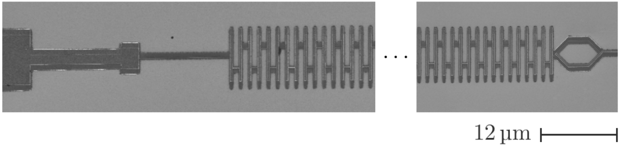
Quantum (analog) simulation of the boundary sine-Gordon model in superconducting circuits

Serge Florens [Néel Institute - Grenoble]



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Léger *et al.*, *"Revealing the finite-frequency response of a bosonic quantum impurity"*, SciPost 2023

Fraudet *et al.*, *"Direct detection of down-converted photons spontaneously produced at a quantum impurity"*, 2024

Simulating large-α QED in superconducting circuits

[Léger *et al.*, SciPost 2023]
[Kuzmin *et al.*, PRL 2021 and arxiv 2023]
[Murani *et al.*, PRX 2020]
[Léger *et al.*, Nat. Comm. 2019]
[Kuzmin *et al.*, npj Quantum 2019]
[Puertas-Martinez *et al.*, npj Quantum 2019]
[Magazzu *et al.*, Nat. Comm. 2018]
[Forn-Diaz *et al.*, Nat. Phys. 2017]
[Snyman & Florens, PRB 2015]
[Peropadre, Zueco, Porras, & García-Ripoll, NJP 2013]
[Goldstein, Devoret, Houzet & Glazman, PRL 2013]
[LeHur, PRB 2012]

Fiat Lux!

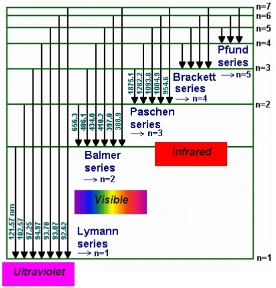
Another fine Feynman quote:

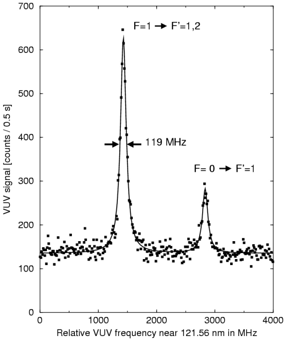
"God's hand wrote α, and we don't know how He pushed his pencil"

$$\alpha_{\rm QED} = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137} \quad \text{(small number)}$$

What if $\alpha_{\rm QED}$ were much larger?

- Spectroscopists would hate it:





[Eikema, Walz & Hänsch, PRL 2001]

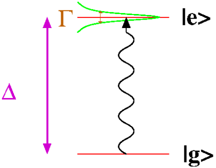
- Natural linewidth Γ for 3D atomic decay way smaller than the transition frequency Δ:

$$\frac{\Gamma}{\Delta} \simeq [\alpha_{\rm QED}]^3 \simeq 10^{-7}$$

Large α is interesting for non-linear optics!

Narrow linewidth of an atomic transition in vacuum:

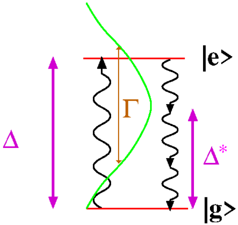
$$\frac{\Gamma}{\Delta} = \left(\frac{P}{e\lambda}\right)^2 \alpha_{\rm QED}$$



- P = transition electric dipole
- λ = wavelength of resonant photon

Ultra-strong coupling of QED:

$$\frac{\Gamma}{\Delta} \simeq 1$$



- Huge Lamb shift Δ^* from bare Δ
  [Leggett *et al.*, RMP 1987]
- Large linewidth $\Rightarrow$ strong photon down-conversion
  [Goldstein, Devoret, Houzet & Glazman, PRL 2013]

Large α is interesting for condensed matter!

Atom in a large α environment: maps to various classic models

- Spin-boson model ⇔ Kondo model

$$H = \frac{\Delta}{2}\sigma_x + \sqrt{\alpha}\sigma_z \nabla\phi(0) + \int \mathrm{d}x\, (\nabla\phi(x))^2$$

- Boundary sine-Gordon (BSG) model ⇔ tunneling in Luttinger liquid

$$H = v \cos[\sqrt{\alpha}\phi(x=0)] + \int \mathrm{d}x\, (\nabla\phi(x))^2$$

Most of the fun happens for $\alpha \simeq 1$:

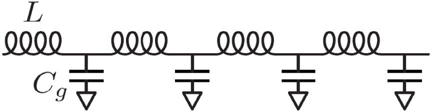
- Kondo-like physics
- Quantum phase transitions
- Algebraic correlations (Luttinger liquids)

High impedance medium is the way to go

Alternative expression: $\alpha_{\mathrm{QED}} = \frac{Z_0}{2R_K}$

- $Z_0 = \sqrt{\mu_0/\epsilon_0} \simeq 376\Omega$: vacuum impedance
- $R_K = h/e^2 \simeq 25812\Omega$: resistance quantum

Telegraph equation for LC waveguide:



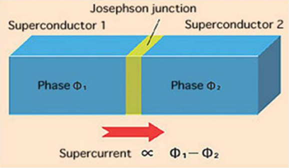
$$\frac{\partial V}{\partial x} = -\ell \frac{\partial I}{\partial t}, \quad \frac{\partial I}{\partial x} = -c_g \frac{\partial V}{\partial t}$$

$$\Rightarrow \quad I(x,t) = I_+ e^{i\omega[t-\sqrt{\ell c_g}x-t]} + I_- e^{i\omega[t+\sqrt{\ell c_g}x]}$$

$$\Rightarrow \quad V(x,t) = \sqrt{\ell/c_g}[I_+(x,t) - I_-(x,t)]$$

Impedance $Z = \sqrt{\ell/c_g}$ and velocity $v = 1/\sqrt{\ell c_g}$

Josephson junction as a high inductance element



Josephson relations: Φ is phase difference across a junction

$$I = I_c \sin\Phi \simeq I_c \Phi \quad \text{(linear regime)}$$

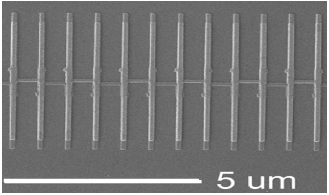
$$V = \frac{\hbar}{2e} d\Phi/dt$$

$$\Rightarrow V = \frac{\hbar}{2eI_c} dI/dt = L_J dI/dt$$

- Putting numbers: $L_J \simeq 1\ \text{nH}/\mu\text{m} \simeq 10^4 L_{\text{geometric}}$
- Effective coupling constant: $\alpha_{\text{chain}} = \frac{(2e)^2}{e^2}\frac{Z_{\text{chain}}}{2R_K} \simeq 1$
- "Light" with slow velocity: $v = 1/\sqrt{\ell c_g} \simeq c/100$

Josephson arrays

Let's consider a chain of tunnel-coupled superconducting islands:

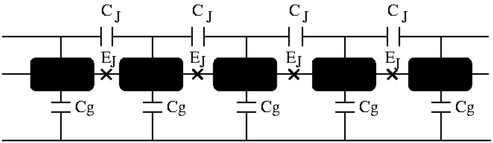


Generic Hamiltonian: valid for $T \ll T_c \simeq$1K

$$H = \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left(C^{-1}\right)_{ij} n_j - E_J \cos(\Phi_i - \Phi_{i+1})$$

$n - \Phi$ are conjugate variables: quantum fluctations are controlled by the ratio of $E_C \sim (2e)^2/C$ and $E_J = \hbar^2/[(2e)^2 L_J]$

Waveguide engineering



Harmonic regime:

- For $E_J \gg (2e)^2/(2C_J + C_g)$, weak phase fluctuations:

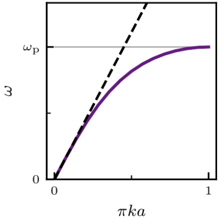
$$H_{\text{chain}} \simeq \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left(C^{-1}\right)_{ij} n_j + \sum_i \frac{E_J}{2}(\Phi_i - \Phi_{i+1})^2 = \sum_k \omega_k a_k^\dagger a_k$$

Spectrum:

$$\omega_k = 2\sin(\tfrac{k}{2})\sqrt{\frac{(2e)^2 E_J}{C_g + 4C_J \sin^2(k/2)}}$$

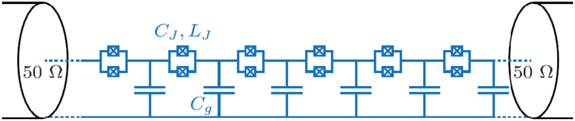
$k = \frac{\pi n}{N}$ with $n = 1 \ldots N$

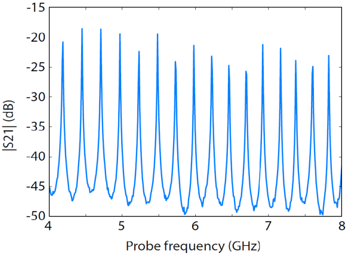
N = number of junctions

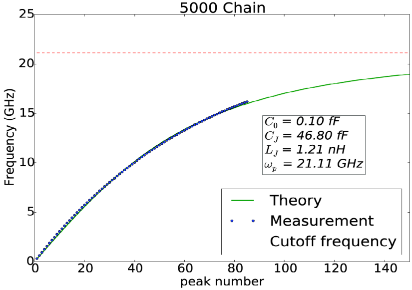


Seeing the modes

Finite chain coupled to 50 Ω lines: "giant Fabry-Perot cavity"







$$Z_{\text{chain}} = \sqrt{L_J/C_g} \simeq 3k\Omega \Rightarrow \alpha_{\text{chain}} = 2Z_{\text{chain}}/R_K \simeq 0.4$$

Speed of light: $v \simeq c/60$

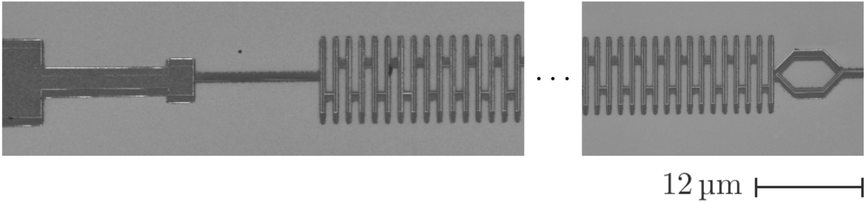
Simulators of bosonic quantum impurity models

