Slave Rotor approach to the strong correlation problem

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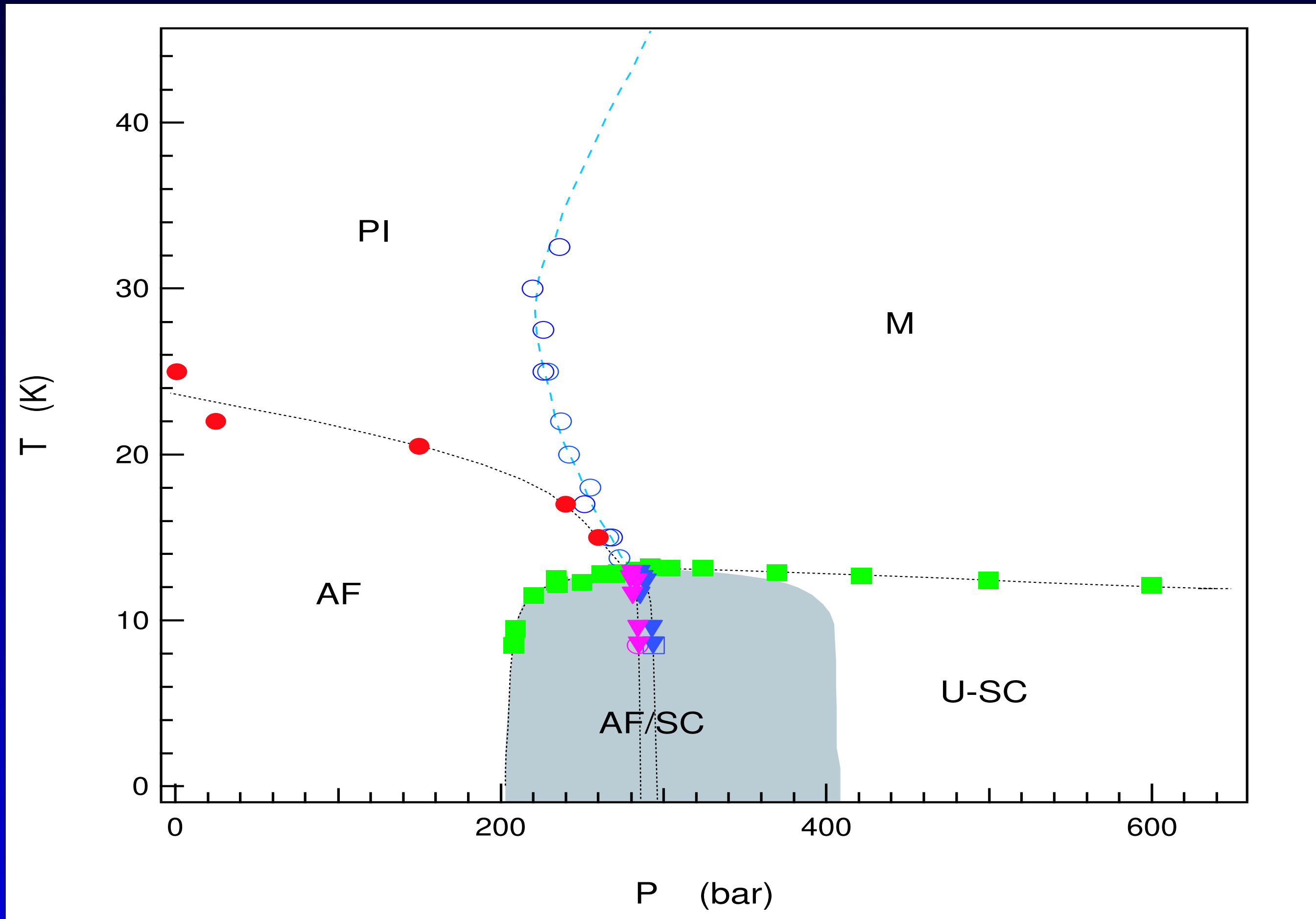
- New technique to deal with electronic interactions
- Common aspects to strongly correlated materials and mesoscopic devices
- How is metallic coherence restored?

