangle}{\sqrt{2}}$$

Singlet-like Ansatz imposed by tunneling process

[Emery&Luther, PRB (1974) ; Silbey&Harris JChemPhys 1984]



True charge displacement : variational optimization gives

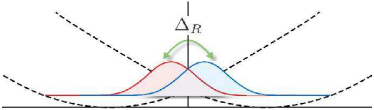
$$f_k = (1/2) g_k / (\omega_k + \Delta_R)$$

with $\Delta_R = \Delta \langle f | -f \rangle \simeq \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)} \ll \Delta$ Giant Lamb shift

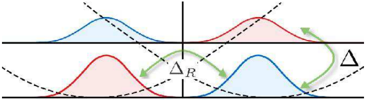
What's missing at large α : the anti-dressed cloud

Physics at play : how to maximize the tunneling amplitude $\Delta\sigma_x$?

- An increase of the overlap is energetically unfavorable :



- Quantum superposition with an anti-cloud does the trick !



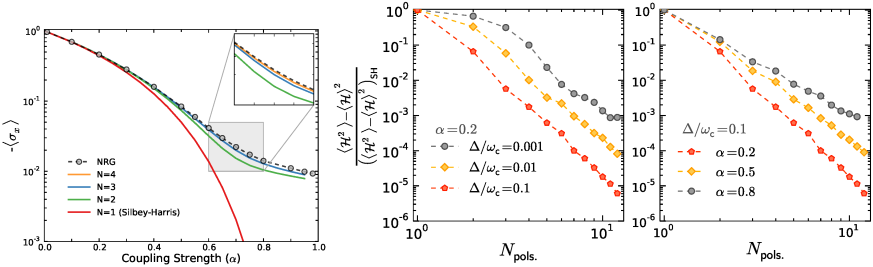
Turning this into a powerful machinery : variational cat states

$$\left|\Psi_\uparrow\right\rangle = \sum_{n=1}^{N_{\mathrm{cats}}} p_n \left|f^{(n)}\right\rangle = \sum_{n=1}^{N_{\mathrm{cats}}} p_n e^{\sum_k f_k^{(n)}(a_k^\dagger - a_k)}|0\rangle$$

Optimize numerically the weights p_n and displacements $f_k^{(n)}$

Checking the cat states expansion

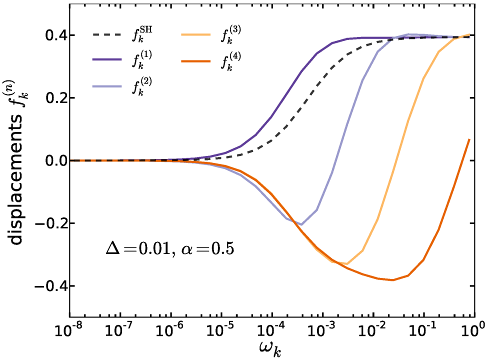
Tunneling amplitude $\langle \sigma_x \rangle$ and energy variance :

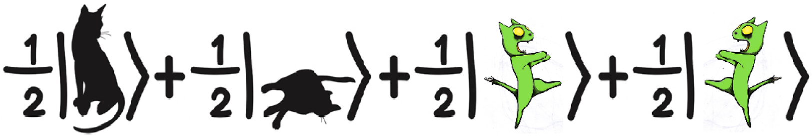


- Fast convergent expansion : error vanishes for small cat number
- Main numerical difficulty : reaching efficiently the global minimum in a very flatish landscape

Structure of the many-body cat $|\Psi_\uparrow\rangle$

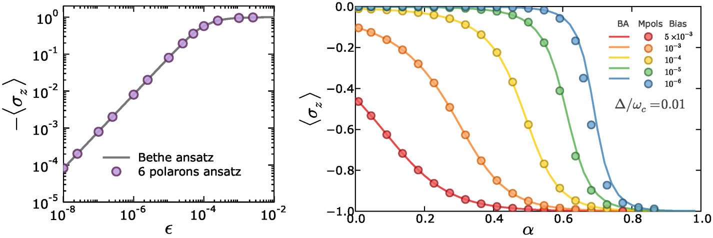
Full form of displacement in momentum space : 4-component cat





Variational cat states : benchmark

Qubit charge $\langle \sigma_z \rangle$ vs local gate ϵ and coupling α :



- Agreement with exact Bethe Ansatz [Ponomarenko PRB 1993]