[Léger *et al.*, SciPost 2023]

[Kuzmin et al. PRL 2021]

Adding a boundary condition

Device: chain of 4250 identical large junctions coupled to a SQUID



Boundary sine-Gordon model:

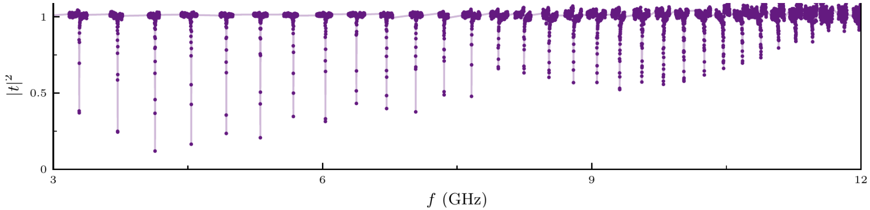
$$H = \sum_k \omega_k a_k^\dagger a_k - E_J(\Phi_B)\cos[\phi(x=0)]$$

with $\phi(x=0) = \sum_k g_k(a_k^\dagger + a_k)$

- Important QFT (Bethe ansatz solution...)
- Flux-tunable non-linearity: $E_J(\Phi_B) = E_J|\cos(\Phi_B)|$

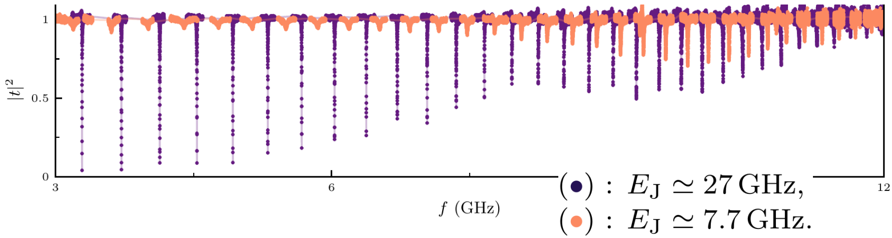
Transmission of the device

Measurement at zero flux: $E_J(\Phi_B = 0)$ is large $\rightarrow$ linear regime



- Eigenmodes are clearly resolved as sharp anti-resonances
- Very high quality factor
- Level spacing decreases at high frequency: UV cutoff ω_P

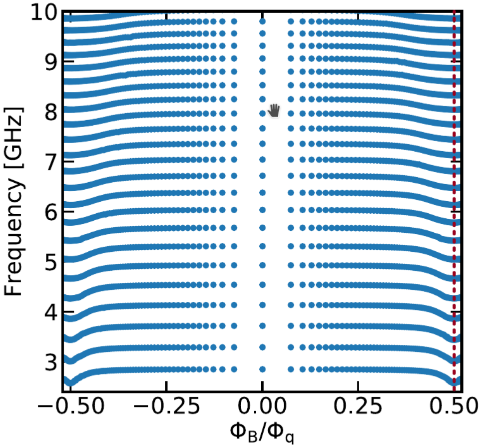
Impact of the boundary on the chain spectrum

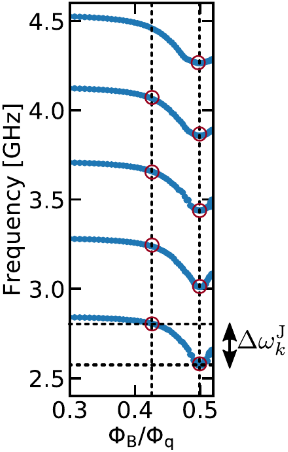


Two clear effects by decreasing E_J:

- Peaks shifts $\rightarrow$ Re$[\Sigma(\omega)]$ = dispersive response
- Peaks broaden $\rightarrow$ Im$[\Sigma(\omega)]$ = dissipative response
- How do we extract $\Sigma(\omega)$ from the data?

Dispersive response: phase shift spectroscopy



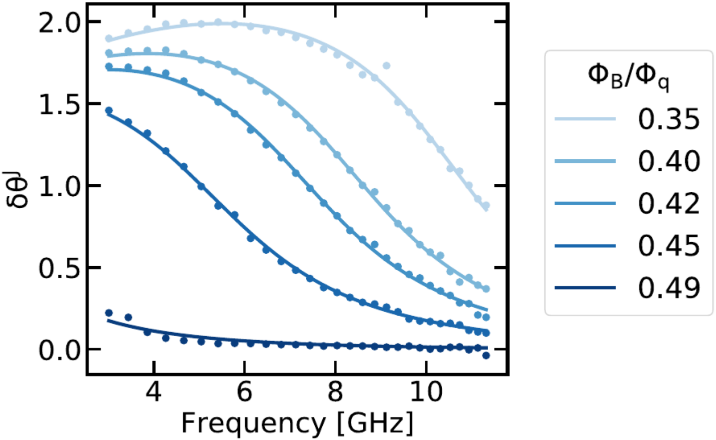


Phase shift: $\delta\theta_k = \pi \frac{\Delta\omega_k}{\omega_{k+1} - \omega_k}$

