Measurement method of the antiproton gravitational mass using the single electron transistor

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We propose a non-destructive method to measure the trajectory of a single antiproton in a drift tube using position sensors based on the single electron transistor. We show that this recently developed device has sufficient sensitivity to detect the electric field of a moving charged particle. Comparing the trajectories of antiprotons and H^- ions could allow a reliable determination of the gravitational mass of the antiproton.

1. Introduction

The measurement of the gravitational mass of an antiparticle has been the subject of interest and considerable experimental effort over the past thirty years [1–4], without success until now. Pioneering experiments on the electron by Witteborn and Fairbank [1,2] aimed at ultimately measuring the gravitational mass of the positron. However, the original result of Witteborn and Fairbank [1,2] has been partly retracted [5] and, for reasons which will be analyzed below, we are probably still very far from a measurement on the electron or the positron.

On the other hand, the Low Energy Antiproton Ring (LEAR) at CERN has been providing for the last ten years an intense beam of low-energy antiprotons. The PS200 collaboration [6] has thus proposed to measure the gravitational mass of the antiproton using, similarly to Witteborn and Fairbank, a time-of-flight technique in a vertical drift tube where the antiprotons would be confined along the vertical axis using a strong magnetic field. The technical difficulties of such an experiment are enormous. Because of the large annihilation cross-section of an ultralow-energy antiproton, a vacuum of extremely high quality is required for the antiparticles to survive the cooling and measurement stages. But the main experimental problem probably resides in the stray electric fields [7] which easily overwhelm the tiny gravitational force on the antiproton. Whereas a differential measurement is in principle possible using a comparison with the time-of-flight of H^- ions, two conditions should be achieved for this comparison to be meaningful: firstly, the conditions of release of antiprotons and H^- ions in the drift tube should be kept identical to a high degree of accuracy and, secondly, the stray electric fields should be kept at a value constant in time and sufficiently small compared to the gravitational force.

We propose here to replace the time-of-flight measurement by the direct monitoring of a single trajectory. More precisely, we propose to use ultra-sensitive electrometers distributed along the drift tube to determine the passage times of a single particle trapped in the tube. Such non-destructive and repeated measurements would considerably relax the constraints mentioned above. Electrometers approaching the requirements for single particle monitoring have recently been developed. They are based on the Single Electron Transistor (SET) first operated by Fulton and Dolan in 1987 [8].

2. Discussion of existing and proposed experiments

2.1. Existing experiments

The measurement of the gravitational mass of individual particles has been achieved only on neutral particles, namely photons [9,10], neutrons [11] and atoms [12]. Although several techniques have been proposed [1,4,13,14], no measurement has been successfully carried out on charged particles. Only indirect determinations have been deduced from measurements on bulk matter [15]. As mentioned previously, the situation is even more dramatic for antiparticles since cooling them at ultra-low energies is made difficult by their large annihilation cross-sections. State-of-the-art developments on the antiproton include the Gabrielse trap [16] and the Holzscheiter trap [4]. These traps have been shown to capture and hold up to 10^6 particles for several months under optimum vacuum conditions using cryo-pumped cavities [17]. In the Gabrielse trap [16], the slowing down of individual antiprotons down to velocities of a few hundreds m/s, typical of the velocities we will consider in the following, has been achieved. On the other hand, the Holzscheiter group [18] has demonstrated the feasibility of extracting antiprotons from the trap and transferring them to other experiments.

2.2. Residual electric field problems

It should be remembered that the action of gravity on a proton can be counterbalanced by an electric field $\mathcal{E} = m_\mathrm{p} g/e \approx 10^{-7}$ V/m. For an electron, this field is of course approximately 2000 times smaller and of the order of 5×10^{-11} V/m. This gravitational force is so weak that the attraction force of an electron to the Earth is equivalent to the electrostatic force applied by a single electron 5 m away. For this reason, the measurement of the gravitational mass of the antiproton appears more realistic than that of the positron. Even in this case, however, all nongravitational forces must be suppressed with a high degree of accuracy.