Advantage of the method :

- Conceptually simple and numerically fast
- One shot computation of observables ($|\Psi\rangle$ is known !) for the true bath spectrum (no discretization)

$\Longrightarrow$ More powerful than state-of-the art techniques

Quantum dynamics : spontaneous emission at large α

[Gheeraert, Bera & Florens, New J. Phys. (2017)]

Dirac-Frenkel variational dynamics at $T = 0$

Procedure :

- Introduce time-dependent weights and displacements :

$$\big|\Psi(t)\big\rangle = \sum_{n=1}^{N_{\rm cats}} \Big[p_n(t)\big|f^{(n)}(t)\big\rangle \otimes |\uparrow\rangle + q_n(t)|h^{(n)}(t)\rangle \otimes |\downarrow\rangle \Big]$$

Note : all terms are complex, and $\mathbb{Z}_2$ symmetry is fully broken

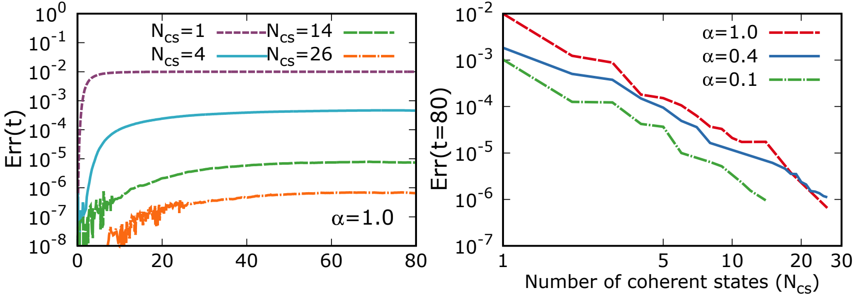
- Construct Lagrangian : $\mathcal{L} = \big\langle \Psi(t)\big| \frac{i}{2}\overleftrightarrow{\partial_t} - \mathcal{H} \big|\Psi(t)\big\rangle$
- Solve Hamilton-Jacobi equations : $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{f}_k^{(n)}} = \frac{\partial \mathcal{L}}{\partial f_k^{(n)}}, \ldots$
- Classical-like equations of motion (after manipulation) :

$$\frac{\mathrm{d}}{\mathrm{d}t} f_k^{(n)} = \sum_{k'=1}^{N_{\rm modes}} \sum_{n'=1}^{N_{\rm cats}} C_{k,k'}^{(n,n')}[f, h, p, q]$$

Norm and energy are conserved by construction

Checking convergence for a quantum quench

Measure of the error : $\mathrm{Err}(t) = \left|\left|(i\partial_t - H)|\Psi(t)\rangle\right|\right|^2$



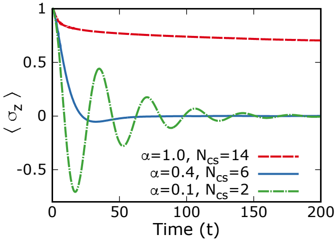
Convergence is independent of α : non-perturbative method

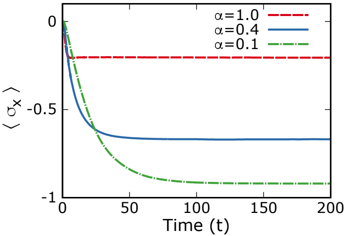
Qubit dynamics at ultra-strong coupling

Protocol : prepare state $\left|\Psi\right\rangle = \left|0\right\rangle \otimes \left|\uparrow\right\rangle = \left|0\right\rangle \otimes [\left|g\right\rangle + \left|e\right\rangle]/\sqrt{2}$

Quantum optics regime ($\alpha \ll 1$) : $T_2 = 2T_1$

- T_2 = time scale for decoherence $\Leftrightarrow$ decay of $\langle \sigma_z(t) \rangle$
- T_1 = time scale for energy relaxation $\Leftrightarrow$ decay of $\langle \sigma_x(t) \rangle$





Ultra-strong coupling regime ($\alpha \simeq 1$) : $T_2 \gg 2T_1$

Relaxation and decoherence time scales decouple !