[DeWitt, Phys. Rev. (1956); Puertas et al., npj Quantum Inf. (2019)]

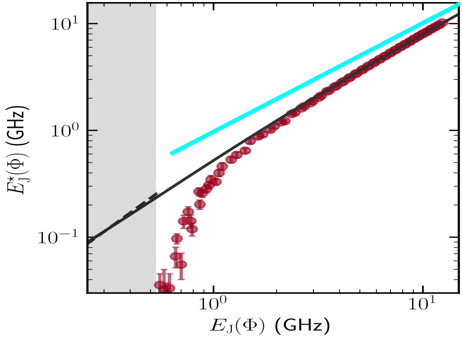
Phase shift crossover curves

Fit the data from a linearized model:



$\Longrightarrow$ Extract from fit the effective Josephson energy $E_J^*(\Phi_B)$

Renormalized Josephson energy E_J^*



Red dots = measured E_J^*
Blue line = bare E_J
Black line = SCHA E_J^*

SCHA: microscopic self-consistent harmonic approximation
$E_J^*(\Phi_B) = E_J(\Phi_B)e^{-\langle\hat{\varphi}_0^2\rangle/2}$ **[Schön & Zaikin, Phys. Rep. (1990)]**

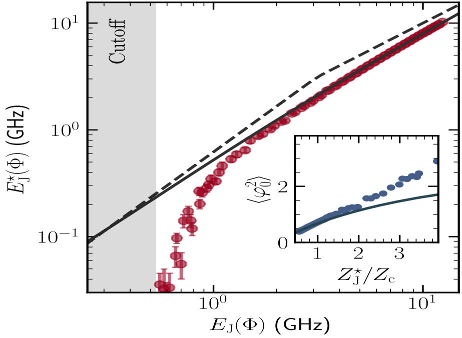
Fit of unknown parameters: $E_J(0) = 27\text{GHz}$
$E_J(\Phi_q/2)/E_J(0) = 3\%$ (SQUID asym.)

Note: Ambegaokar-Baratoff gives $E_J(0) = 26\text{GHz}$, OK!

Can we see scaling law of E_J^*? (almost)

Expected exponent: $E_J^{*\mathrm{scaling}} \propto E_J^{1/(1-\alpha)}$

[Panyukov & Zaikin Physica B 1988, Hekking & Glazman PRB 1997]



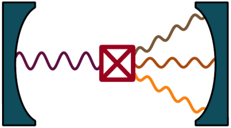
Dots = E_J^* measurement

Dashed = Min$(E_J, E_J^{*\mathrm{scaling}})$

Black line = E_J^* from SCHA

IR cutoff: thermal effects spoil the scaling regime...

Dissipation from photon down-conversion

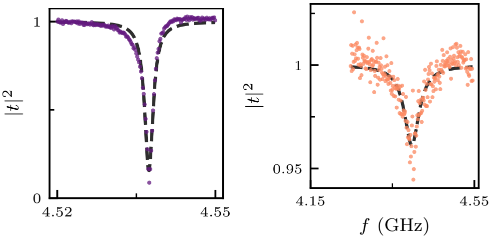


[Léger *et al.*, SciPost 2023]

[Kuzmin et al. PRL 2021]

Dissipative response: quality factor spectroscopy

Analysis of two given resonances at small and large Φ_B:



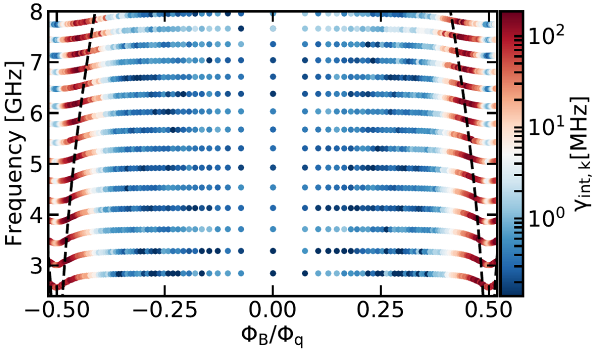
Extract external and internal quality factors Q_e and Q_i:

- Peak total width $= \gamma_k = \left[\frac{1}{Q_e} + \frac{1}{Q_i}\right]\omega_k = \gamma_k^{\text{external}} + \gamma_k^{\text{internal}}$
- Peak depth $= \frac{1}{1+Q_e/Q_i}$

Finite $Q_i \Rightarrow$ photons are lost somewhere inside the circuit

Internal losses for all modes

Peak linewidth due to internal losses: $\gamma_k^{\text{internal}} = \omega_k / Q_i(\omega_k)$



Low flux regime: $\Phi_B = 0$

$-E_J(\Phi_B)\cos(\hat{\varphi}_0) \simeq E_J(0)\hat{\varphi}_0^2/2 \rightarrow$ low-loss linear regime

High flux regime: $\Phi_B = \pi/2$

$E_J(\pi/2) \neq 0$ due to SQUID asymmetry $\Rightarrow$ losses persist!