The experimental problems raised by the gravitational mass measurement of individual charged particles have been reviewed by Darling et al. [20]. The main experimental difficulty appears to be due to the patch effect [7]. To reduce the ambient electrostatic forces, a metallic drift tube is used in all existing or proposed experiments. However, although textbooks state that the electric field is zero inside a conducting cavity, the non-uniformity of the dipole density at the surface of crystal domains induces a residual field. This effect, called patch effect, is due to variations of the work function from one domain to another by a fraction of eV. For some metallic surfaces with amorphous coatings, the variations of the surface voltage Φ_Σ integrated over the area of a Kelvin probe appear to be reduced close to its sensitivity [7] which lies in the mV range. The size of these patch domains is typically $l \approx 1$ μm. The residual potential fluctuation along the axis of a cylinder with radius ρ has a root mean square value $\Phi_{\rm rms} \approx 0.6\ \Phi_\Sigma l/\rho$. This would correspond to potential variations less than 10^{-5} V in a tube with $\rho = 0.1$ m. In order for the patch effect to be negligible with respect to the gravity force, a reduction of such variations by two orders of magnitude seems then required. Therefore, these stray electric fields which can very easily overwhelm the effect of gravity constitute the most stringent constraint on any measurement of gravity on a charged particle.

In these conditions, the measurement of the gravitational mass of a charged particle appears extremely problematic since existing and proposed experiments can only give access to a single quantity, i.e., time-of-flight or position of the particle, after which the particle is either irretrievably perturbed or even annihilated. Repeated weakly perturbative measurements thus appear necessary to disentangle the gravity and electric field contributions. We now discuss whether position sensors based on SETs could meet these two requirements.

3. The single electron transistor

3.1. Description

Various single electron devices in which the current results from subsequent transfers of single electrons have been operated during the last ten years [24]. The SET is the basic active device of single electronics. It consists of two ultrasmall tunnel junctions in series, with a small intermediate electrode (island) capacitively coupled to a gate electrode (see fig. 1). This device is characterized by the tunnel resistance $R_{\rm t}$ of each junction and by the total capacitance of the island $C = C_{\rm g} + 2C_{\rm j}$, where $C_{\rm g}$ and $C_{\rm j}$ act respectively for the gate and junction capacitances. In a voltage biased SET, the current is periodically modulated by the gate voltage $V_{\rm g}$, the period corresponding to one electron charge induced in the island.

Two conditions are required for operating a SET:

1) The tunnel resistance $R_{\rm t}$ of each junction should be of the order of or larger than the resistance quantum $R_{\rm K} = h/e^2 \approx 25.6$ kΩ.

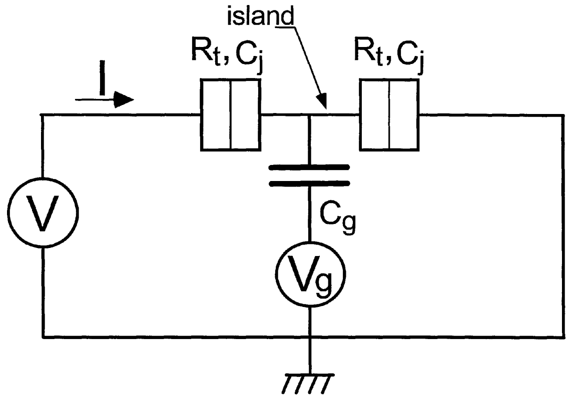

Fig. 1. Schematic diagram of a voltage biased single electron transistor. The intermediate electrode between the two tunnel junctions is an "island" whose charge is quantized in units of e. R_t and C_j are respectively the tunnel resistance and the capacitance of each junction. The gate voltage induces a polarization charge on the island which modulates the current.

2) The island capacitance C must be small enough and the temperature must be low enough so that the energy $E_c = e^2/2C$ required to add a single electron charge to the island exceeds by far the available energy of thermal fluctuations, i.e., $E_c \gg k_B T$. In practice, $C \approx 1$ fF, which requires sub-Kelvin temperatures.

When these conditions are satisfied, the island charge corresponds to an integer number of extra electrons and the current results from the sequential tunneling of electrons one by one through both junctions. Each tunnel event occurs at a rate which depends on the change in electrostatic energy that it induces. Since the electrostatic energy of the island depends on the gate voltage, the current through the device is modulated by the gate voltage.

3.2. A highly sensitive electrometer

Current modulation curves of a typical SET with metallic tunnel junctions are shown in fig. 2. A maximum modulation depth of the order of $e/(R_t C)$ is obtained for a bias voltage $V \approx e/C$. Any variation of the electric field near the island will induce in the island a polarization charge that will act in the very same way as a similar change of the gate charge $q = C_g V_g$. Since a SET is a sub-electron sensitivity electrometer for the induced polarization charge on the island, it is worth noticing that SETs can already provide an important improvement in the measurement of the residual electric field in a metallic drift tube as compared to Kelvin probes [7].

