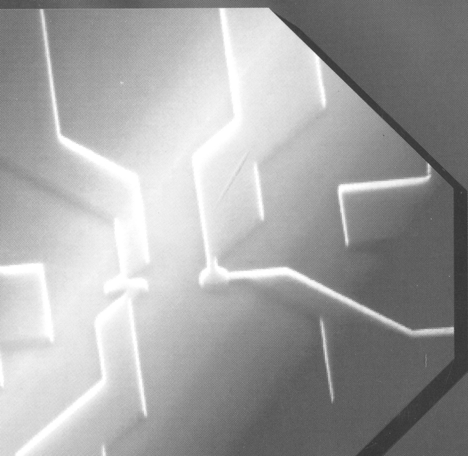
Quantum Devices *and* Circuits



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Electronics with a single Cooper pair

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1. Introduction

Unlike the tunneling of electrons through a junction which is an incoherent and irreversible process, the tunneling of Cooper pairs (i.e. the Josephson effect) is coherent and reversible. In single electron devices, electron tunneling is in competition with charging effects, resulting in the phenomenon of Coulomb blockade. In superconducting devices, such a competition will take place between Cooper pair tunneling which tends to make the number of Cooper pairs fluctuate quantum mechanically, and charging effects, which tend on the contrary to make the number of Cooper pair a good quantum number. In this paper, we present two experiments in which the interplay between Cooper pair tunneling and charging effects leads a quantum superconducting device to adopt a macroscopic quantum superposition of two charge states.

2. The superconducting box

The superconducting box consists of a small piece of superconductor, nicknamed "island", connected to a superconducting reservoir by an ultrasmall Josephson junction (see Fig. 1). The island can be electrostatically polarized by a gate capacitance C_g connected to a voltage source U[1]. This island is characterized by its number n of excess Cooper pairs whose average value is measured by an electrometer[2].

The metallic superconducting island and the Josephson junction are realized in our laboratory using electron beam lithography followed by depositions of Aluminum through a suspended mask. A controlled oxidation of the aluminum layer between two depositions allows us to fabricate the tunnel barrier. With this process, Josephson junctions Al/AlO$_x$/Al of typical size $0.1\times0.1\mu m^2$ are obtained. The total capacitance of the island $C_\Sigma = C_j + C_g$ determines the characteristic energy : $E_c = (2e)^2 / (2C_\Sigma)$ which corresponds to the cost of a Cooper pair entering the island. E_c is here dominated by the capacitance C_g which is of the order of $1n$F ($E_c/k_B \approx 1$K) .

The electrostatic Hamiltonian H_{el} associated with the quantum variable n can be modified by varying the voltage U on the gate capacitance C_g :

$$H_{el} = E_c\left(n - C_gU/(2e)\right)^2$$

The associated eigenvalues can be represented as a function of U by a set of parabolas indexed by the integer n. These parabolas cross at voltages $U = (2n+1)\,e/C_g$ (see Fig. 2a).

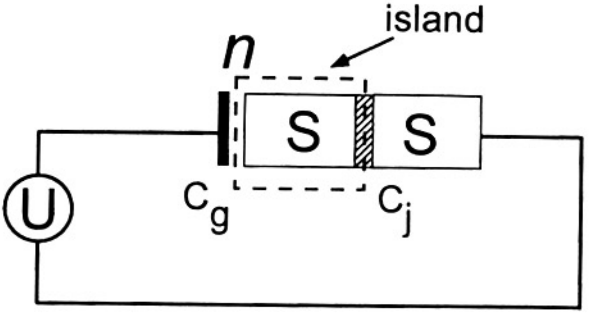

Figure 1: *Schematic diagram of the single Cooper pair box. The island in the dotted frame is characterized by its number n of excess Cooper pairs that have entered the island through the Josephson junction of capacitance C_j. The island can be polarized by the capacitance C_g connected to the voltage source U.*

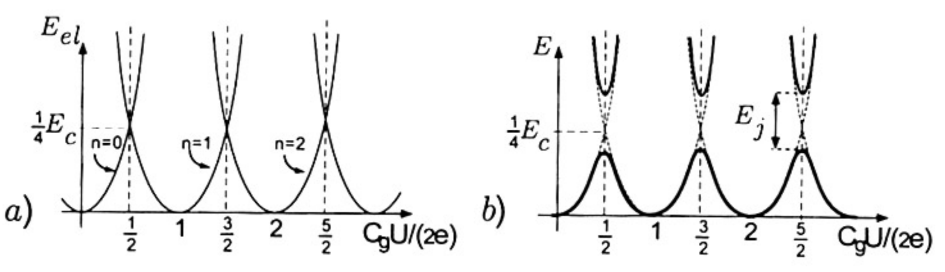

Figure 2: *a) Electrostatic energy of the box for given numbers n of Cooper pairs in excess in the island as a function of the reduced voltage C_gU/e. b) Energy levels of the total energy of the system when both electrostatic and Josephson Hamiltonians are taken in account. The gap between the ground level and the first excited level is equal to the Josephson energy E_J.*