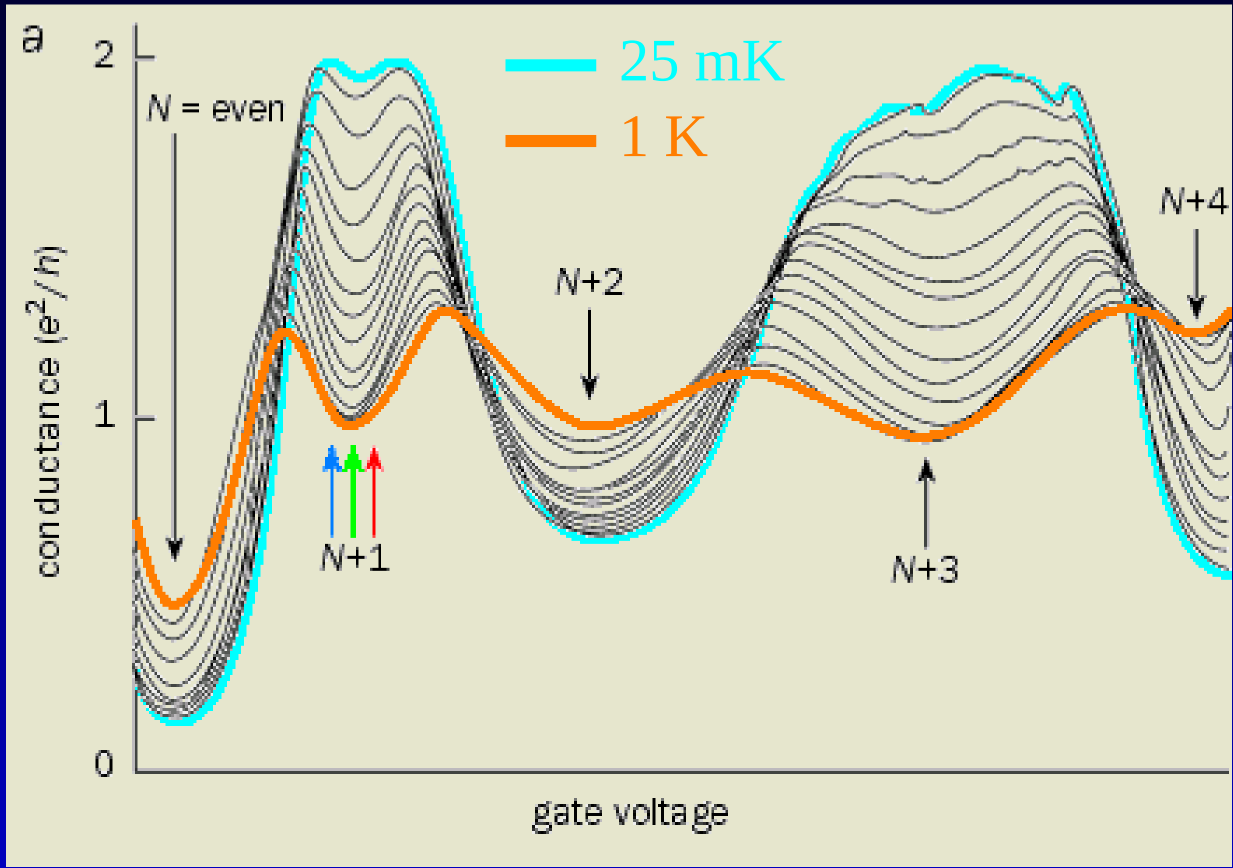
Mott physics in 2d organics

κ-$(BEDT)_2Cu[N(CN)_2]Cl$



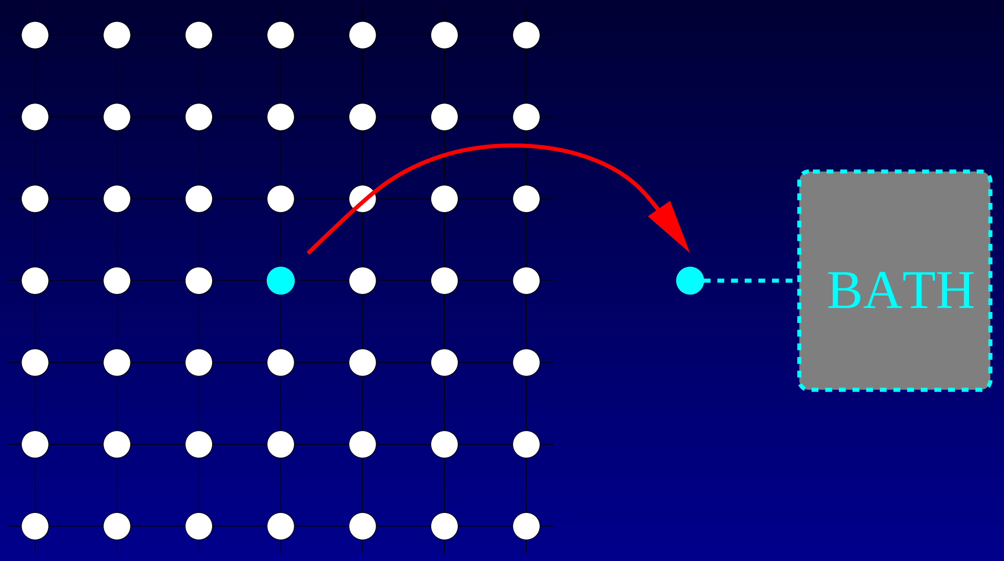
S. Lefebvre *et al.*, PRL **85** 5420 (2000)