Diagramatic approach for BSG

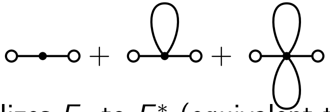
Hamiltonian:

$$H = \int \mathrm{d}k\, \omega(k) a_k^\dagger a_k - E_J \cos(\phi_0)$$

$$\phi_0 = \int \mathrm{d}k\, g(k) \left(a_k^\dagger + a_k \right)$$

$\cos(\phi_0)$ contains $a_{k1}^\dagger a_{k2}^\dagger a_{k3} a_{k4} + \ldots \Rightarrow$ frequency conversion

Expansion: Φ_B close to $\pi/2 \Longrightarrow E_J$ small

$\Sigma(t) =$  $+ \ldots = E_\mathrm{J} \delta(t) e^{-\frac{1}{2} G_F(0)}$

$\rightarrow$ renormalizes E_J to E_J^* (equivalent to "SCHA")

$\Sigma(t) =$  $+ \ldots = E_\mathrm{J}^2 \left[\sin(G(t)) - G(t) \right]$

$\rightarrow$ provides dissipative response $\mathrm{Im}[\Sigma(\omega)]$

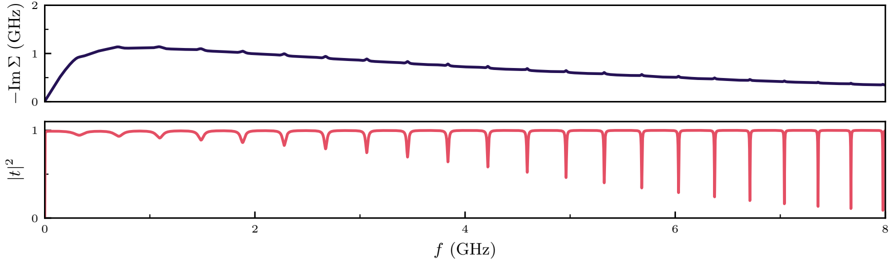
Computed self-energy

Resum bold (skeleton) diagrams:

$\Sigma(t) =$  $+ \ldots$

Small $\omega_k \simeq v.k \Rightarrow$ many near degeneracies in many-body spectrum

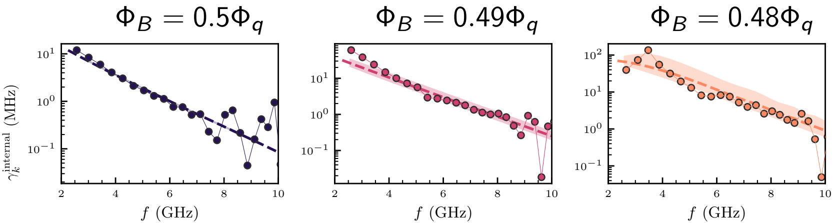
$\Rightarrow$ self-consistency provides level repulsion



- Smooth $\mathrm{Im}[\Sigma(\omega)] \Rightarrow$ true dissipation
- Leads to Lorentzian transmission peaks as seen experimentally

Many-body losses: theory vs experiment

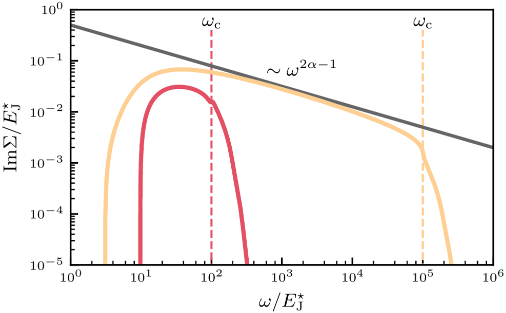
Predicted internal linewidth at high flux:



- Fit for $\Phi_B = 0.5\Phi_q$ and $0.49\Phi_q$:
  $\Rightarrow E_J(0) = 25\text{GHz}$ and SQUID asym. 2.5%
  Agreement with theory at small flux
- Losses at $\Phi_B = 0.48\Phi_q$ well described (no fitting)
- Losses are a smooth function: many-body dissipation

Can one see scaling laws of losses? (no)

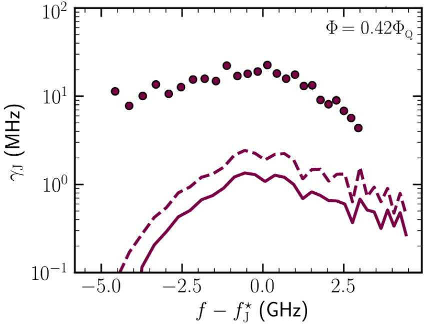
Known result: $\Sigma(\omega) \simeq \omega^{2\alpha-1}$ (Luttinger liquid analogy)



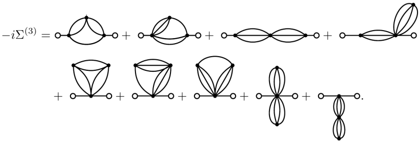
- Diagrammatics does reproduce the scaling laws
- Scaling is only found if $E_J^* \lll$ UV cutoff
  Limitation of Josephson platforms w.r.t. electronic circuits