We also have to consider Josephson tunneling, characterized by the energy E_J, which couples neighboring charges states and which gives the following contribution to the total Hamiltonian :

$$H_J = -\frac{E_J}{2}\sum_n \left(|n+1\rangle\langle n| + |n\rangle\langle n+1|\right)$$

With standard large capacitance Josephson junctions, H_J dominates over H_{el}, since E_c is small compared to E_J. The ground state of the system is therefore a coherent superposition of a large number of charge states :

$$|\Psi\rangle = \sum_{k=-\infty}^{+\infty} e^{-\frac{k^2}{2N^2}} |n_0 + k\rangle \quad \text{with } N = \frac{1}{\sqrt{2}}\left(\frac{E_J}{E_c}\right)^{\frac{1}{4}} \tag{1}$$

However, for small capacitance junctions, H_{el} can dominate over H_J. At temperatures such $E_J/k_B \ll T \ll E_c/k_B$, the island adopts the number n of extra Cooper pairs which minimizes E_{el}. If one sweeps the voltage U, Cooper pairs will enter the island one by one at the degeneracy voltages : $U = (2n+1)\ e/C_g$.

Our device lies in an intermediate case : E_J is smaller but not negligible compared to E_c and $k_B T$ (typically E_J=0.3E_C). As the Josephson coupling comes into play only when the electrostatic energy difference between two neighboring charge states is of the order of E_J, the ground level of the system follows the lowest electrostatic eigenvalue except in the vicinity of the transition between two charge states. There, a gap given by the Josephson energy E_J forces levels to anticross (see Fig. 2b). In this region, n is no longer a good quantum number and for $U = (2n+1)\ e/C_g$, the system is in a coherent superposition of two charge states : $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|n\rangle + |n+1\rangle\right)$. Theory predicts that the average value of n at the transition between two charge states takes the shape of a rounded step. At the middle of the transition, the slope is equal to E_c/E_J and directly reflects the competition between charging and Josephson effects.

In our experiment, we have measured the time-averaged number of Cooper pairs in the island by coupling it electrostatically to a SET electrometer. The variation of the average value of n as a function of the polarization C_gU/e has the shape of a rounded staircase. At temperatures above 100mK, the rounding is well explained by thermal fluctuations. Below 100 mK, the curve becomes temperature-independent and the shape of transition steps agrees with the theoretical prediction at T=0 based on the coherent superposition of two states (see Fig. 4):

$$\langle n\rangle = \frac{C_gU/e - 1}{\sqrt{\left(C_gU/e-1\right)^2 + \left(E_j/E_c\right)^2}} \tag{2}$$

This observation is radically different from the case of a superconducting island connected to a non-superconducting reservoir[3]. In such a case, the slope diverges at low temperature since without Josephson effect, thermal fluctuations remain the only smearing process.

The superconducting box experiment demonstrates that, unlike the Josephson effect in a standard junction which involves a large number of Cooper pairs, a single Cooper pair can be transferred through a Josephson junction at a given time.

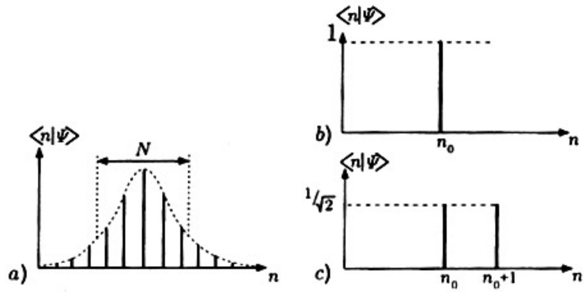

Figure 3: *Box ground state amplitude for respectively large capacitance junctions a) and for ultrasmall capacitance junctions b),c). The gaussian wave-packet of case a) is given by formula (1). Amplitudes of cases b) and c) are represented respectively for voltages* $U = 2n\ e/C_g$ *and* $U = (2n+1)e/C_g$.

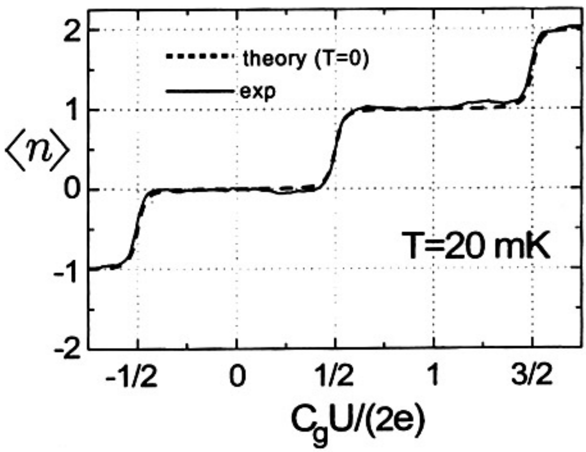

Figure 4: *Time averaged number of excess Cooper pairs in the island as a function of the reduced voltage* C_gU/e. *The experimental curve (solid line) is compared to the zero-temperature theoretical prediction (dotted line) obtained using formula (2).*