Decoupling of the dynamics

Fast energy relaxation :

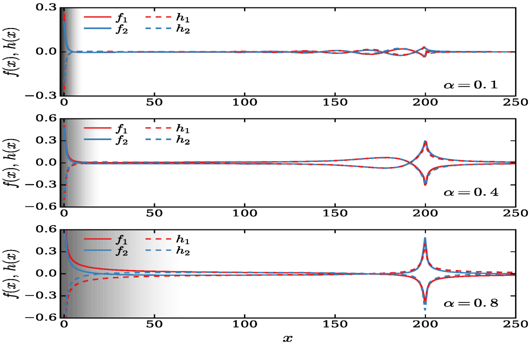
- $|e\rangle$ is very strongly damped at increasing α
- Relaxation time as short as $T_1 \simeq 1/\omega_p$ for $\alpha \simeq 1$

Slow decoherence :

- The system must equilibrate to the many-body ground state
- This can take an exponential time $T_2 \propto L_K \propto 1/\Delta_R$
- Quantum information is transferred slowly from qubit to environment (due to extended size of dressed cloud)

Spatial profile of electromagnetic modes

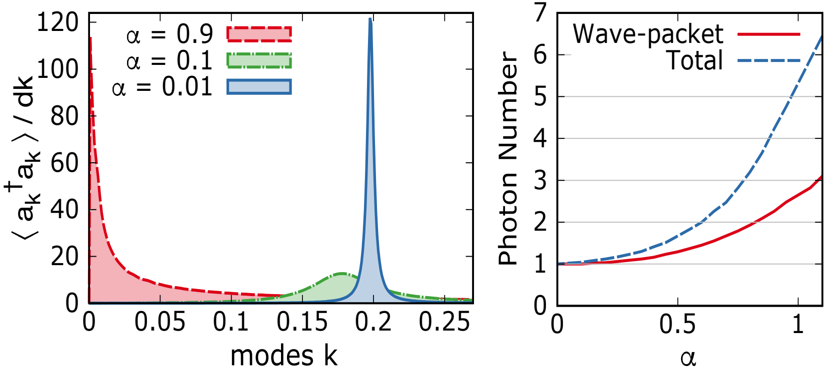
Initial state : $|\Psi\rangle = |0\rangle \otimes |e\rangle = |0\rangle \otimes (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$



For large α :

- Entanglement spreads within the waveguide (T_2 long)
- Wavepacket is dominated by wavefront (T_1 short)
- Displacements in the wavepacket increase

Distribution of emitted photons at large α



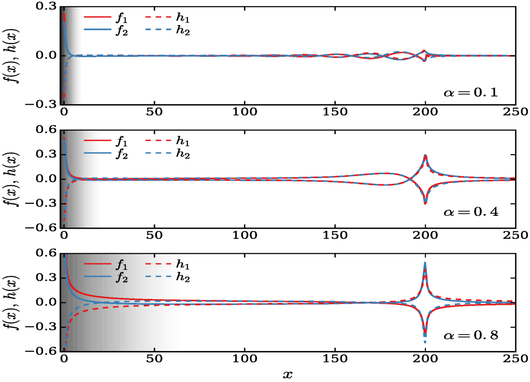
For increasing α :

- Fast energy relaxation $\Rightarrow$ broad spectral distribution
- $\Gamma/\Delta \simeq 1 \Rightarrow$ Photon number $\gg 1$

Emission : from photon to cat

A photon = small Schrödinger kitten

$$|f\rangle - |-f\rangle = \sum_k 2 f_k a_k^\dagger |0\rangle \ \text{for } f_k \to 0$$

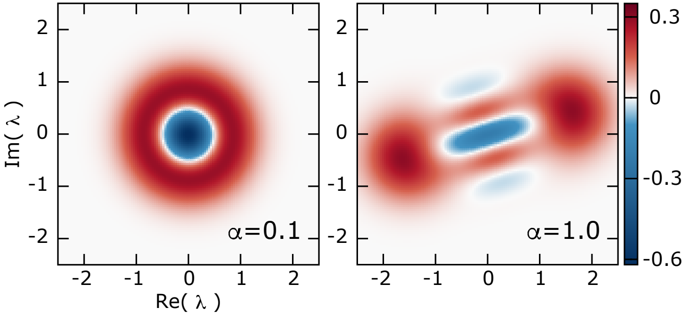


Kitten size grows at increasing α since $n = |f|^2$

Emission : from photon to cat

A photon = small Schrödinger kitten

$$|f\rangle - |-f\rangle = \sum_k 2 f_k a_k^\dagger |0\rangle \quad \text{for } f_k \to 0$$