Non perturbative regime of the experiment

Losses at intermediate flux: $\Phi_B = 0.42\Phi_q$



Compares experiment to $O(E_J^2)$ and $O(E_J^3)$ diagrams

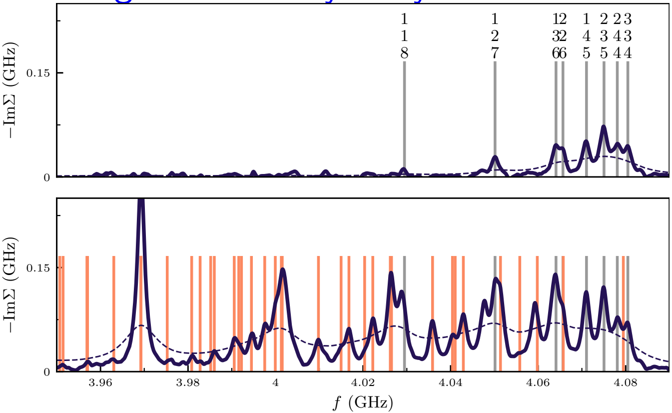


- Losses have a peak at $\omega = \omega_J^*$
- Diagrammatic theory underestimates the magnitude of losses $\Rightarrow$ requires a truly non perturbative approach

Direct detection of frequency down-converted photons

[Fraudet *et al.*, to appear]

Zooming on the many-body structure in $\mathrm{Im}\Sigma(\omega)$



Mode $= 10$
$Z_{\mathrm{tl}} = 10\Omega$
$T = 0$K
Emission

Mode $= 10$
$Z_{\mathrm{tl}} = 10\Omega$
$T = 20$mK
Emission+
Absorption

At $T = 0$K: 3-photons (or more) resonances at $\omega = \omega_{k_1} + \omega_{k_2} + \omega_{k_3}$

[Goldstein et al. PRL 2013; Gheeraert et al. PRA 2018; Houzet and Glazman PRL 2020; Burshtein et al., PRL 2021; Kuzmin et al. PRL 2021]

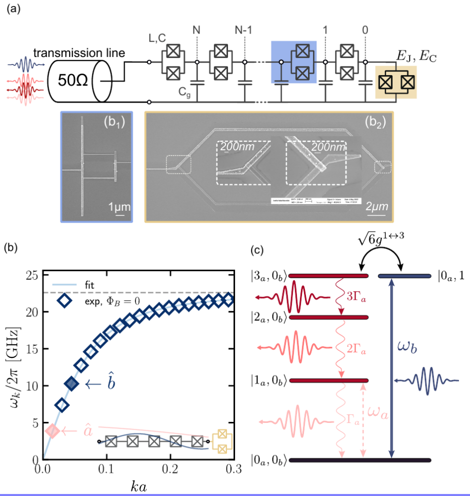
At finite T: also absorption resonances at $\omega = \omega_{k_1} \pm \omega_{k_2} \pm \omega_{k_3}$

[Léger et al. SciPost 2023]

BSG: the few-body version

New sample: only 200 junctions

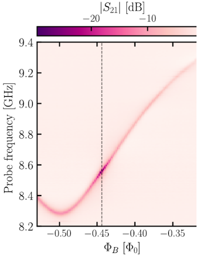
- circuit designed for down-converting mode 3 only into mode 1

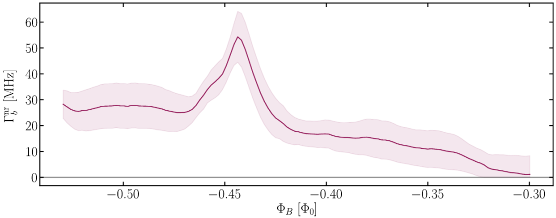


Losses: the few-body version

Resonance condition:

- the losses show a peak for the flux Φ_B where $\omega_3 \simeq 3\omega_1$

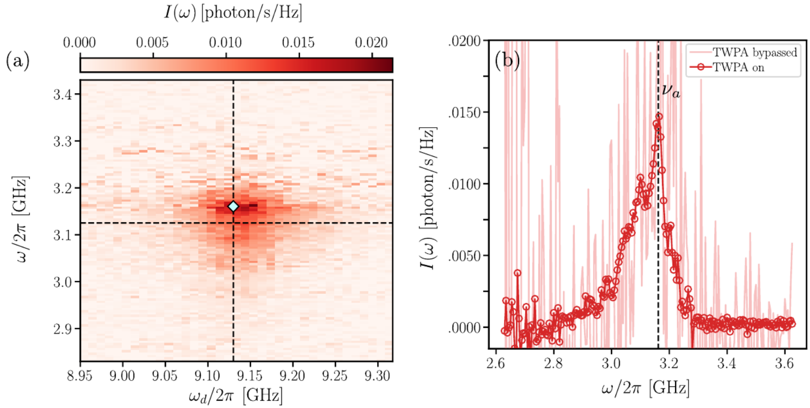




- this is no accident: the same behavior is seen by changing the resonance condition (flux in the chain)

Direct evidence for spontaneous down-conversion

- Pump: driven at $\omega_d \simeq$ 9GHz and low power
- Probe: measured at frequency $\omega_d \simeq 3GHz$