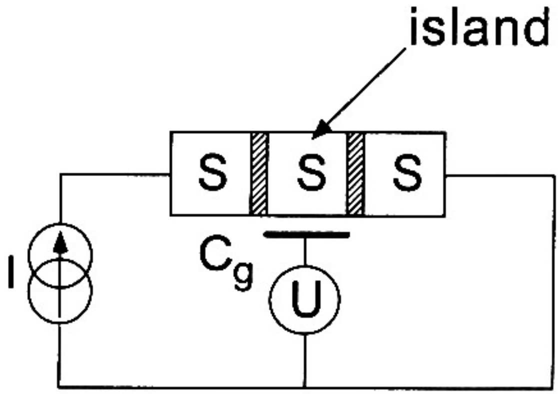

Figure 5: *Schematic diagram of the superconducting transistor. The island is connected to two superconducting electrodes by Josephson junctions. The transistor is biased by a current source. As for the superconducting box, the island can be polarized by a capacitance connected to a voltage source U.*

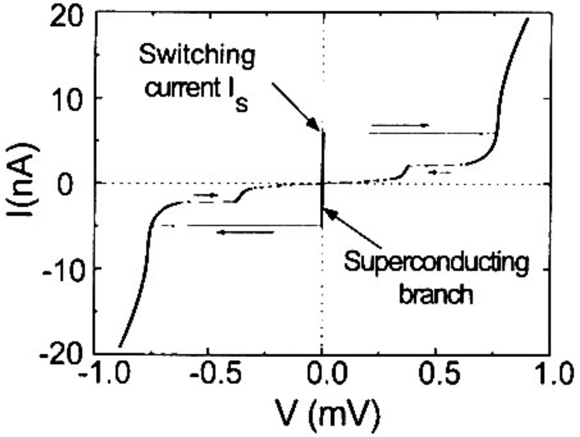

Figure 6: Experimental current-voltage characteristic of the superconducting transistor biased with a quasi-ideal current source. The zero voltage branch corresponds to the superconducting state of the transistor. For $I > I_s$, the transistor switches to the finite voltage branch.

3. The superconducting transistor : an electrostatically controlled quantum switch

In this second type of device, a superconducting island is connected through Josephson junctions to two superconducting electrodes and operated in a transport mode (Fig. 5)[4,5]. As in the superconducting box, the island is polarized by a gate capacitance C_g and we have thus a three terminals device. This superconducting transistor is biased by a current source.

The $I-V$ characteristic of the transistor is presented in Fig. 6. It shows a zero-voltage branch which corresponds to a supercurrent flow through the junctions. As the current increases, the system switches to a finite voltage branch for which conduction is dominated by the quasi-particule current. Switching occurs for a current I_s which is the main experimental feature of the device.

We have measured the variations of I_s as a function of the gate voltage (Fig. 7). The curve is $2e$-periodic with respect to the polarizing charge C_gU on the island. It has sharp maxima whose values reach the theoretical prediction $I_{\max} = (e/\hbar)\ E_J$ in a case of a well controlled electromagnetic environment connected to the device. These maxima occur for voltages $U = (2n+1)e/C_g$ corresponding to the middle of the steps in the superconducting box experiment.

The cusp shape of the peaks in the $I_s(U)$ curve provides a signature of the coherent superposition of two charge states for the island : if the island was in an incoherent state, the shape of the peaks would be parabolic[6].

On the other hand, the switching current has smooth minima when the polarizing charge on the gate is an integer number of Cooper pairs. There, the number of excess Cooper pair in the island can fluctuate quantum mechanically only through a second order effect in the Josephson coupling. This cannot efficiently counterbalance charging effects and lead to a reduced switching current given by :

$$I_{\min} = I_{\max}\ (E_J/E_c) = (e/\hbar)\ E_J^2/E_c$$

The supercurrent modulation by the gate voltage is reminiscent of the current modulation in a field effect transistor (FET). However, unlike the FET whose principle is based on statistical variations of a macroscopic number of carriers, the modulation arises here from quantum fluctuations involving a single Cooper pair.

4. Conclusion

We have presented here two experiments which implement at a solid state a two-level quantum system. The coherence time of a quantum state in such systems, limited by the dissipation of the charge in the electromagnetic environment, have to be measured if one wants to use them for further applications such as quantum computing .

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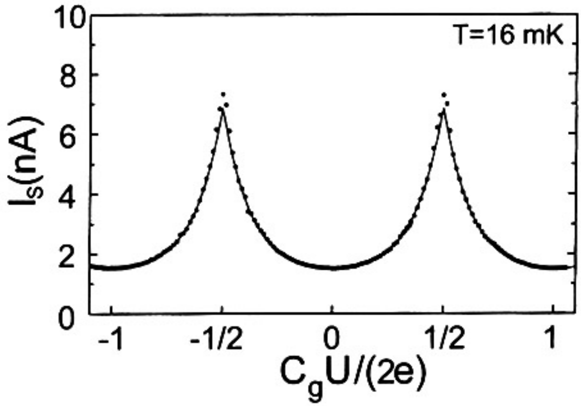

Figure 7: *Experimental switching current I_s of the superconducting transistor as a function of the reduced gate voltage C_gU/e at 16mK (dots). Sharp maxima are obtained for odd integer values of CgU/e. Theory is shown by a continuous line.*