On the Relation between Electrical Noise Spectra and AC Conductivity in Disordered Systems

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Abstract. We show that the low frequency, f, spectra of current fluctuations, S(f), and complex AC conductivity $\sigma(f)=\sigma'(f)+i\sigma''(f)$, are linked by the relationship following from the fluctuation-dissipation, FD, theorem, $\sigma'(f)/S(f)\propto f^2$. We measured σ(f) and S(f) in impurity conduction in lightly doped semiconductors, where at sufficiently low temperatures, σ'(f) and σ''(f), follow a power function of f. At higher temperatures, in a mixed, hopping and extended state transport regime, noise becomes very strong and $S(f) \propto 1/f^2$, with a flat σ(f), implied by the FD theorem.

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We address in this paper the relationship between the low frequency noise and complex conductivity, $\sigma(\omega)=\sigma'(\omega)+i\sigma''(\omega)$, where $\omega=2\pi f$, and f is frequency. First, we show that the power spectral density, PSD, of current fluctuations in a conducting medium, denoted S(ω), and σ'(ω) follow $\sigma'(\omega)/S(\omega)\propto\omega^2$ relationship. In disordered systems, where transition rates, w, are exponential functions of a random variable x,

$$w = w_0 \exp(-x)\,, \qquad (1)$$

where x can be a distance, an excitation energy etc., the cited $S(\omega)\leftrightarrow\sigma(\omega)$ relationship can be obtained by summing up the contributions from individual transitions. Finally, we discuss some results of measurements on S(ω) and σ(ω) in Ge and Si in the impurity conduction regime. It should be pointed out that σ'(ω) and σ''(ω) are linked by the Kramers-Kroning relations, and σ'(ω) to S(ω), thus the information obtained by a measurement of either of the three, in large enough span of f, is essentially the same.

AC CONDUTION AND NOISE SPECTRUM

General Case

As shown by Callen and Welton[1)] and Pytte and Imry[2)], the fluctuation-dissipation, FD, theorem links the PSD of current fluctuations, S(ω), and the electrical susceptibility function, χ(ω), by the relation,

$$S(\omega) = \hbar \coth(\frac{\hbar\omega}{2kT}) \operatorname{Im} \chi(\omega) \ , \qquad (2)$$

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where $\chi(\omega)=\chi'(\omega)+i\,\chi''(\omega)$. This expression applies to any sufficiently weak, periodic signal for which $\hbar\omega << kT$. From the Taylor series expansion of coth(x) we get,

$$S(\omega)=\frac{kT}{\omega}\chi''. \tag{3}$$

From the linear response theory we obtain a relation between $\chi(\omega)$ and $\sigma(\omega)$,

$$\chi(\omega)=\frac{\sigma(\omega)}{i\omega\varepsilon_0}. \tag{4}$$

Separating the real and imaginary parts in Eq. (4) we obtain,

$$\omega\varepsilon_0\chi''(\omega)=-\sigma'(\omega). \tag{5}$$

Using χ'' from Eq. (3) we readily obtain the relation between $S(\omega)$ and and $\sigma'(\omega)$,

$$S(\omega)=\frac{kT}{\omega^2\varepsilon_0}\sigma'(\omega). \tag{6}$$

The PSD of current fluctuations, $S(\omega)$, can be obtained from $\sigma'(\omega)/\omega^2$. Actually, theory accounts only for the $\sigma'(\omega)-\sigma'(0)$ function, which we use discussing our data.