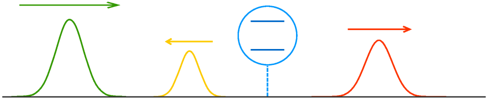


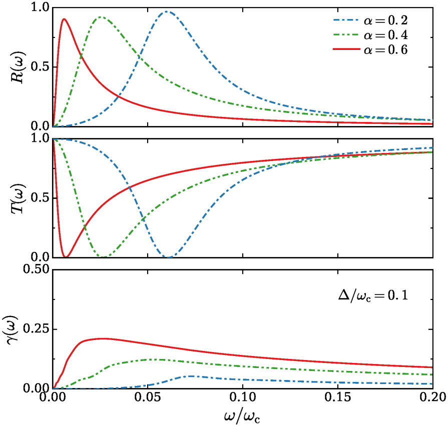
- Cat states with $n = 3$ are radiated for $\alpha \simeq 1$

Strong inelastic effects in photon scattering

[Gheeraert *et al.*, in preparation]
[Bera, Baranger & Florens, PRA 2016]
[Goldstein, Devoret, Houzet & Glazman, PRL 2013]

Huge inelastic losses at large α



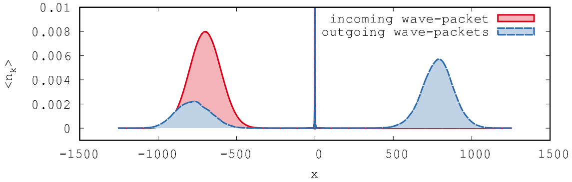


Reflection $R(\omega)$

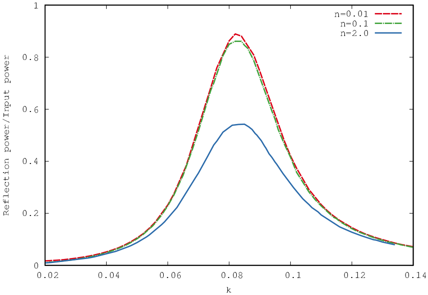
Transmission $T(\omega)$

Losses $1 - R(\omega) - T(\omega)$

Checking qubit saturation at high intensity

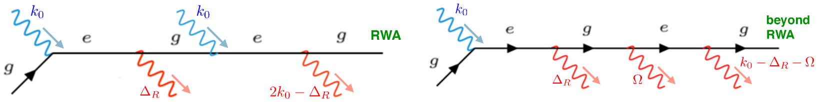


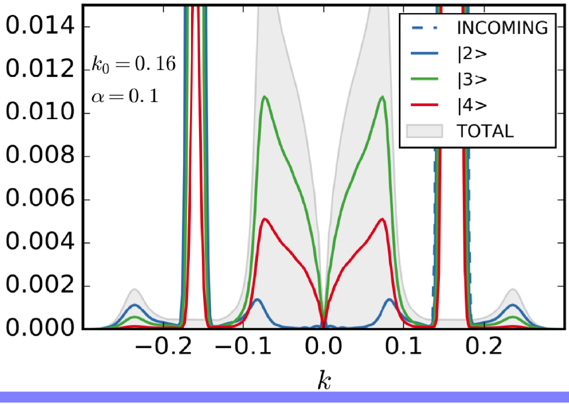
Reflected power : for various beam intensity n



Single and multi-photon non-linearities

Frequency conversion spectra : [Gheeraert *et al.* (in preparation)]





Conclusion and perspectives

- Quantum optics expectations change at ultra-strong coupling :
  - Vacuum is non-trivial : atom is dressed by cloud of photons
  - Atom spontaneously emit spectrally broad cat states
  - Inelastic cross-sections for particle production are huge
- Perspectives :
  - Experimental investigations of these effects
  - Extensions of cat state ideas to metallic nano-circuits
  - Extensions of cat state ideas to lattice models

Acknowledgments

- Experimental side : Sébastien Léger, Javier Puertas-Martinez, Nicolas Roch (NEEL)

- Theory side : Harold Baranger (Duke University), Soumya Bera (NEEL, now IITB), Alex Chin (Cavendish Lab), Nicolas Gheeraert (NEEL), Ahsan Nazir (Manchester University), Izak Snyman (Wits University)