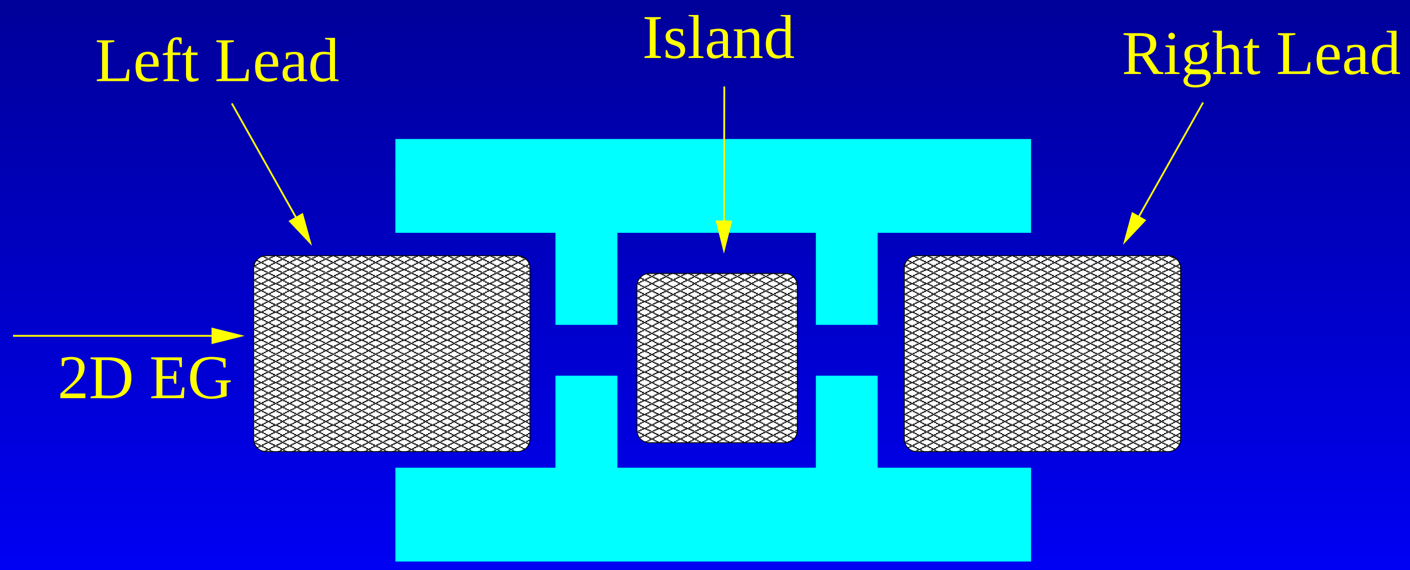
Correlations in mesoscopic devices



Cf. review by Kouwenhoven and Glazman

→ Quantum impurity models





Impurity solver

Ideally, one wants:

- non perturbative solution
- real frequency computation
- multiorbital
- fast and accurate...

Slave Rotor representation

Generalized SU(N) impurity $\sigma = 1 \ldots N$:

$$H = \frac{U}{2}\left[\sum_\sigma \left(d^\dagger_\sigma d_\sigma - \frac{1}{2}\right)\right]^2 = \frac{U}{2}\hat{L}^2$$

with the equivalence:

$$d^\dagger_\sigma = f^\dagger_\sigma e^{i\theta},\ \hat{L} = -i\partial/\partial\theta$$

Necessary constraint:

$$\hat{L} = \sum_\sigma \left(d^\dagger_\sigma d_\sigma - \frac{1}{2}\right)$$

Applications

- Anderson impurity model
- Mott transition
  - DMFT
  - Beyond DMFT
- Mesoscopic
  - Single Electron Box
  - Granular systems

Anderson model

$$
\begin{aligned}
H \;=\; & \sum_{\sigma} \epsilon_0 f^{\dagger}_{\sigma} f_{\sigma} + \frac{U}{2}\hat{L}^2 + \sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} \\
& + \sum_{k\sigma} V \left(c^{\dagger}_{k\sigma} f_{\sigma} e^{-i\theta} + f^{\dagger}_{\sigma} e^{i\theta} c_{k\sigma} \right) \\
& \bigoplus \text{constraint}
\end{aligned}
$$

$$
\begin{aligned}
S \;=\; & \int_0^{\beta} d\tau \sum_{\sigma} f^{\dagger}_{\sigma} (\partial_{\tau} + \epsilon_0 - h) f_{\sigma} + \frac{(\partial_{\tau}\theta + ih)^2}{2U} + \frac{N}{2} h \\
& + \int_0^{\beta} d\tau \int_0^{\beta} d\tau' \Delta(\tau - \tau') \sum_{\sigma} f^{\dagger}_{\sigma}(\tau) f_{\sigma}(\tau') \, e^{i\theta(\tau) - i\theta(\tau')}
\end{aligned}
$$