4. SET as a position sensor for charged particles

It is now clear that a SET can detect, at least in principle, the passage of a charged particle in the vicinity of its island. The operating principle of such a detector is

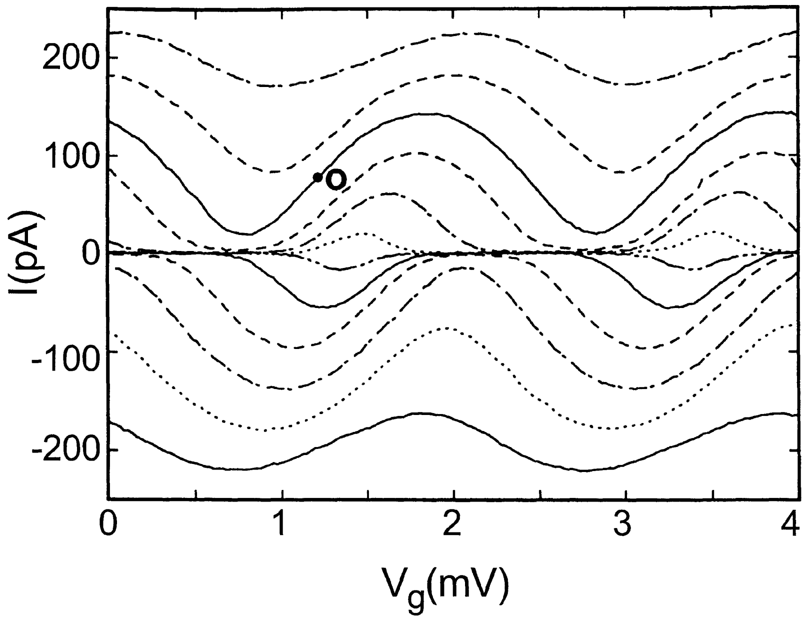

Fig. 2. Current modulation curves of a SET versus the gate voltage at a temperature of 20 mK. The different curves correspond to a set of values of the bias voltage V separated by 25 μV. The island capacitance is $C = 0.6$ fF. The maximum charge sensitivity is obtained at the working point O.

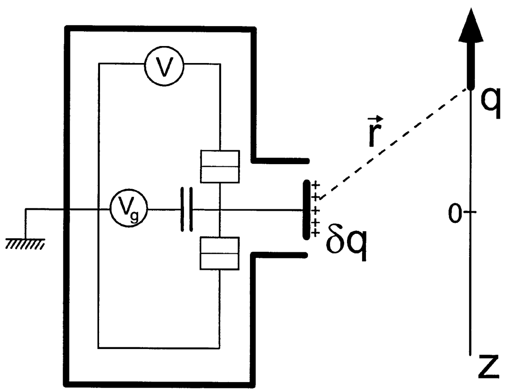

Fig. 3. Schematic diagram of a charged particle position sensor based on a SET. The SET measures the polarization charge δq induced by the charge q moving along the z axis.

sketched in fig. 3. The island of the SET actually behaves like an antenna which probes locally the electric field in the drift tube. Its self-capacitance contributes to the total island capacitance C. The grounded electrode surrounding the SET shields the electric field due to the voltage sources. One should however take care to keep the interaction energy between the particle and this ground electrode sufficiently small to prevent trapping of the particle when passing in front of the detector.