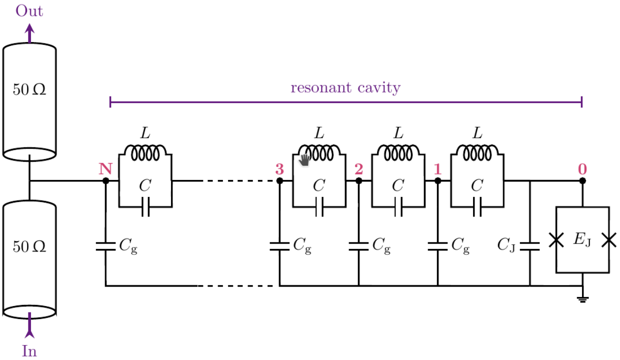
Summary

- Superconducting circuits are a nice platform to simulate many-body systems
- We analyzed the renormalization of the Boundary Sine-Gordon (BSG) model
- The dissipative response of BSG is controlled by “particle production” processes
- A fully microscopic diagrammatic theory was developed to model dissipative losses
- The direct detection of down-converted photons was achieved using a few-body version of BSG

Outlook: correlations between non-linearly converted photons?

Extra slides

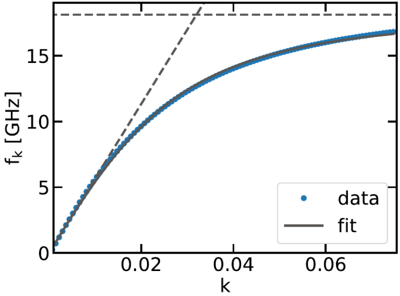
A circuit view of the full device



- Boundary = terminal junction (SQUID) with tunable $E_J(\Phi_B)$
- Chain of microwave resonators = resonant cavity
- AC measurement: $I_{\rm out} e^{i\omega_{\rm out} t}$ vs $I_{\rm in} e^{i\omega_{\rm in} t}$ (in GHz range)

[Léger et al., Nat. Comm. (2019); Kuzmin et al., PRL (2021)]

Properties of the array



Dispersion relation: $\omega_k = \sqrt{\frac{4}{LC_g} \frac{\sin^2(k/2)}{1+4(C/C_g)\sin^2(k/2)}}$

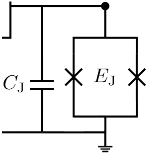
Fitted chain parameters: $L = 0.52\text{nH}$, $C_g = 0.15\text{fF}$, $C = 144\text{fF}$

Chain impedance: $Z_{\text{chain}} = \sqrt{L/C_g} = 1.9k\Omega \Rightarrow \boxed{\alpha = 0.3}$

Plasma frequency: $\omega_P = 18$ GHz (UV cutoff)

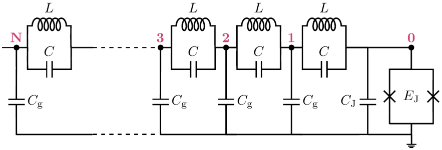
Boundary junction frequency?

Isolated junction:



$$\omega_J = \frac{1}{\sqrt{L_J C_J}} = \sqrt{2E_c E_J(\Phi_B)}$$

Boundary junction + chain:



$E_J(\Phi_B)$ changes the boundary condition
⇒ affects all eigenmodes via a phase shift

Dispersive response: phase shift spectroscopy

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Transition from Discrete to Continuous Spectra*

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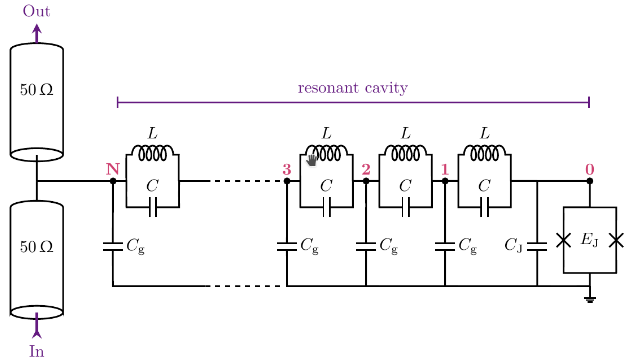
The stationary states of a system bound in a spherical box and additionally subjected to a perturbation of finite range are studied in the limit as the box radius becomes infinite. The transition from formal discrete-spectrum theory to formal scattering theory is carried out explicitly by two different methods. It is shown quite generally (i.e., even when the total Hamiltonian is not separable) that the level shift produced by the perturbation is proportional to the corresponding scattering phase shift.

I. INTRODUCTION

AN attempt has recently been made by Reifman and Newton, in collaboration with the author,[1] to justify a procedure of Brueckner[2] which attempts to deal with nuclear many-body bound-state problems in the language of scattering theory, by imagining that the nuclear radius is sufficiently large so that the stationary two-body states are quasi-continuous. In particular, the attempt was made to justify Brueckner's cases that the level shift produced on a quasi-continuous state by a perturbation of finite range becomes, in the limit as boundary walls recede to infinity, proportional simply to the corresponding phase shift, *not* to its tangent.