Impurity Hopping Conduction

For a uniform distribution of sites in a disordered systems, the rate distribution function takes a particularly simple form, N(w)=const/w. Pollak and Geballe[3], PG, have shown that a direct summation of Miller-Abrahams[4] transition rates between charged and empty impurity states (pairs) leads to an expression,

$$\mathrm{Re}(\sigma(\omega)) \propto \int N(w)w(\omega/w)^2\frac{1}{1+(\omega/w)^2}dw. \tag{7}$$

Similarly the calculation of the autocorrelation function gives,

$$S(\omega)\propto\int N(w)\frac{1}{1+(\omega/w)^2}dw. \tag{8}$$

With N(w)=const/w, the integration in Eq. (7) and (8) is analytic, and

$$\begin{aligned}&\mathrm{Re}(\sigma(\omega))\propto\omega,\\ &S(\omega)\propto 1/\omega.\end{aligned} \tag{9}$$

The ratio $S(\omega)/\mathrm{Re}(\sigma)\propto 1/\omega^2$, in agreement with Eq. (6). It should be noted that Eq.(8) is similar to the well-known expression of McWhorter[5], for noise generation. It follows generally from a summation of random telegraph noise and, in MOSFETs, from processes of trapping/release of charges on localized states in the gate dielectric. In the pair approximation of PG, the tunnel transitions occur in impurity pairs. For completeness, one should mention here a model of 1/f noise in hopping conduction proposed by Shklovskii[6] and an experimental work on 1/f noise in hopping conduction in Ge, by Shlimak et al.[7]. A publication by Burin et al[8], on noise in variable range hopping in Si at very low temperatures, contains numerous references to a more recent work.

EXPERIMENT ON AC CONDUTION AND NOISE IN Ge AND Si BULK SAMPLES AT LOW TEMPERATURES

Disk shaped Ge:P and Si:P bulk samples with 6mm diameter dimensions, about 0.5mm thick, i.e. having geometry adapted for capacitance measurements, were used for both AC conduction and LFN measurements at low T (in a continuous flow LqHe cryostat). In both materials the impurity concentration was selected below the metal non-metal transition, but close, in order to reduce the samples impedance. At $kT \ll E_i$, the latter being impurity ionization energy, the carriers are localized and transport proceeds via phonon-assisted tunneling from neutral to ionized impurities[4)]. In Fig.1 the real and imaginary parts of σ, measured in a Si:P sample at various temperatures, are shown. One observes that $\sigma'(\omega)$ and $\sigma''(\omega)$ are conjugated, by Kramers-Kronig relations, as it turns out. At the lowest T (4.2K) both parts show a ω^s dependence, with $s \approx 0.8$. That region was explored in depth by Pollak and Geballe[3)].

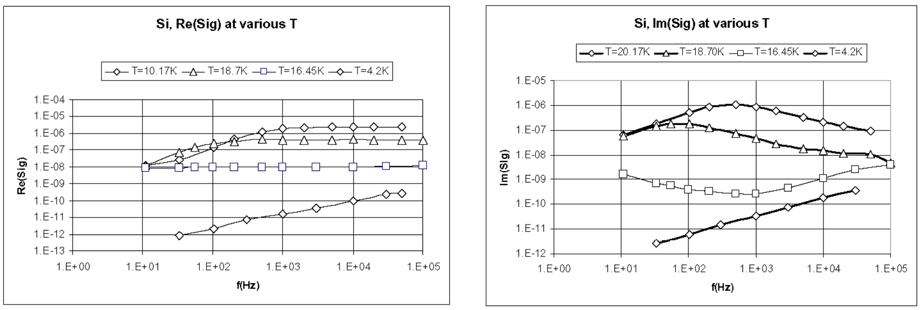

FIGURE 1. Si:P. AC conductivity $\sigma'(\omega)$ and $\sigma'(\omega)$. The straight lines at the bottom correspond to the NNH regime. Peaked features in σ''(inflection in σ') at higher T is du to the P impurity ionization.