Approximation scheme

Define $X \equiv e^{i\theta}$
Spherical limit: $|X(\tau)|^2 = 1$ in average

Green's functions:

$$\begin{aligned} G_f(i\omega_n)^{-1} &= i\omega_n - \epsilon_0 + h - \Sigma_f(i\omega_n) \\ G_X(i\nu_n)^{-1} &= \frac{\nu_n^2}{U} + \lambda - \frac{2ih\nu_n}{U} - \Sigma_X(i\nu_n) \\ G_X(\tau = 0) &= 1 \end{aligned}$$

Self-energies:

$$\begin{aligned} \Sigma_X(\tau) &= N\Delta(\tau)G_f(\tau) \\ \Sigma_f(\tau) &= \Delta(\tau)G_X(\tau) \end{aligned}$$

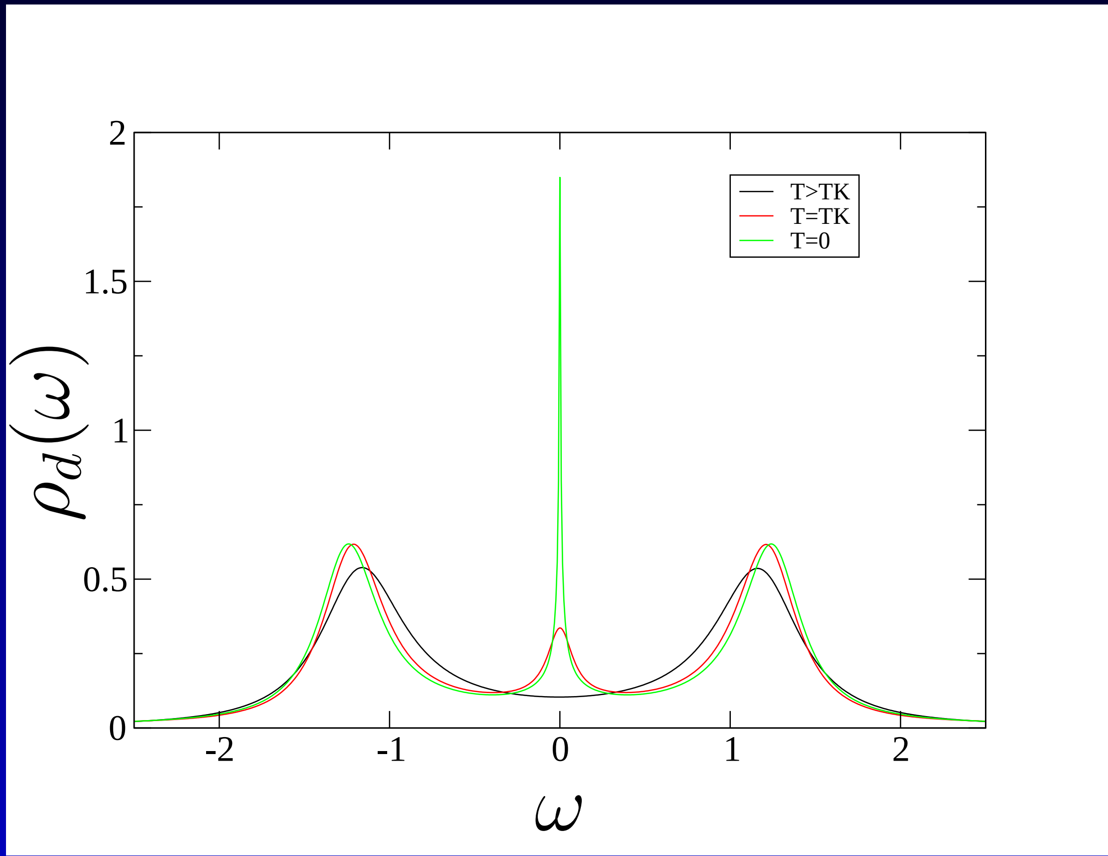
Similarities with U-NCA:

- Integral Equations
- Overscreening aspect (NFL)

Differences with U-NCA:

- Pinning for all U (not exact value)
- Causality
- Limit U=0 exact
- Complexity of the sigma-model

Kondo effect



Generation of an exponentially small Kondo scale

High-energy features

DMFT for the Hubbard model

Effective Anderson model with self-consistency:

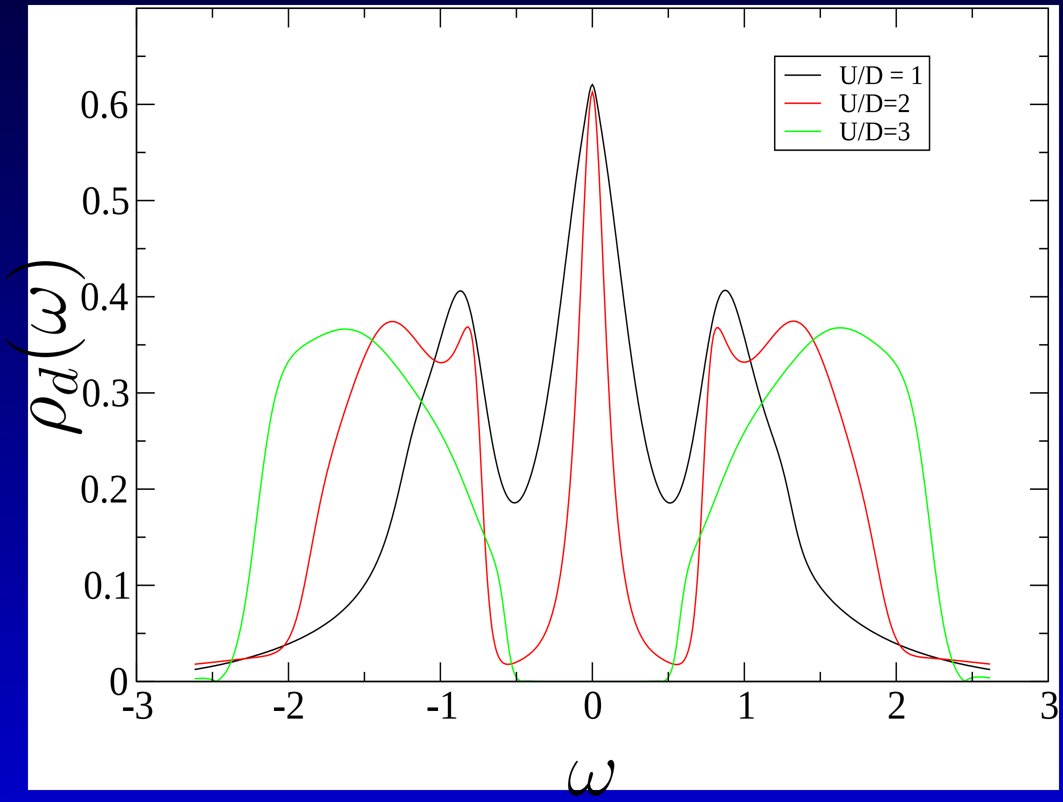
$$G_d(i\omega_n) = \sum_{\vec{k}} \frac{1}{\Delta(i\omega_n) + G_d(i\omega_n)^{-1} - \varepsilon_{\vec{k}}}$$

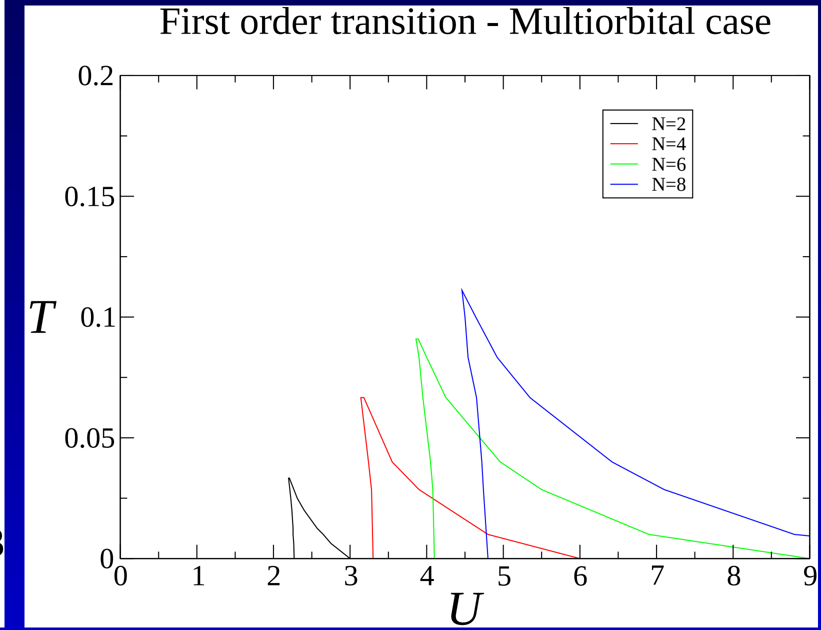
Locality of the self-energy

Pinning$\Rightarrow$ Brinkman-Rice picture of the MIT:

$$m/m^* \sim Z \to 0$$

Mott transition in DMFT





Beyond DMFT

Hubbard model revisited:

$$S = \int_0^\beta d\tau \sum_{\mathrm{i}\sigma} f^\dagger_{\mathrm{i}\sigma}(\partial_\tau + \epsilon_0 - h_\mathrm{i}) f_{\mathrm{i}\sigma} + \sum_\mathrm{i} \frac{(\partial_\tau \theta_\mathrm{i} + i h_\mathrm{i})^2}{2U}$$
$$- \sum_{\mathrm{ij}\sigma} t_{\mathrm{ij}} f^\dagger_{\mathrm{i}\sigma} f_{\mathrm{j}\sigma} e^{i\theta_\mathrm{i} - i\theta_\mathrm{j}}$$

Bond mean-field: $\overline{Q} =< f^\dagger_{\mathrm{i}\sigma} f_{\mathrm{j}\sigma} >$ and $Q =< X_\mathrm{i} X^*_\mathrm{j} >$

analogous to $Q(\tau - \tau') =< X(\tau) X^*(\tau') >$

$$G_f(i\omega_n,\vec{k})^{-1} = i\omega_n - \epsilon_0 + h - Q\varepsilon_{\vec{k}}$$

$$G_X(i\nu_n,\vec{k})^{-1} = \frac{\nu_n^2}{U} + \lambda - \overline{Q}\varepsilon_{\vec{k}}$$

$$G_X(\tau = \vec{x} = 0) = 1$$

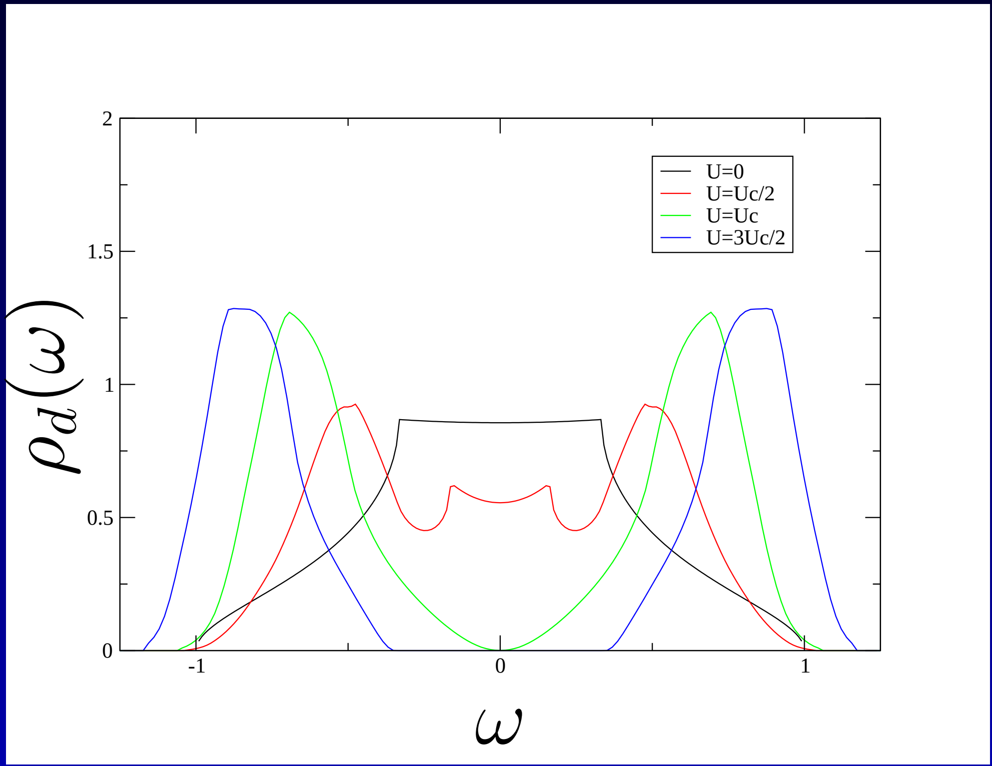
Metallic phase:

$$Z = < X >^2$$

$$\frac{m}{m^*} = Q \sim Z + \frac{t}{U}$$

$$G_d(i\omega_n,\vec{k}) = G_f \star G_X = \frac{Z}{i\omega_n - \epsilon_0 + h - Q\varepsilon_{\vec{k}}} + G_d^{\text{incoherent}}$$