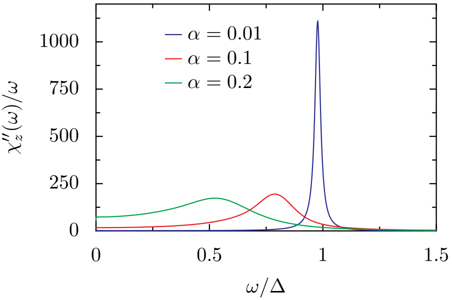

That's all Folks!
SUPERB wallpapers

Extra slides

Weak dissipation regime : $\alpha < 0.4$

Spin dynamics : underdamped Rabi oscillations

- Bosonic NRG "solves" the model [Bulla *et al.* PRL (2003)]
- Spin-spin dynamical correlation functions for arbitrary dissipation strength [Florens *et al.* PRB (2011)]

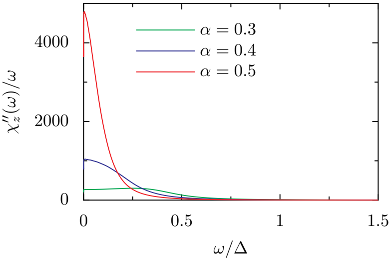


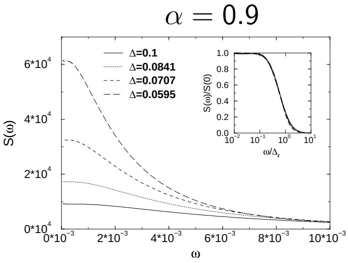
- Peak at renormalized scale $\Delta_R < \Delta$
- Non-lorentzian lineshape for $\alpha > 0.1$

Strong dissipation regime : $\alpha > 0.4$

Spin dynamics : overdamped Rabi oscillations

- Linewidth $\Gamma > \Delta_R$: incoherent qubit
- Boring ? No ! : universal (Kondo) regime for $\alpha \lesssim 1$
  $\rightarrow$ strongly correlated many-body photonic state





Exact mapping to Kondo model

Spin boson Hamiltonian :

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Unitary transformation : $U_\gamma = \exp\{-\gamma\sigma_z \sum_k \frac{g_k}{2\omega_k}(a_k^\dagger - a_k)\}$

$$U_\gamma H U_\gamma^\dagger = \frac{\Delta}{2}\sigma^+ e^{-\gamma \sum_k \frac{g_k}{\omega_k}(a_k^\dagger - a_k)} + h.c. + (\gamma - 1)\sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

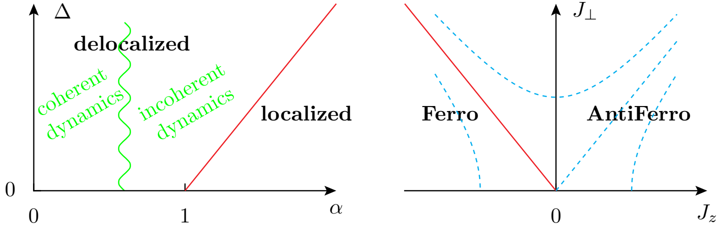
Bosonization (ohmic bath only) : [Guinea, Hakim, Muramatsu PRB (1985)]

$$U_\gamma H U_\gamma^\dagger = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \Delta\sigma^+ \sum_{kk'} c_{k\downarrow}^\dagger c_{k'\uparrow} + h.c. \quad \rightarrow J_\perp = \Delta$$

$$+(1-\sqrt{\alpha})\omega_c \sigma^z \sum_{kk'} [c_{k\uparrow}^\dagger c_{k'\uparrow} - c_{k\downarrow}^\dagger c_{k'\downarrow}] \quad \rightarrow J_z \propto 1 - \sqrt{\alpha}$$

Relating Kondo to spin-boson

Phase diagram :



Spatial correlations : an exact identity connects bosonic to fermionic Kondo cloud

$$\left\langle \sigma_z [c^\dagger_\uparrow(x) c_\uparrow(x) - c^\dagger_\downarrow(x) c_\downarrow(x)] \right\rangle \sim \chi(x) + \cos(2k_F x)\chi_{2k_F}(x)$$