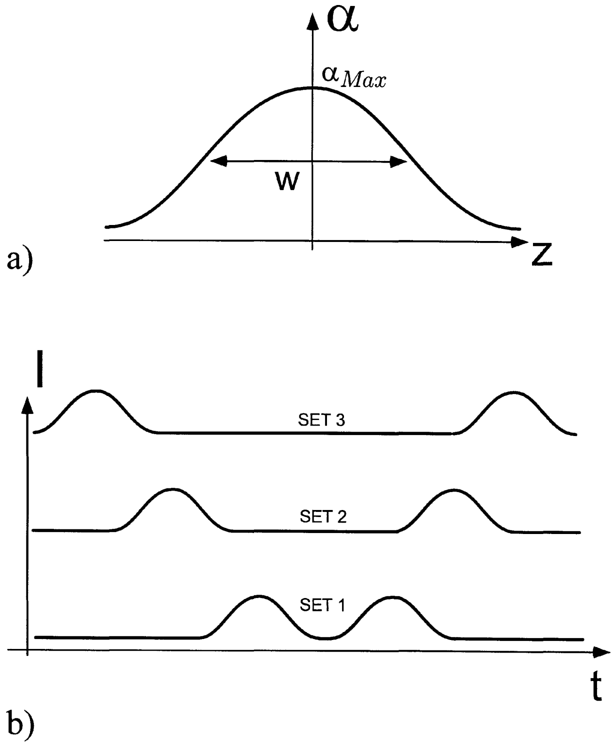

Fig. 4. (a) Expected dependence of the electrostatic coupling constant α with the particle position z. The width w is of the order of the minimal distance between the particle and the island. (b) Expected time dependence of the current in three SETs distributed along a drift tube containing a trapped antiproton moving back and forth between two reflectors. The effect of gravity is deduced from the analysis of the passage times.

The SET measures the polarization charge δq induced in the island by the charge q moving along the z axis of the drift tube. The Gauss reciprocity identity shows that $\delta q = -qV(\vec{r})/V_0$, where $V(\vec{r})$ is the potential at point $\vec{r}$ when the island is at potential V_0. The coupling constant $\alpha(\vec{r}) = -\delta q/q = V(\vec{r})/V_0$ is a geometrical factor which depends on the position of the particle and on the shape of the electrodes. Expected variations of α when a charged particle follows the z axis of a drift tube are shown in fig. 4(a). The characteristic width w of this interaction curve is of the order of the minimal distance between the SET island and the particle. It is worth noticing such a position sensor could also be used to monitor the position of an antiproton in a trap.

The reciprocity theorem also implies that the measuring SET will have a back-action on the particle. As seen from the particle, the fluctuating electric field due to the variations of the number $n(t)$ of extra electrons on the island is given by:

$$\vec{\mathcal{E}}(\vec{r}, t) = n(t)\,\big(e/C\big)\,\vec{\nabla}\alpha(\vec{r}). \tag{1}$$

The observation of the moving particle will thus induce a modification of the kinetic energy of the particle.

4.1. Detection sensitivity of the SET electrometer

Two kinds of noise limit the accuracy of the SET as a charge detector.

4.1.1. Shot noise

The first source of noise is intrinsic to the device: the successive transfers of single electrons are uncorrelated and constitute a Poisson process. At the optimal working point of the SET, the characteristic time τ of this Poisson process is $\tau \approx R_t C$. The current noise is equivalent to a white noise in the charge to be measured with a spectral density $q_N \approx \sqrt{2\tau}e/\sqrt{\text{Hz}}$. This noise figure is $q_N \approx 10^{-5}e/\sqrt{\text{Hz}}$ for an optimized SET with a bias current of about 1 nA.

4.1.2. Background charge noise

The second source of noise is extrinsic: since the SET measures the polarization charge induced on the island, any displacement of charges in the vicinity of the island results in a parasitic signal. It has been observed that this noise originates from a collection of charges randomly jumping between two positions. Each of them acts as a two-state fluctuator producing a telegraphic noise with a given characteristic switching time. The superposition of all these fluctuators results in a $1/f$ noise [21]. The amplitude of this $1/f$ charge noise currently observed in SETs regardless of the composition of the substrate is $q'_N \approx 3 \times 10^{-4}e/\sqrt{\text{Hz}}$ at 10 Hz. It dominates the intrinsic shot noise up to a crossover frequency of the order of 10 kHz and results in a long time drift of the SET working point. A low-frequency feed-back on the gate voltage is necessary to maintain the working point at the optimal gain and to prevent the $1/f$ noise from acting on the moving charged particles. The determination of the passage time of the particles is then not affected by the $1/f$ noise provided that the measuring time is less than 10^{-4} s. In this case, the passage time of a particle with velocity v_0 and charge e can be determined with an accuracy w/v_0 provided that the following condition is satisfied:

$$\alpha_{\text{Max}}\sqrt{\frac{w}{v_0}} > \frac{q_N}{e}. \tag{2}$$

4.1.3. Operation of a SET at high frequencies

The measurement bandwidth of SETs is usually limited by the cut-off frequency of the filtering circuitry connecting the device to the room temperature amplifiers. Coupling the SET to a cryogenic amplifier that provides impedance matching is necessary to obtain a large bandwidth. One realization [22] has been achieved by bonding a SET to an InP High Electron Mobility Transistor (HEMT) thus leading to a cut-off frequency of 700 kHz at the expense of an increased noise of $3\times10^{-4}e/\sqrt{\text{Hz}}$. Direct fabrication of a SET directly on top of a HEMT is also promising [23].