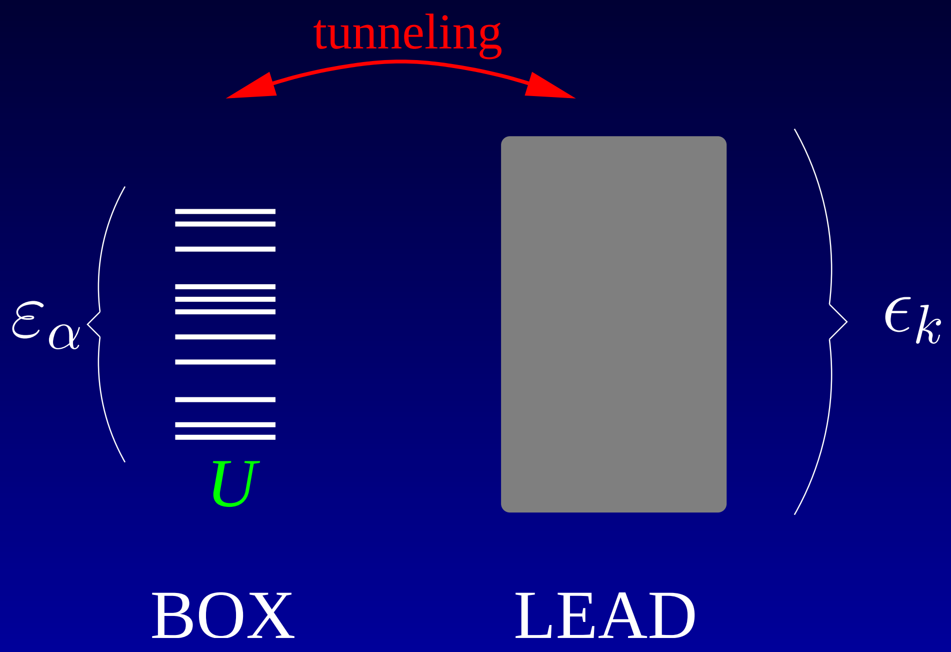
Mott transition



Collapse of q.p. peak (vs. pinning)

Collective modes (vs. preformed gap)

Tunnel junction



$$H = \sum_{\sigma\alpha} \varepsilon_\alpha f^\dagger_{\sigma\alpha} f_{\sigma\alpha} + \frac{U}{2}\hat{L}^2 + \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$
$$+ \sum_{k\sigma\alpha} \frac{V}{\sqrt{N_B}} \left(c^\dagger_{k\sigma} f_{\sigma\alpha} e^{-i\theta} + f^\dagger_{\sigma\alpha} e^{i\theta} c_{k\sigma} \right)$$

Model

$$S = \int_0^\beta d\tau \sum_{\sigma\alpha} f^\dagger_{\sigma\alpha}(\partial_\tau + \varepsilon_\alpha) f_{\sigma\alpha} + \frac{(\partial_\tau\theta)^2}{2U}$$
$$+ \int_0^\beta d\tau \int_0^\beta d\tau' \frac{\Delta(\tau-\tau')}{N_B} \sum_{\sigma\alpha\alpha'} f^\dagger_{\sigma\alpha}(\tau) f_{\sigma\alpha'}(\tau')\, e^{i\theta(\tau)-i\theta(\tau')}$$

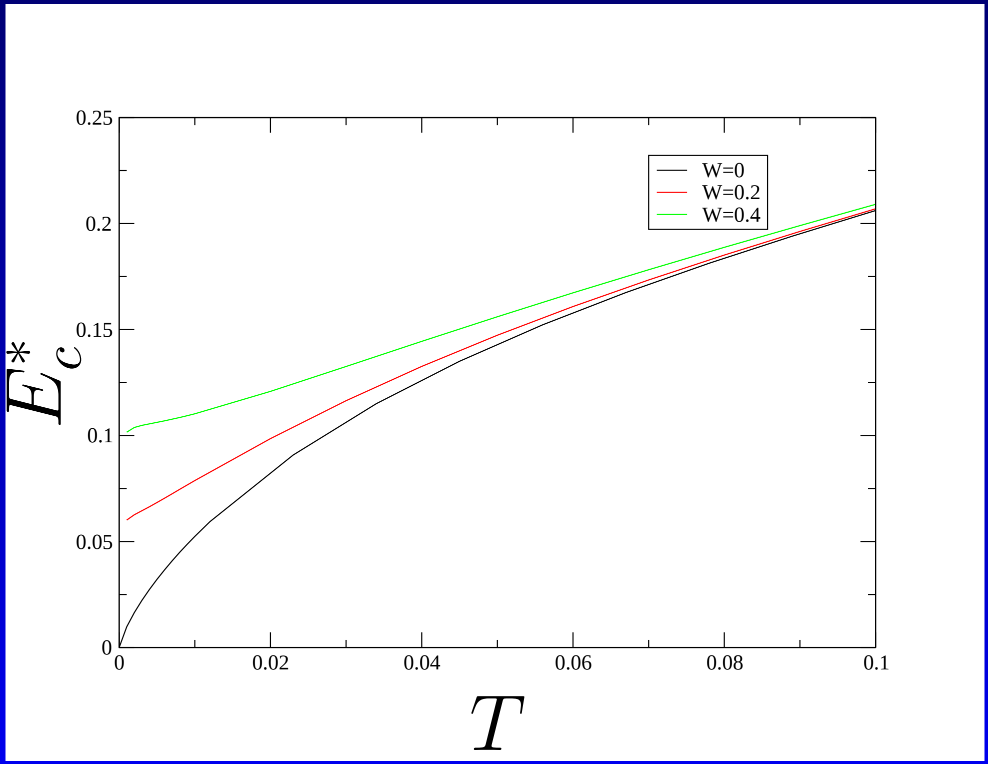
Usually, phase-only treatment, valid for:

- weak tunneling or many channels
- incoherent regime

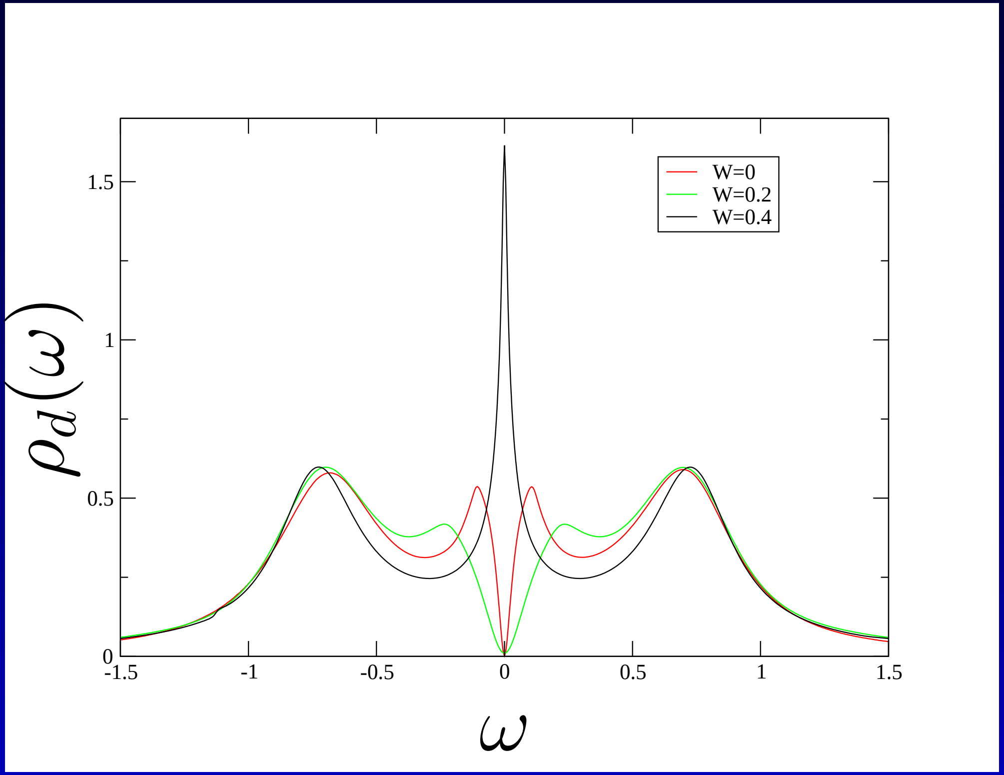
Cf. review Schön and Zaikin

$$G_X^{-1} = \left(-\frac{\partial_\tau^2}{U} + \lambda\right)\delta_{\tau\tau'} + \frac{\Delta_{\tau\tau'}}{N_B}\sum_{\sigma\alpha\alpha'}\langle f_{\sigma\alpha}^\dagger(\tau) f_{\sigma\alpha'}(\tau')\rangle$$

$$\left(G_f^{-1}\right)_{\alpha\alpha'} = (\partial_\tau + \varepsilon_\alpha)\delta_{\tau\tau'}\delta_{\alpha\alpha'} + \frac{\Delta_{\tau\tau'}}{N_B}\langle X(\tau)X^*(\tau')\rangle$$



Coulomb blockade



W = bandwidth of the metallic grain

Outlook

Description of Coulomb blockade and Kondo effect on an equal footing

Useful for coherent tunneling [failure of phase-only approaches]:

- Discrete spectrum
- Metal-insulator transition in extended granular system

Work in progress (Florens, Georges, Guinea, San Jose)

Conclusion

- New representation to deal with the electron-electron interactions
- General method to attack the strong correlation problem
- Non local interactions
- Application to mesoscopic devices and granular systems

Unrelated work

- Transport calculations for 2d organics
- Theory for spectroscopy at surfaces in $TaSe_2$
- Coulomb interaction in metals (Extended DMFT)