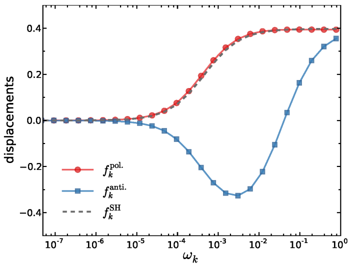
Unveiling the Kondo anti-cloud

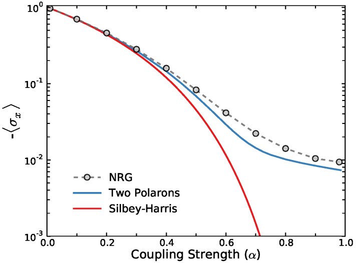
New Ansatz with two coherent states :

$$|\Psi\rangle = |\uparrow\rangle \otimes \left[|+f_k^{\mathrm{pol.}}\rangle + p|+f_k^{\mathrm{anti.}}\rangle \right] - |\downarrow\rangle \otimes \left[|-f_k^{\mathrm{pol.}}\rangle + p|-f_k^{\mathrm{anti.}}\rangle \right]$$

$p =$ weight of the anti-cloud relative to the main polarization

Variationally determined displacements and $\langle \sigma_x \rangle$:



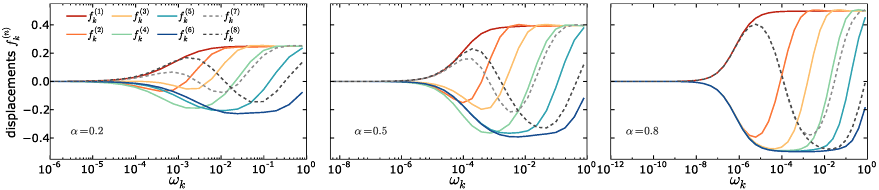


These correlations drastically improve the tunneling amplitude !

General many-body coherent states expansion

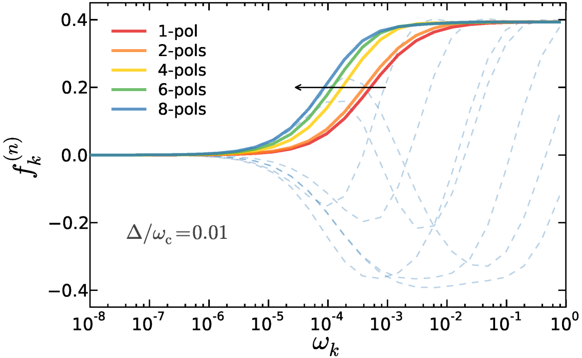
Idea : we expand the wavefunction in the coherent state "basis"

$$|\Psi\rangle = \sum_{n=1}^{N_{\rm cats}} p_n \Big[|+f^{(n)}\rangle \otimes |\uparrow\rangle - |-f^{(n)}\rangle \otimes |\downarrow\rangle \Big]$$



- Development of an efficient numerical algorithm : $f_k^{(n)}$ parametrized by $(N_{\rm cats})^2$ coefficients $\Longrightarrow$ Cheap !

Kondo scale (extra) renormalization



Exact Kondo scale : $\boxed{T_K = C_\alpha (\Delta/\omega_c)^{\alpha/(1-\alpha)}}$

The renormalization factor C_α can be important at strong coupling

Status of theory

Theoretical challenges :

- Fast and accurate impurity solvers still in demand (DMFT)
- Some challenges to extend available methods (NRG, DMRG, QMC, fRG,...) to non-equilibrium dynamics and strong correlations

Experimental challenges :

- Probe regimes beyond linear-response (finite voltage bias, strong AC drive)
- Measure correlations not only in the time-domain (*e.g.* probe the spatial structure of the Kondo cloud)

Our philosophy here :

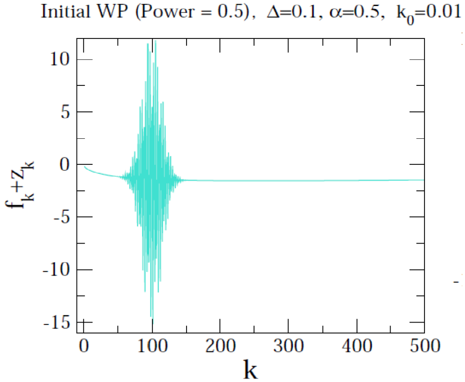
Progress comes with a better understanding of the environment (we won't trace out the bath !)

