

Bethe-Briegls correction

$$\begin{aligned} \textcircled{1} \quad P(\sigma_0, \sigma_1, \dots, \sigma_N) &\stackrel{\approx}{=} \exp \left[\beta J \sum_{\langle ij \rangle} \sigma_i \sigma_j \right] \\ &\stackrel{\approx}{=} \exp \left[\beta J \sigma_0 \sum_{i=1}^N \sigma_i \right] \exp \left[\beta J \sum_{i,j \geq 1} \sigma_i \sigma_j \right] \\ &\stackrel{\approx}{=} \exp \left[\beta J \sigma_0 \sum_{i=1}^N \sigma_i \right] P_0(\sigma_1, \dots, \sigma_N) \end{aligned}$$

$$\textcircled{2} \quad P_0(\sigma_1, \dots, \sigma_N) \stackrel{\approx}{=} \exp \left[\beta h \sum_{i=1}^N \sigma_i \right]$$

champ autocohérent
↓

$$\begin{aligned} P(\sigma_0) &= \sum_{\sigma_2 = \pm 1} - \sum_{\sigma_p = \pm 2} P(\sigma_0, \dots, \sigma_N) \\ &\stackrel{\approx}{=} \sum_{i=1}^N \sum_{\sigma_i = \pm 1} e^{\beta J \sigma_0 \sum_{i=1}^N \sigma_i} \underbrace{e^{\beta h \sum_{i=1}^N \sigma_i}}_{P_0} \end{aligned}$$

ATTENTION:
typo dans
l'énoncé sur
la définition
de $P(\sigma_0)$

$$\begin{aligned} &\stackrel{\approx}{=} \left[\sum_{\sigma_2 = \pm 1} - \sum_{\sigma_p = \pm 2} \exp \left(\beta (h + J \sigma_0) \sum_{i=1}^p \sigma_i \right) \right] \\ &\quad \times \left[\sum_{\sigma_{p+1} = \pm 1} - \sum_{\sigma_N = \pm 1} e^{\beta h \sum_{i=p+1}^N \sigma_i} \right] \end{aligned}$$

part dans la constante
de normalisation
(independant de σ_0)

$$\stackrel{\approx}{=} \left[2 \cosh \beta (h + J \sigma_0) \right]^p$$

$$\textcircled{3} \quad \text{En l'absence de } (\sigma_2), P_2(\sigma_0) \stackrel{\approx}{=} e^{\beta h \sigma_0}$$

Mais en utilisant le résultat de la question

précédente $P_2(\sigma_0) \approx [2 \operatorname{ch} \beta(h + J\sigma_0)]^{p-1}$

$$e^{\beta h \sigma_0} = [2 \operatorname{ch} \beta(h + J\sigma_0)]^{p-1}$$

Maintenant: $\operatorname{ch} \beta(h + J\sigma_0) = (\operatorname{ch} \beta h)/(\operatorname{ch} \beta J) + (\operatorname{sh} \beta h)(\operatorname{sh} \beta J \sigma_0)$
 $= (\operatorname{ch} \beta h)(\operatorname{ch} \beta J) \left[1 + o_0((h\beta h)(h\beta J)) \right]$

$(h\beta h)(h\beta J) \in [-1, 1] \Rightarrow \exists! u = u(\beta, h, J)$ tel que

$$(h\beta h)(h\beta J) = h(\beta u)$$

$$\Rightarrow \operatorname{ch} \beta(h + J\sigma_0) = \text{cste} \left[1 + o_0(h\beta u) \right] \approx e^{\beta u \sigma_0}$$

indep de σ_0

Donc $e^{\beta h \sigma_0} \approx e^{\beta(p-1)u \sigma_0} \Rightarrow \boxed{h = (p-1)u}$

$$\Rightarrow h \left(\frac{\beta h}{p-1} \right) = (h\beta h)(h\beta J)$$

Nouvelle équation d'autocorrérence sur h qui donne
la température critique

$$\left. \frac{\partial}{\partial h} h \left(\frac{\beta h}{p-1} \right) \right|_{h=0} = \left. \left(\frac{\partial}{\partial h} h\beta h \right) \right|_{h=0} h\beta J$$

$$\beta_c = (p-1) \beta_c h(\beta J) \Rightarrow \beta_c J = \alpha h \left(\frac{1}{p-1} \right)$$

Note

A une dimension, $p=2 \Rightarrow \beta_c J = +\infty \Rightarrow T_c = 0$
on retrouve bien le résultat exact.