It is curious that this result, which seems to have been known more or less privately for some time by various individuals, has not previously achieved the dignity of a special statement in the literature.† In the

Circuitry in the non-linear case



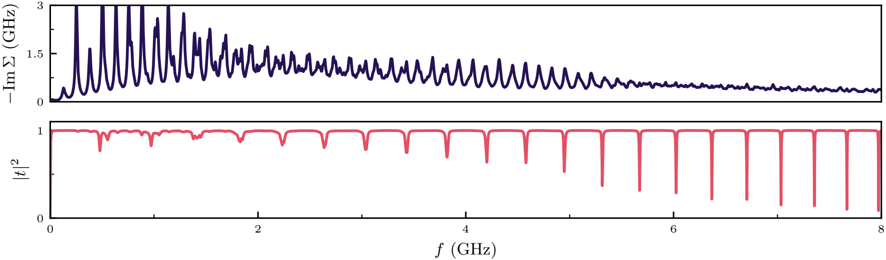
$$\text{Dyson equation}: \left[\omega^2 \hat{C} - 1/\hat{L} + \frac{i2\omega}{Z_{\rm tl}}\hat{\delta}^{(N)} - \Sigma(\omega)\hat{\delta}^{(0)}\right]\hat{G} = 1$$

$Z_{\rm tl} = 50\Omega$: external broadening from transmission line

Transmission from Kubo: $t(\omega) = 2i\omega G_{N,N}(\omega)/Z_{\rm tl}$

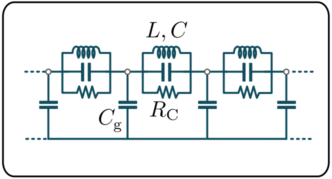
Predicted dissipative response and transmission

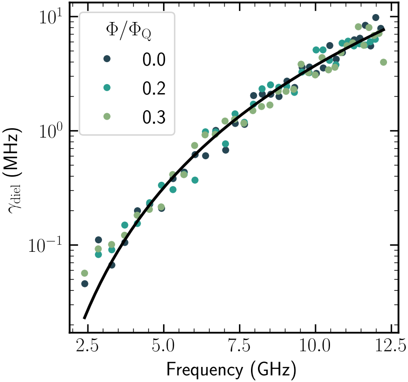
Full microscopic model:



- OK: qualitatively similar to the experimental internal width γ_k
- Not OK: incorrect multiplet structure (not seen experimentally)
  This is due to sharp resonances in the self-energy

Dielectric losses in the chain

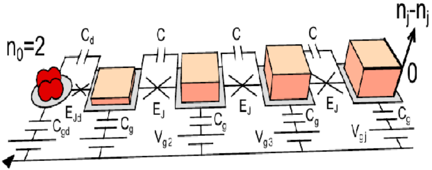


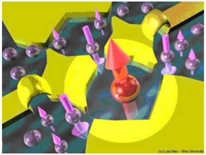


- Dielectric losses dominate at low flux
- Phenomenological fit [Nguyen *et al.* PRX 2019] is subtracted to obtain the intrinsic internal losses

Connection to other physical systems

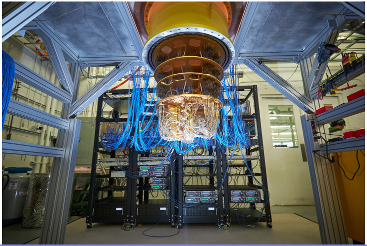
Josephson boundaries bear strong similarities to the Kondo effect:





[Leggett, RMP (1987); Le Hur, PRB (2012); Snyman&Florens, PRB (2015);...]

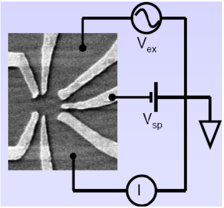
Quantum computing:



Requires full control to prepare complex quantum states

Connection with fermionic quantum impurities

Setup: quantum dot (spin) connected to metallic leads

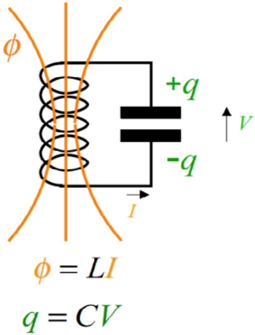


Kondo model:

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J\,\vec{S}.\vec{s}(x=0)$$

- Important brick for strongly correlated fermions
- Renormalized scale $T_K \simeq De^{-D/J} \ll J$ analogous to Δ^*
- Tunable exchange coupling J via gates

Connection with quantum circuits



Classical energy: harmonic LC oscillator

$$H = \frac{Q^2}{2C} + \frac{L}{2} I^2 = \frac{Q^2}{2C} + \frac{L}{2} (\dot{Q})^2$$

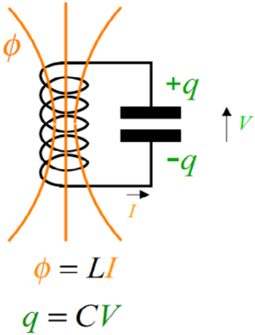
Conjugate classical variables: charge/flux

$$\frac{\partial H}{\partial \dot{Q}} = L\dot{Q} = LI = \phi$$

Quantizing the LC circuit: $\boxed{[\hat{Q}, \hat{\phi}] = i\hbar}$ What does it mean?

- Tiny electromagnetic signals generated by charge fluctuations of order $\delta Q \simeq 2e$ and flux fluctuations of order $\delta\phi \simeq h/2e$
- Vacuum reached when $k_B T \ll \hbar/\sqrt{LC}$ and low loss

Connection with quantum circuits



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