Scattering onto many-body vacuum

Initial state preparation :

Polaron ground state + coherent wavepacket z_k with momentum k_0

$$|\Psi(t=0)\rangle = [|\uparrow\rangle \otimes |+f\rangle - |\downarrow\rangle \otimes |-f\rangle] \otimes |+z\rangle$$



- $f_k = (1/2) g_k / (\omega_k + \Delta_R)$
- $z_k \propto e^{-(k-k_0)^2/\sigma^2} e^{ikx_0}$
- Compute elastic transmission from wavepacket photon number :

$$T(k_0) = \frac{\sum_k |z_k^{(\text{out})}|^2}{\sum_k |z_k^{(\text{in})}|^2}$$

Direct computation of the fermionic Kondo cloud

Full fermionic cloud (including Friedel oscillations) :

$$\chi^{\parallel}(x) = \left\langle \sigma_z S_z^{\text{el.}}(x) \right\rangle \sim \chi_0^{\parallel}(x) + \cos(2k_F x)\chi_{2k_F}^{\parallel}(x)$$

$$\chi_0^{\parallel}(x) = \frac{\partial_x}{2\pi} \left\langle \sigma_z \left[\varrho^{\dagger}(x) + \varrho(x) \right] \right\rangle \text{ (same as previously)}$$

$$\chi_{2k_F}^{\parallel}(x) = \frac{2}{\pi x} \mathrm{Im} \left\langle \sigma_z e^{i\varrho^{\dagger}(x)} e^{i\varrho(x)} \right\rangle$$

with $\varrho(x) = 2 \sum_{q>0} \sqrt{\frac{\pi}{Lq}} e^{-aq/2} \sin(qx) a_q$

Spin-flip component of the cloud :

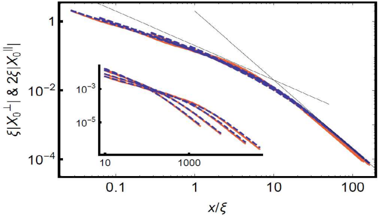
$$\chi^{\perp}(x) = \left\langle \sigma_x S_x^{\text{el.}}(x) + \sigma_y S_y^{\text{el.}}(x) \right\rangle = \chi_0^{\perp}(x) + \cos(2k_F x)\chi_{2k_F}^{\perp}(x)$$

which can be similarly bosonized

All these observables are readily computed in the coherent state expansion of the ground state (trivial algebra)

Results for the fermionic cloud on SU(2) line ($J^{\parallel} = J^{\perp}$)

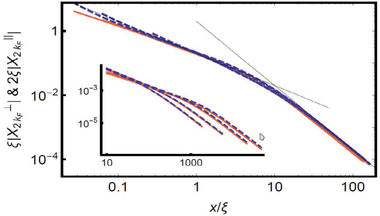
Enveloppe component of the fermionic cloud :



Red solid line : $\chi_0^{\parallel}(x)$
Blue dashed line : $\chi_0^{\perp}(x)$

Isotropy and universality are satisfied !

Friedel component of the fermionic cloud :



Red solid line : $\chi_{2k_F}^{\parallel}(x)$
Blue dashed line : $\chi_{2kF}^{\perp}(x)$

Useful concept : fermionic "coherent" state

Idea : the generic many-body coherent state
$|f\rangle = e^{\sum_k (f_k a_k^\dagger - f_k^\star a_k)}|0\rangle$ is the ground state of :

- Bosonic Hamiltonian : $H_0 = \sum_k k(a_k^\dagger - f_k^\star)(a_k - f_k)$
- Fermionic Hamiltonian : $H_0 = \int dx \psi^\dagger(x)[-i\partial_x + V(x)]\psi(x)$
  with $V(x) = -2Re\left[\sum_k \sqrt{k} e^{ikx} f_k\right]$

Fermionic "coherent" state = phase-shifted Fermi Sea $|FS[V]\rangle$

Relates to early ideas by [Anderson Phys.Rev. 1967]

Insight : fermionic coherent states expansion formulated from the Fermi sea overlaps $\langle FS[V]|FS[V']\rangle = e^{-\sum_k [|f_k|^2 + |f_k'|^2 - 2 f_k^\star f_k']/2}$

The full Kondo wavefunction thus reads :

$$|\Psi_{\mathrm{Kondo}}\rangle = \sum_{n=1}^{N_{\mathrm{cats}}} p_n \left[|FS[V^{(n)}]\rangle \otimes |\uparrow\rangle - |FS[-V^{(n)}]\rangle \otimes |\downarrow\rangle \right]$$

Outlook : application to other impurity models and DMFT