Our initial objective was to measure LFN on the same Si sample and compare the slopes of $\sigma(\omega)$ and $S(\omega)$. We expected to observe $S(\omega) \propto \omega^{-(2-0/8)}$, implied by Eq. (6). Unfortunately, the resistance of lightly doped Si in that temperature range turned out to be prohibitively high for meaningful LFN measurements. At higher T, $\sigma''(\omega)$ exhibits a peaked feature and $\sigma'(\omega)$ a corresponding inflection point. The peak in $\sigma''(\omega)$ moves towards higher f as T is increased, following $\exp(-E_i/kT)$ law, with E_i=47meV, which is close to the E_i for P in Si (equal to 44meV). The observed features in $\sigma(\omega)$ can be attributed to the presence of pockets filled with free carries. Measurements of noise at higher T region revealed a dramatic increase in S, by several orders of magnitude, with $1/f^2$ (black noise) f dependence, consistent with that model. Extremely strong noise was observed in Ge:P at 6K which is shown in Fig. 2B.

Figure 2A shows the noise data taken at 4.2K on a Ge:P sample in the hopping regime. Figure 3A shows $\sigma'(\omega)$, $\sigma''(\omega)$, and $S(\omega)$, the latter comprising the data shown in Fig. 2A. At T=4.2K conduction in lightly doped Ge is known to be dominated by hopping. In most cases $\sigma''(\omega) \propto \omega^s$ dependence in Ge holds down to very low f, while $\sigma'(\omega)$ and $\sigma'(\omega)-\sigma(0)$ flatten down as f is decreased [8)]. That is demonstrated in Fig.3A.

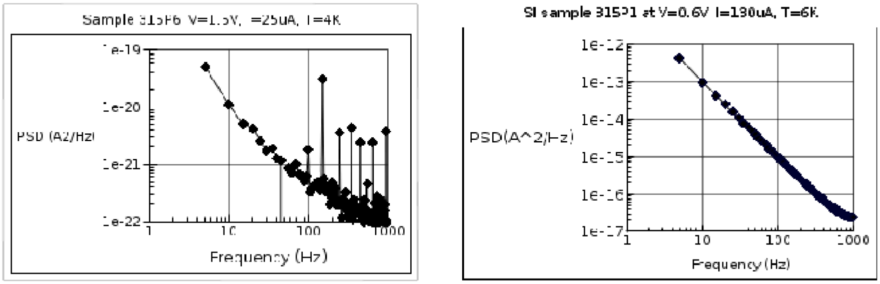

FIGURE 2 A and B. **(A)** S(ω) Ge:P. at 4.2K and **(B)** at 6K. The data were corrected for σ(0) and the system noise, respectively. At 6K the noise is seen to increase by 7 orders of magnitude and $S(f) \propto 1/f^2$.

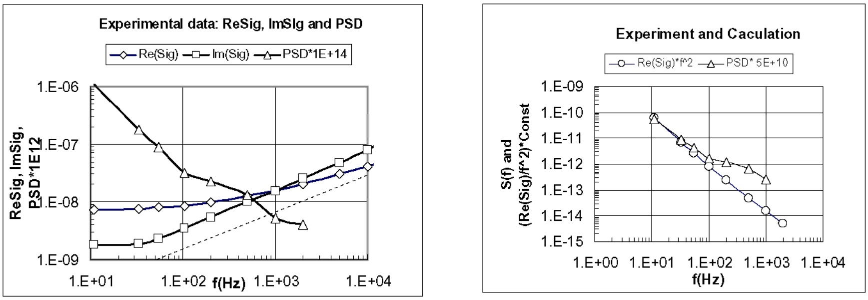

FIGURE 3 A and B. **(A)** Ge:P at 4.2K**:** σ', σ', and S as a function of f, S(f) is shifted to fit the plot. Broken line shows ω^s, with s=0.6, followed by σ''(f).. **(B)** Comparison of $(\sigma'-\sigma_{DC})/f^2$ with S(f), the latter shifted by a factor 10^{+10} so as the data and the calculation results coincide at 10Hz.

The concave shapes of σ'(f) (diamonds) and S(f) (triangles), shown in Fig. 3A, are compatible with predictions of Eq. (6), i.e. the flattening at low f in σ' corresponds to a steeper slope in S and vice versa. However, the values of S(ω) calculated using σ'(ω)-σ(0) and Eq. (6), do not fully account for the data, as demonstrated in Fig. 3B.

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