4.2. Back-action of the measurement on the particle

The electric field produced by the island charge at the particle position modifies its energy. The change δE of the particle kinetic energy E_0 after one measurement is $\delta E = e \int \mathcal{E}_z(z = v_0 t, t) v_0 \, \mathrm{d}t$. Using the expression (1) of the electric field, we obtain:

$$\delta E = 2E_\mathrm{c} \int n(t) \, \frac{\partial \alpha}{\partial z} \, (z = v_0 t) v_0 \, \mathrm{d}t. \tag{3}$$

The systematic part of δE cancels out because the temporal average of $n(t)$ only depends on the position of the particle through the coupling coefficient $\alpha(z)$:

$$\overline{\delta E} = 2E_\mathrm{c} \int_{\text{trajectory}} \bar{n}(\alpha) \, \frac{\partial \alpha}{\partial z} \, \mathrm{d}z = 0.$$

The assumption that the velocity of the particle remains almost constant during the measurement requires, however, that:

$$2E_\mathrm{c} \int_0^{\alpha_\mathrm{Max}} \bar{n}(\alpha) \, \mathrm{d}\alpha \approx E_\mathrm{c} \alpha_\mathrm{Max}^2 \ll E_0. \tag{4}$$

The standard deviation δE^* of the fluctuating part of δE is readily calculated assuming that $\tau \ll w / v_0$:

$$\delta E^* \approx E_\mathrm{c} \sqrt{v_0 \tau \int_{\text{trajectory}} \left(\frac{\partial \alpha}{\partial z} \right)^2 \mathrm{d}z} \approx E_\mathrm{c} \alpha_\mathrm{Max} \sqrt{\frac{v_0 \tau}{w}}. \tag{5}$$

Ensuring that the effect of gravity on the particle trajectory is not washed out by the back-action thus implies the following constraint on the design parameters:

$$E_\mathrm{c} \alpha_\mathrm{Max} \sqrt{\frac{v_0 \tau}{w}} \ll m_\mathrm{p} g L. \tag{6}$$

This constraint limits the maximum coupling α_Max and thus prevents from benefiting of the full sensitivity of the SET. This constraint is not, however, a fundamental limitation imposed by quantum mechanics. In particular, the randomness in the charge-discharge cycles of the island could be avoided if the island state corresponded to a coherent quantum superposition of two charge states. SETs based on 2D electron gases could be, in principle, operated in this regime, but the issue of back-action noise has not yet been investigated. On the other hand, the large magnetic field in the drift tube forbids the use of superconducting single Cooper pair electrometers which might also present a smaller back-action noise.

5. Design of a SET-based $\bar{\mathrm{p}}$ gravitational mass measurement

5.1. Description

The proposed geometry for the gravitational mass measurement of a charged particle is schematically represented on fig. 5. The particle (proton, antiproton, H^- or ion) is

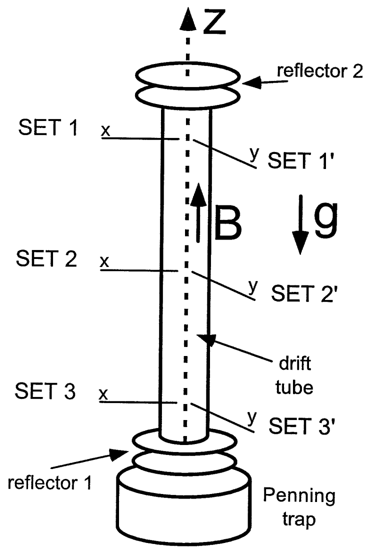

Fig. 5. Sketch of the antiproton gravitational mass measurement based on SETs. Antiprotons in a Penning trap are injected in a drift tube and move back and forth between two reflectors. X and Y position sensors based on SETs are distributed along the tube in order to monitor the trajectory.

released from a Penning trap and confined along the vertical axis of a drift tube by a magnetic field of typically 1 T. For the velocities considered here, typically 100 m/s, the cyclotron radius $R = m_{\mathrm{p}}v_0/(eB)$ of the trajectory is of the order of 1 μm so that the motion can be considered to be one-dimensional.

After the particle has been released from the Penning trap in the measurement region, the potential at the electrode reflectors is increased so that the particle bounces back and forth and is confined in the central region where the potential is kept as constant as possible.

Since the measurement is non-destructive, several SETs can be placed along the drift tube to monitor the trajectory (see fig. 5). In the configuration proposed, three measurement positions are used. Two SETs have been placed at each measurement position in order to estimate both the x and y transverse coordinates of the particle. Since obviously the gravitational mass measurement requires a very small residual electric field in the measurement region, the trajectory of the particle will slowly drift over a timescale of seconds. The measurement of this drift will provide a further estimate of stray electric fields together with the gravitational force. A differential measurement comparing the trajectories of antiprotons and H^- ions, with the same electric charge, could provide a determination of the difference of gravitational mass of these particles. In the case of H^- ions, the Stark effect in the residual electric field also contributes to modify the trajectory. However, this effect, compared to the effect of gravity, is negligible for the residual field amplitudes that have to be obtained in the experiment.

5.2. Determination of the gravitational mass

The potential energy $U(z)$ of the particle in the tube is the sum of an electrostatic and of a gravitational contribution. Assuming that $U(z)$ is small compared to the total energy of the particles $E = \frac{1}{2}mv_0^2$, where m is the inertial mass, the velocity v is:

$$v = \sqrt{\frac{2}{m}\big(E - U(z)\big)} \approx v_0\left(1 - \frac{U(z)}{mv_0^2}\right),$$

which gives for the passage time $t(z)$ at position z:

$$t(z) \approx \frac{z}{v_0} + \frac{\int_0^z U(z')\,\mathrm{d}z'}{mv_0^3}.$$

From the comparison of the passage times of antiprotons and of H^- particles at three heights z_i along the tube, it is possible to get rid of the common electrostatic contribution through the differences $\int_{z_\mathrm{i}}^{z_\mathrm{f}} U_{\bar{\mathrm{p}}}(z')\,\mathrm{d}z' - \int_{z_\mathrm{i}}^{z_\mathrm{f}} U_{\mathrm{H}^-}(z')\,\mathrm{d}z' = (m_{\bar{\mathrm{p}}} - m_{\mathrm{H}^-})g(z_\mathrm{f} - z_\mathrm{i})$, where $m_{\bar{\mathrm{p}}}$ and m_{H^-} are the gravitational masses to be compared. Repeated measurements would increase the accuracy by averaging the back-action of the measuring SETs on the particles.

5.3. Operating parameters

From the previous discussion, we propose here a realistic set of parameters that make it possible to determine the gravitational mass of the antiproton:

- The PS196 experiment [16,17] has shown that antiprotons can be slowed down to velocities $v_0 \approx 100$ m/s (i.e., with an energy $E_0 \approx 50$ μeV).
- We assume that a $L \approx 1$ m long drift tube with sufficiently low residual electric field can be fabricated. A drift tube with a similar length is presently used by the PS200 experiment [19]. The corresponding gravitational energy change for a proton over this distance is $\delta E_\mathrm{G} = m_\mathrm{p} gL \approx 0.1$ μeV.
- The charging energy of each SET is chosen to be $E_\mathrm{c} \approx 20$ μeV which corresponds to a total island capacitance $C \approx 5$ fF. This requires an operating temperature of 50 mK. The tunnel resistance is chosen such as $\tau \approx R_\mathrm{t}C \approx 10^{-10}$ s.
- The chosen SET coupling parameters are $\alpha_\mathrm{Max} \approx 0.05$ and $w \approx 500$ μm. These values appear to be consistent with the chosen island capacitance C. With this set of parameters, the characteristic energy change δE^* due to back-action given by (3) is of the order of 0.005 μeV, which corresponds to 5% of δE_G.
- Assuming that the SET sensitivity is only limited by the shot-noise $q_\mathrm{N} \approx 10^{-5}e/\sqrt{\mathrm{Hz}}$ and that the bandwidth is 1 MHz, the signal-to-noise ratio obtained from eq. (2) is $e\alpha_\mathrm{Max}/(q_\mathrm{N}\sqrt{w/v_0}) \approx 10$. The expected time resolution is therefore better than the interaction time $w/v_0 \approx 5$ μs.

6. Conclusions and perspectives

From the previous discussion, we conclude that SET based technology offers an alternative for charge sensing that could be used to monitor the trajectory of a charged antiproton in a trap or in a drift tube. The measurement of the gravitational mass of a single proton or antiproton appears to be feasible by placing state-of-the-art SETs in a 1 m long drift tube provided that residual electric fields are kept low enough.

Acknowledgements

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